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Research Article

Properties of Intuitionistic β-Open Mappings

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Abstract. The concept of intuitionistic fuzzy set and intuitionistic fuzzy topological space were defined by Atanssov. Later Coker introduced the concept of intuitionistic set and intuitionistic points. He also introduced the concept of intuitionistic topological space and investigated basic properties of continuous functions and compactness. In a recent paper, the concept of intuitionistic β -open, intuitionistic β-closure and intuitionistic β-interior in intuitionistic topological space were defined by Singaravelan. Also some basic properties of intuitionistic β-open set were discussed. The purpose of this paper is to introduce and study the concept of intuitionistic β-open mappings and study its properties.

Keywords. $I\beta$-open sets; $I\beta$-closed sets; $I\beta$-closure; $I\beta$-interior; $I\beta$-continuous; $I\beta$-open mapping; $I\beta$-closed mapping

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1. Introduction

In 1986, Atanassov [4] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Later in 1996, Coker [9] introduced the concept of intuitionistic set and intuitionistic points. This is a discrete form of intuitionistic fuzzy sets where all the sets are crisp set. In 2000, Coker [11] also introduced the concept of "intuitionistic topological space" and investigated basic properties of continuous functions and compactness. In general topological space (Levine [16]) introduced semi open sets and semi continuity and Abd El. Monsef *et al*. [1] introduced "β-open sets and β-continuous mapping" and discussed some of their basic properties. Andrijevic [3] introduced and discussed some more properties of semi pre open set in topological space. Csaszar [5, 6] introduced and discussed generlazied open set, γ-interior and γ-closure in topological space.

Recently Gnanambal Ilango and Selvanayaki [14], introduced and studied generalized pre regular closed sets in intuitionistic topological spaces. Singaravelan [21] introduced intuitionistic β-open sets in intuitionistic topological space.

In this paper, properties of intuitionistic β-open mappings and intuitonistic β-closed mappings are discussed.

2. Preliminaries

Let us recall some basic definitions and results which are useful for this sequel. Throughout the present study, a space X means an intuitionistic topological space.

Definition 2.1 ([9]). Let X is a non empty set. An intuitionistic set (IS for short) A is an object having the form $A = \langle X, A_1, A_2\rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

Definition 2.2 ([9]). Let X be a non empty set and let A, B are intuitionistic sets in the form $A = \langle X, A_1, A_2\rangle$, $B = \langle X, B_1, B_2\rangle$, respectively. Then

(a) $A \subseteq B$ iff $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \langle X, A_2, A_1\rangle$

(d) $[]A = \langle X, A_1, (A_1)^c\rangle$

(e) $A - B = A \cap B^C$.

(f) $\phi_\sim = \langle X, \phi, X\rangle$, $X_\sim = \langle X, X, \phi\rangle$,

(g) $A \cup B = \langle X, A_1 \cup B_1, A_2 \cap B_2\rangle$,

(h) $A \cap B = \langle X, A_1 \cap B_1, A_2 U B_2\rangle$.

Furthermore, let $\{A_\alpha : \alpha \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_\alpha = \langle X, A_\alpha^{(1)}, A_\alpha^{(2)}\rangle$. Then

(i) $\cap A_\alpha = \langle X, \cap A_\alpha^{(1)}, UA_\alpha^{(2)}\rangle$.

(j) $\cup A_\alpha = \langle X, \cup A_\alpha^{(1)}, \cap A_\alpha^{(2)}\rangle$.

Definition 2.3 ([11])**.** An intuitionistic topology (for short IT) on a non empty set X is a family of IS's in X satisfying the following axioms.

(i) $\phi_\sim, X_\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$.

(iii) $\cup G_\alpha \in \tau$ for any arbitrary family $\{G_i : G_\alpha/\alpha \in J\} \subseteq \tau$ where (X,τ) is called an intuitionistic topological space (for short ITS(X)) and any intuitionistic set in is called an intuitionistic open set (for short IOS) in X. The complement A^c of an IOS A is called an intuitionistic closed set (for short ICS) in X.

Definition 2.4 ([11])**.** Let (X,τ) be an intuitionistic topological space (for short ITS(X)) and $A = \langle X, A_1, A_2\rangle$ be an IS in X. Then the interior and closure of A are defined by

$$Icl(A) = \cap\{K : K \text{ is an ICS in } X \text{ and } A \subseteq K\},$$

$$Iint(A) = \cup\{G : G \text{ is an IOS in } X \text{ and } G \subseteq A\}.$$

It can be shown that $Icl(A)$ is an ICS and $Iint(A)$ is an IOS in X and A is an ICS in X iff $Icl(A) = A$ and is an IOS in X iff $Iint(A) = A$.

Definition 2.5 ([9])**.** Let X be a non empty set and $p \in X$. Then the ISP defined by $p = \langle X, \{p\}, \{p\}^c\rangle$ is called an intuitionistic point (IP for short) in X. The intuitionistic point p is said to be contained in $A = \langle X, A_1, A_2\rangle$ (i.e., $p \in A$) if and only if $p \in A_1$.

Definition 2.6 ([14])**.** Let (X,τ) be an $ITS(X)$. An intuitionistic set A of X is said to be

(i) Intuitionistic semiopen if $A \subseteq Icl(Iint(A))$.

(ii) Intuitionistic preopen if $A \subseteq Iint(Icl(A))$.

(iii) Intuitionistic regular open if $A = Iint(Icl(A))$.

(iv) Intuitionistic α-open if $A \subseteq Iint(Icl(Iint(A)))$.

The family of all intuitionistic pre open, intuitionistic regular open and intuitionistic α-open sets of (X,τ) are denoted by IPOS, IROS and $I\alpha OS$, respectively.

Definition 2.7 ([21])**.** A subset A of anintuitionistic topological space X is intuitionistic β-open, if there exists a intuitionistic preopen set U in X, such that $U \subseteq A \subseteq Icl(U)$. The family of all intuitionistic β-open sets in X will be denoted by $I\beta OS(X)$. The complement of intuitionistic $I\beta$-open set is $I\beta$-closed set.

Definition 2.8 ([9, 11])**.** Let A, A_i $(i \in J)$ be IS's in X, B, B_j $(j \in K)$ IS's in Y and $f : X \to Y$ a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

(c) $A \subseteq f^{-1}(f(A))$ and if f is 1-1,then $A = f^{-1}(f(A))$.

(d) $f(f^{-1}(B))$ and if f is onto, then $f(f^{-1}(B)) = B$.

(e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$.

(f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$.

(g) $f(\cup A_i) = \cup f(A_i)$.

(h) $f(\cap A_i) \subseteq \cap f(A_i)$ and if f is 1-1, then $f(\cap A_i) = \cap f(A_i)$.

(i) $f^{-1}(Y_\sim) = X$.

(j) $f^{-1}(\phi_\sim) = \phi_\sim$.

(k) $f(X_\sim) = Y_\sim$. If f is onto.

(l) $f(\phi_\sim) = \phi_\sim$.

(m) If f is onto, then $\overline{f(A)} \subseteq f(\bar{A})$: and if furthermore, f is 1-1, we have $\overline{f(A)} \subseteq f(\bar{A})$.

(n) $f^{-1}(\bar{B}) = \overline{f^{-1}(B)}$

(o) $B_1 \sqsubset B_2 \Rightarrow f^{-1}(B_1) \sqsubset f^{-1}(B_2)$.

Definition 2.9 ([11])**.** Let (X,τ) and (Y,Φ) be two ITS's and let $f : X \to Y$ be a function. Then f is said to be continuous iff the preimage of each intuitionistic open in Φ is an intuitionistic open in τ.

Definition 2.10 ([5])**.** Let (X,τ) and (Y,Φ) be two ITS's and let $f : X \to Y$ be a function. Then f is said to be open iff the preimage of each intuitionistic open in τ is an intuitionistic open in Φ.

Definition 2.11. Let (X,τ) and (Y,Φ) be two ITS's and let $f : X \to Y$ is called intuitionistic semi continuous if for every intuitionistic open V of Y, $f^{-1}(V)$ is semi open in X.

Definition 2.12. Let (X,τ) and (Y,Φ) be two ITS's and let $f : X \to Y$ is called intuitionistic regular continuous if for every intuitinistic open set V of Y, $f^{-1}(V)$ is regular open in X.

Definition 2.13. Let (X,τ) and (Y,Φ) be two ITS's and let $f : X \to Y$ is called intuitionistic pre continuous if for every intuitinistic open set V of Y, $f^{-1}(V)$ is pre open in X.

Definition 2.14. Let (X,τ) and (Y,Φ) be two ITS's and let $f : X \to Y$ is called intuitionistic α-continuous if for every intuitinistic open set V of Y, $f^{-1}(V)$ is α-open in X.

Definition 2.15 ([11])**.** Let (X,τ_1) and (Y,τ_2) be two ITS on X. Then τ_1 is said to be contained in τ_2 (in symbols, $\tau_1 \subseteq \tau_2$), if $G \in \tau_2$ for each $G \in \tau_1$. In this case, we also say that τ_1 is coarser than τ_2.

Definition 2.16 ([21])**.** Let (X,τ) be an intuitionistic topological space and let $A = \langle X, A_1, A_2\rangle$ be the subset of X. Then $I\beta\text{-}cl(A) = \cap\{F : F$ is intuitionistic β-closed in X and $A \subseteq F\}$.

Definition 2.17 ([21])**.** Let (X,τ) be an intuitionistic topological space and let $A = \langle X, A_1, A_2\rangle$ be the subset of X. Then $I\beta\text{-}int(A) = U\{F : F$ is intuitionistic β-open in X and $F \subseteq A\}$.

Proposition 2.18 ([21]). *A subset $A = \langle X, A_1, A_2\rangle$ of an ITS(X) is intuitionistic β-open set iff $A \subseteq Icl(Iint(Icl(A)))$.*

Lemma 2.19 ([21]). *Let A and B be subsets of $ITS(X)$, then the following results are obvious.*

(i) $I\beta\text{-}cl(X) = X$ *and* $I\beta\text{-}cl(\varnothing_\sim) = \varnothing_\sim$.

(ii) *If* $A \subseteq B$, *then* $I\beta\text{-}cl(A) \subseteq I\beta\text{-}cl(B)$

(iii) $I\beta\text{-}cl(I\beta\text{-}cl(A)) = I\beta\text{-}cl(A)$.

3. Properties of $I\beta$-Open and $I\beta$-Closed Mappings

Definition 3.1. A mapping $f : X \to Y$ is said to be $I\beta$-open, if the image of each open set in X is $I\beta$-open in Y.

Definition 3.2. A mapping $f : X \to Y$ is said to be $I\beta$-closed, if the image of each closed set in X is $I\beta$-closed in Y.

Definition 3.3. A mapping $f : X \to Y$ is said to be IP-closed, if the image of each closed set in X is IP-closed in Y.

Definition 3.4. A mapping $f : X \to Y$ is said to be IS-closed, if the image of each closed set in X is IS-closed in Y.

Definition 3.5. A mapping $f : X \to Y$ is said to be Ir-closed, if the image of each closed set in X is Ir-closed in Y.

Definition 3.6. A mapping $f : X \to Y$ is said to be $I\alpha$-closed, if the image of each closed set in X is $I\alpha$-closed in Y.

Lemma 3.7. *Let $A = \langle X, A_1, A_2\rangle$ be a subset of intuitionistic topological space X, then the following conditions are equivalent.*

(i) $A \in I\beta O(X)$

(ii) $A \subseteq Icl(Iint(Icl(A)))$

(iii) $A \subseteq Isint(Iscl(A))$

Proof. Obvious. □

Theorem 3.8. *Let (X,τ) and (Y,σ) be intuitionistic topological spaces. Then the following statements are equivalent.*

(i) $f : (X,\tau) \to (Y,\sigma)$ *is a* $I\beta$-*closed function.*

(ii) $I\beta\text{-}cl(f(A)) \subseteq f(I\beta\text{-}cl(A))$ *for each* $I\beta$-*closed set* A *in* X.

Proof. (a)⇒(b): Let $A = \langle X, A_1, A_2\rangle$ be any $I\beta$-closed set in X, clearly $I\beta\text{-}cl(A)$ is an $I\beta$-closed in X. Since f is $I\beta$-closed function, $f(I\beta\text{-}cl(A)) \subseteq I\beta\text{-}cl(f(I\beta\text{-}cl(A))) = f(I\beta\text{-}cl(A))$.

$\Rightarrow f(I\beta\text{-}cl(A)) \subseteq f(I\beta\text{-}cl(A))$.

(b)⇒(c): Let A be any $I\beta$-closed set in X, then $I\beta\text{-}cl(A) = A$, by (b)

$\Rightarrow I\beta\text{-}cl(f(A)) \subseteq f(I\beta\text{-}cl(A)) = f(A) \subseteq I\beta cl(f(A))$. Thus $f(A) = I\beta\text{-}cl(f(A))$ and hence $f(A)$ is an $I\beta$-closed set in Y. Therefore f is an $I\beta$-closed function. □

Theorem 3.9. *Let $f : (X,\tau) \to (Y,\sigma)$ be a intuitionistic continuous and intuionistic open ,then for each $I\beta$-open set A of X, $f(A)$ is $I\beta$-open subset of Y.*

Proof. Let $A = \langle X, A_1, A_2\rangle$ be any $I\beta$-open set. Then $A \subseteq Icl(Iint(Icl(A)))$,

$\Rightarrow f(A) \subseteq f(Icl(Iint(Icl(A)))) \subseteq Icl(Iint(Icl(f(A))))$

$\Rightarrow f(A) \subseteq Icl(Iint(Icl(f(A)))))$

Therefore $f(A)$ is $I\beta$-open subset of Y. □

Theorem 3.10. *Let $f : (X,\tau) \to (Y,\sigma)$ be intuitionistic topological space, then the followings are equivalent*

(i) $f : (X,\tau) \to (Y,\sigma)$ *is $I\beta$-open.*

(ii) $f(I\beta\text{-}int(A)) \subseteq I\beta\text{-}int(f(A))$ *for each intuitionistic set A in X.*

(iii) $I\beta\text{-}int(f^{-1}(B)) \subseteq f^{-1}(I\beta\text{-}int(B))$ *for each intutionistic set B in Y.*

Proof. (i)⇒(ii): Let f be an $I\beta$-open function. Since $f(I\beta\text{-}int(A))$ is an $I\beta$-open set contained in $f(A)$, $f(I\beta\text{-}int(A)) \subseteq I\beta\text{-}int(f(A)))$ by definition $I\beta$-interior.

(ii)⇒(iii): Let B be any $I\beta$-set in Y. Then $f^{-1}(B)$ is an $I\beta$-set in X, by (ii), $f(I\beta\text{-}int(f^{-1}(B))) \subseteq I\beta\text{-}int(f(f^{-1}(B)))) \subseteq I\beta\text{-}int(B)$,

$\Rightarrow I\beta\text{-}int(f^{-1}(B)) \subseteq f^{-1}(\beta\text{-}int(B))$.

(iii)⇒(i): Let A be any $I\beta$-open in X. Then $I\beta\text{-}int(A) = A$ and $f(A)$ is an $I\beta$-open in Y by (iii), $A = I\beta\text{-}int(A) \subseteq I\beta\text{-}int(f^{-1}(f(A))) \subseteq f^{-1}(I\beta\text{-}int(f(A)))$. Hence we have $f(A) \subseteq f(f^{-1}(I\beta\text{-}int(f(A))) \subseteq I\beta\text{-}int(f(A)) \subseteq f(A)$. Thus $f(A) = I\beta\text{-}int(f(A))$ and hence $f(A)$ is an $I\beta$-open set in Y. Therefore f is an $I\beta$-open function. □

Theorem 3.11. *Let $f : (X,\tau) \to (Y,\sigma)$ be a $I\beta$-continuous and $I\alpha$-open function then the inverse image of each intuitionistic open set in Y is $I\beta$-open in X.*

Proof. Let $A = \langle X, A_1, A_2\rangle$ is a $I\beta$-open, then $A \subseteq Icl(Iint(Icl(A)))$ and so

$$f^{-1}(A) \subseteq f^{-1}(Icl(Iint(Icl(A)))) \subseteq Icl(f^{-1}(Iin(Icl(A)))$$

as f is $I\alpha$-open and $Iint(Icl(A))$ is intuitionistic preopen. Since f is $I\beta$-continuous,

$$\Rightarrow \quad f^{-1}(A) \subseteq Icl(Iint(Icl(f^{-1}(Iin(Icl(A)))) \subseteq Icl(Iint(Icl(f^{-1}(Icl(Iin(Icl(A))))))$$

$$\Rightarrow \quad f^{-1}(A) \subseteq Icl(Iint(Icl(f^{-1}(A)))), \text{ because } f \text{ is } I\alpha\text{-open function.}$$ □

Theorem 3.12. *Let $f:(X,\tau)\to(Y,\sigma)$ be a $I\beta$-continuous and intuitionistic open function then the following statements are hold.*

(a) *The inverse image of each intuitionistic preopen in Y is $I\beta$-open in X.*

(b) *The inverse image of each intuitionistic semi open in Y is $I\beta$-open in X.*

Proof. (a): Let $A=\langle X,A_1,A_2\rangle$ is intuitionistic preopen in Y, $A\subseteq Iint(Icl(A))$. Then $f^{-1}(A)\subseteq f^{-1}(Iint(Icl(A)))\Rightarrow f^{-1}(A)\subseteq f^{-1}(Iint(Icl(A)))\subseteq Icl(Iint(Icl(f^{-1}(Iint(Icl(A)))),$ as $f^{-1}(Iint(Icl(A)))$ is $I\beta$-open being f is $I\beta$-continuous. That is $f^{-1}(A)\subseteq Icl(Iint(Icl(f^{-1}(Iint(Icl(A))))))$, $f^{-1}(A)\subseteq Icl(Iint(Icl(f^{-1}(Icl(A))))$, $f^{-1}(A)\subseteq Icl(Iint(Icl(f^{-1}(A))))$, as f is open function. Therefore inverse image of intuitionistic preopen in Y is $I\beta$-open in X.

(b): Let $B=\langle X,B_1,B_2\rangle$ is an intuitionistic semi open in Y, $B\subseteq Icl(Iint(B))$. Then

$$f^{-1}(B)\subseteq f^{-1}(Icl(Iint(B)))\subseteq Icl(f^{-1}(Iint(B))) \text{ (as } f \text{ is intuitionistic open mapping)}$$

$$f^{-1}(B)\subseteq Icl(Iint(Icl(f^{-1}(Iint(B))))) \text{ (as } f \text{ is } I\beta\text{-continuous)}$$

$$f^{-1}(B)\subseteq Icl(Iint(Icl(f^{-1}(B)))).$$

Therefore inverse image of intuitionistic preopen in Y is $I\beta$-open in X. □

Theorem 3.13. *A intuitionistic bijective function is $I\beta$-open iff it is $I\beta$-closed.*

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic bijective $I\beta$-open function and let F be any intuitionistic closed subset of X. Then X-F is intuitionistic open and hence $f(X-F)=X-f(F)$ is $I\beta$-open implies $f(F)$ is $I\beta$-closed. Therefore f is $I\beta$-closed function.

Conversely, let $f:(X,\tau)\to(Y,\sigma)$ is an intuitionistic bijective $I\beta$-closed and U be intuitionistic open and subset of X. Then $X-U$ is intuitionistic closed subset of X and hence $f(X-U)=X-f(U)$ is $I\beta$-closed implies $f(U)$ is $I\beta$-open. Therefore f is $I\beta$-open function. □

Theorem 3.14. *Let $f:(X,\tau)\to(Y,\sigma)$ be bijective $I\beta$-continuous and $g:(Y,\sigma)\to(Z,\Psi)$ be bijective continuous function then $g\circ f:(X,\tau)\to(Z,\Psi)$ is $I\beta$-continuous function.*

Proof. Let $A=\langle X,A_1,A_2\rangle$ be any intuitionistic open subset of Z, then $g^{-1}(A)$ be open in Y and as f is $I\beta$-continuous, $f^{-1}(g^{-1}(A))$ is $I\beta$-open in X, $(g\circ f)^{-1}(A)$ is $I\beta$-open in X implies $g\circ f$ is $I\beta$-continuous function. □

Theorem 3.15. *Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\Psi)$ be two mappings. If f is intuitionistic continuous and onto and $g\circ f:(X,\tau)\to(Z,\Psi)$ is $I\beta$-closed mappings, then g is intuitionistic $I\beta$-closed mapping.*

Proof. Let $A = \langle X, A_1, A_2\rangle$ be a intuitionistic closed set in Y. Then $f^{-1}(A)$ is intuitionistic closed in X. Since f is intuitionistic continuous, now $g \circ f$ is $I\beta$-closed and f is onto, $(g \circ f)^{-1}(f^{-1}(A)) = g(A)$ is intuitionistic β-closed in Z. Hence g is a intuitionistic β-closed mapping. □

Theorem 3.16. *A mapping $f : (X,\tau) \to (Y,\sigma)$ is $I\beta$-open if and only if $f(Iint(A)) \subseteq I\beta\text{-}int(f(A))$, for every intuitionistic set A of X.*

Proof. (Necessity): If f is $I\beta$-open mapping, then $f(Iint(A)) \in I\beta O(Y)$. Hence $f(Iint(A)) = I\beta\text{-}int(f(Iint(A)))) \subseteq I\beta\text{-}int(f(A)) \Rightarrow f(Iint(A)) \subseteq I\beta\text{-}int(f(A))$.

(Sufficiency): Let $A = \langle X, A_1, A_2\rangle$ be a intuitionistic open set of X. then by hypothesis, $f(A) = f(Iint(A)) \subseteq I\beta\text{-}int(f(A)) \Rightarrow f(A) \subseteq I\beta\text{-}int(f(A))$. Hence $f(A)$ is $I\beta$-open set in Y. □

Theorem 3.17. *A mapping $f : (X,\tau) \to (Y,\sigma)$ is $I\beta$-closed if and only if $I\beta\text{-}cl(f(A)) \subseteq f(Icl(A))$, for every intuitionistic set A of X.*

Proof. (Necessity): If f is $I\beta$-closed mapping, then $f(Icl(A))$ is $I\beta$-closed set containing $f(A)$ and therefore $I\beta\text{-}cl(f(A)) \subseteq f(Icl(A))$.

(Sufficiency): Let $A = \langle X, A_1, A_2\rangle$ be a intuitionistic closed set of X. Then by hypothesis, $I\beta\text{-}cl(f(A)) \subseteq f(Icl(A)) = f(A)$. By the definition of $I\beta$-closure, we have $f(A) \subseteq I\beta\text{-}cl(f(A))$ and so $f(A)$ is $I\beta$-closed in Y. Hence f is a $I\beta$-closed mapping.

Theorem 3.18. *A mapping $f : (X,\tau) \to (Y,\sigma)$ is intuitionistic open mapping (res. intuitionistic closed) and $g : (Y,\sigma) \to (Z,\Psi)$ is $I\beta$-open mapping (res. $I\beta$-closed) then $g \circ f$ is $I\beta$-open mapping (res. $I\beta$-closed).*

Proof. Obvious. □

Theorem 3.19. *Let $f : X \to Y$ be a $I\beta$-open mapping. If $A = \langle X, A_1, A_2\rangle$ is a intuitionistic set in Y and $B = \langle X, B_1, B_2\rangle$ is intuitionistic closed set in X containing $f^{-1}(A)$, then there exists a intuitionistic β-closed set C in Y such that $A \subseteq C$ and $f^{-1}(C) \subseteq B$.*

Proof. Let $C = Y - f(X - B)$. Since $f^{-1}(A) \subseteq B$, we have $f(X - B) \subseteq (Y - A)$. Since f is $I\beta$-open, then C is a $I\beta$-closed set of Y and $f^{-1}(C) = X - f^{-1}(f(X - B)) \subseteq X - (X - B) = B \Rightarrow f^{-1}(C) \subseteq B$. □

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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