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Research Article

Regular Interval-Valued Intuitionistic Fuzzy Graphs

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Abstract. In this paper, we introduce *Regular Interval-Valued Intuitionistic Fuzzy Graphs* (RIVIFG) and investigate some of their attributes. We talk about some conditions for regularity of an interval-valued intuitionistic fuzzy graph and obtain f-morphism on an interval-valued intuitionistic fuzzy graph and regular interval-valued intuitionistic fuzzy graph. $(2,k)$-regular and totally $(2,k)$-regular interval-valued intuitionistic fuzzy graphs are some elegant properties.

Keywords. Intuitionistic fuzzy graph(IFG); f-morphism; $(2,k)$-regular graph

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1. Introduction

The difference between probability and possibility were encountered by Zadeh [14] and established the concept of fuzzy sets. The researchers emphasized on this concept because it provides the method of finding uncertainty of any problem containing linguistic parameters. Applications in different areas namely, in computer science, electrical engineering, system analysis, mathematical modeling, economics, medical science, social networks, transportation, etc., shows it's worth. A trend continues to deal this imprecise information more appropriately. In 1986, Atanassov [4] introduced intuitionistic fuzzy set. This enhanced idea of fuzzy sets looks more appropriate to quantify uncertainty. It provides an opportunity to model the problems precisely based on existing knowledge and observations. After three years in 1989, Atanassov and Gargov [3] extended the concept to *Interval-Valued Intuitionistic Fuzzy Set* (IVIFS). It is more efficient than previous theories and helps to materialize the problem containing imprecise information.

Influential reforms got visible in classical graph theory when Rosenfeld [10] introduced the concept of fuzzy graphs in 1975. Fuzzy graphs seem useful to deal uncertainty-bearing in relationships which differ greatly from classical graphs. In 2011, Akram and Dudek [1] introduced *Interval-Valued Fuzzy Graphs* (IVFG). Afterward, the concept proposed by Atanassov [5] of intuitionistic fuzzy relations and *Intuitionistic Fuzzy Graphs* (IFG) handle uncertainty more accurately among the relational objects and in their relationships. In fact, to extend the theory of fuzzy graph we use interval-valued fuzzy graphs and interval-valued intuitionistic fuzzy graphs which are two different models to handle vague objects. Mishra and Pal [6] introduced the product of interval-valued intuitionistic fuzzy graphs. Akram and Davvaz [2] introduced *Strong Intuitionistic Fuzzy Graphs* (SIFG).

Nowadays many researchers contributed much more in this field they obtained many relations in fuzzy graphs and in intuitionistic fuzzy graphs. The concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs were introduced by Atanassov [5] but Parvathy and Karunambigai [9] introduced the concept more elaborately and define it properly. Nagoor Gani and Radha introduced some special properties of a fuzzy graph like, regular fuzzy graphs, total degree and totally regular fuzzy graphs in [7]. Alison Northup [8] introduced some properties on $(2,k)$-regular fuzzy graphs. Santhi Maheswari and Sekar [11] introduced d_2 of a vertex in fuzzy graphs [12] and also obtained some properties. Seethalakshmi and Gnanajothi [13] introduced the notion of f-morphism on intuitionistic fuzzy graphs and study their action on strong regular intuitionistic fuzzy graphs.

2. Regular Interval-valued Intuitionistic Fuzzy Graph

Throughout this paper we assume $D[0,1]$ be the set of all closed sub-intervals of the interval $[0,1]$ and elements of this set are denoted by uppercase letters. If $M \in D[0,1]$ then this interval can be represented as $M = [M_L, M_U]$, where M_L and M_U are the lower and upper limits of M when these subintervals are membership of the elements of any set A then the membership values are denoted by M_A and by N_A we mean the non-membership values.

Definition 2.1. An interval-valued intuitionistic fuzzy graph with underlying graph $G^* = (V,E)$ is defined to be a pair $G = (A,B)$, where

(i) the functions $M_A : V \to D[0,1]$ and $N_A : V \to D[0,1]$ denote the degree of membership and non membership of the element respectively, such that $0 \le M_A + N_A \le 1$ for all $x \in V$.

(ii) the functions $M_B : E \subset V \times V \to D[0,1]$ and $N_B : E \subset V \times V \to D[0,1]$ are defined by

$$M_{BL}(x,y) \le \min(M_{AL}(x), M_{AL}(y)) \quad \text{and} \quad N_{BL}(x,y) \ge \max(N_{AL}(x), N_{AL}(y)),$$

$$M_{BU}(x,y) \le \min(M_{AU}(x), M_{AU}(y)) \quad \text{and} \quad N_{BU}(x,y) \ge \min(M_{AU}(x), M_{AU}(y)).$$

Such that $0 \le M_{BU}(x,y) + N_{BU}(x,y) \le 1$, $\forall\ (x,y) \in E$.

Definition 2.2. The interval-valued intuitionistic fuzzy graph is said to be strong if

$$M_{BL}(u_i,v_j) = \min\{M_{AL}(u_i), M_{AL}(v_j)\},$$

$$M_{BU}(u_i,v_j) = \min\{M_{AU}(u_i), M_{AU}(v_j)\}$$

and

$$N_{BL}(u_i,v_j) = \max\{N_{AL}(u_i), N_{AL}(v_j)\},$$

$$N_{BU}(u_i,v_j) = \max\{N_{AU}(u_i), N_{AU}(v_j)\}.$$

Definition 2.3. An interval-valued intuitionistic fuzzy graph G is said to be regular if the absolute degree of each vertex of an interval-valued intuitionistic fuzzy graph is constant. If the absolute degree of each vertex is k, then we say the graph is k-regular interval-valued intuitionistic fuzzy graph.

Definition 2.4. Absolute degree $d(u)$ of any vertex u of an interval-valued intuitionistic fuzzy graph G is

$$d(u) = \left| \sum_{u \ne v, v \in V} M_{BU}(u,v) - \sum_{u \ne v, v \in V} N_{BU}(u,v) \right|.$$

Absolute membership of an edge $e = uv\ \forall\ e \in G$ is defined as $d(e) = |M_{BU} - N_{BU}|$, where $e = (M,N)\ \forall\ e \in G$.

Example 2.5. Let $G^* = (V,E)$ where $V = \{u,v,w,x\}$ and $E = \{uv, vw, wx, ux, vx, uw\}$ shown in Figure 1. Define $G(A,B)$ by

$M_A(u) = [.3,.6]$, $N_A(u) = [.2,.4]$;
$M_A(v) = [.4,.7]$, $N_A(v) = [.1,.3]$;
$M_A(w) = [.3,.7]$, $N_A(w) = [0,.2]$;
$M_A(x) = [.2,.5]$, $N_A(x) = [.3,.5]$;
$M_B(uv) = [.3,.5]$, $N_B(uv) = [.2,.4]$;
$M_B(vw) = [.2,.4]$, $N_B(vw) = [.1,.3]$;
$M_B(wx) = [.2,.4]$, $N_B(wx) = [.3,.5]$;
$M_B(xu) = [.2,.4]$, $N_B(xu) = [.3,.5]$;
$M_B(xv) = [.2,.5]$, $N_B(xv) = [.3,.5]$;
$M_B(uw) = [.3,.6]$, $N_B(uw) = [.2,.4]$.

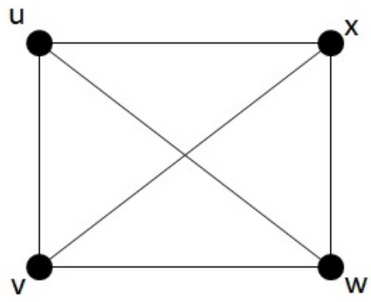

Figure 1. Absolute degree of an interval-valued intuitionistic fuzzy graph

Now absolute degree of the vertices u,v,w,x are:

$$d(u) = |(.6+.5+.4)-(.4+.4+.5)| = |1.5-1.3| = .2;$$

$$d(v) = |(.5+.4+.5)-(.4+.3+.5)| = |1.4-1.2| = .2;$$

$$d(w) = |(.4+.6+.4)-(.3+.4+.5)| = |1.4-1.2| = .2;$$

$$d(x) = |(.5+.4+.4)-(.5+.5+.5)| = |1.3-1.5| = .2$$

Here absolute degree of each vertex is .2. Thus, interval-valued intuitionistic fuzzy graph G is .2-regular.

Definition 2.6. Let $G=(A,B)$ be an interval-valued intuitionistic fuzzy graph on $G^*(V,E)$. The total degree of a vertex u is defined as

$$td(u) = \left| \sum_{u\neq v, v\in V} M_{BU}(u,v) - \sum_{u\neq v, v\in V} N_{BU}(u,v) \right| + |M_{AU}(u) - N_{AU}(u)|$$

$$= d(u) + |M_{AU}(u) - N_{AU}(u)|; \quad \forall\ uv \in E.$$

If each vertex of G has the same total degree k; then G is said to be totally regular interval-valued intuitionistic fuzzy graph of degree k or k-totally regular interval-valued intuitionistic fuzzy graph.

Definition 2.7. Let $G=(A,B)$ be an interval-valued intuitionistic fuzzy graph. The d_2-degree of a vertex $u \in G$ is $d_2(u) = \left|\sum M^2_{BU}(u,v) - \sum N^2_{BU}(u,v)\right|$ and summation runs over all such $v \in V$ which are distance two apart from u; where

$$M^2_{BU}(u,v) = \inf\{M_{BU}(u,u_1), M_{BU}(u_1,v)\}$$

and

$$N^2_{BU}(u,v) = \sup\{N_{BU}(u,u_1), N_{BU}(u_1,v)\}.$$

Also,

$$M_{BU}(uv) = 0 \text{ and } N_{BU}(uv) = 1; \quad \text{for } uv \notin E.$$

The minimum d_2-degree of G is $\delta_2(G) = \wedge\{d_2(v) : v \in V\}$.

The maximum d_2-degree of G is $\triangle_2(G) = \vee\{d_2(v) : v \in V\}$.

Example 2.8. Consider $G^* = (V, E)$, where $V = \{u, v, w, x, y\}$ and $E = \{uv, vw, wx, xy, yu\}$. Define $G = (A, B)$ by

$$M_A(u) = [.3, .6], \quad N_A(u) = [.2, .4];$$
$$M_A(v) = [.4, .7], \quad N_A(v) = [.1, .3];$$
$$M_A(w) = [.3, .7], \quad N_A(w) = [0, .2];$$
$$M_A(x) = [.2, .5], \quad N_A(x) = [.3, .5]$$
$$M_A(y) = [.2, .6], \quad N_A(y) = [.1, .3];$$
$$M_B(uv) = [.3, .5], \quad N_B(uv) = [.2, .4];$$
$$M_B(vw) = [.2, .4], \quad N_B(vw) = [.1, .3];$$
$$M_B(wx) = [.2, .4], \quad N_B(wx) = [.3, .6];$$
$$M_B(xy) = [.2, .4], \quad N_B(xy) = [.3, .5];$$
$$M_B(yu) = [.2, .6], \quad N_B(yu) = [.2, .4].$$

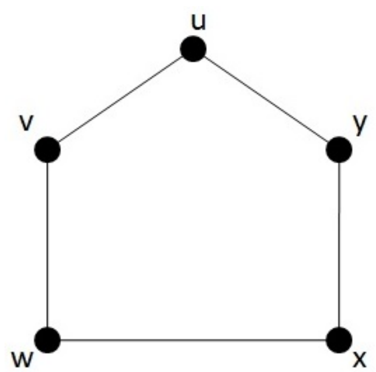

Figure 2. d_2-degree for the vertices of an interval-valued intuitionistic fuzzy graph

Now,

$$d_2(u) = |\inf\{.5, .4\} + \inf\{.6, .4\} - \sup\{.4, .3\} - \sup\{.4, .5\}| = .1$$

$$d_2(v) = |\inf\{.4, .4\} + \inf\{.5, .6\} - \sup\{.3, .6\} - \sup\{.4, .4\}| = .1$$

$$d_2(w) = |\inf\{.4, .4\} + \inf\{.4, .5\} - \sup\{.6, .5\} - \sup\{.3, .4\}| = .2$$

$$d_2(x) = |\inf\{.4, .6\} + \inf\{.4, .4\} - \sup\{.5, .4\} - \sup\{.6, .3\}| = .3$$

$$d_2(y) = |\inf\{.6, .5\} + \inf\{.4, .4\} - \sup\{.4, .4\} - \sup\{.5, .6\}| = .1$$

Theorem 2.9. *Even length interval-valued intuitionistic fuzzy cycle graph is regular or k-regular $\Longleftrightarrow$ absolute membership of e and $d_2(e)$ for each $e \in G$ is equal i.e. $d(e) = d_2(e)$ $\forall$ $e \in G$.*

Proof. Let $G = (A, B)$ is an even length interval-valued intuitionistic fuzzy cycle then if the absolute membership of each edge is same i.e. equal to any real number k then $d(e) = d_2(e)$

$\forall\ e \in G$ thus $d(u) = 2k\ \forall\ u \in G$. Hence the theorem is trivially true. Now if the absolute membership of any two adjacent edges are not equal but $d_2(e)$ is equal then for any $e \in G$

$$d(e_1) = d_2(e_1) = d(e_3) = d_2(e_3) = \ldots = d(e_{2n-1}) = k_1 \quad \text{(say)}$$

Similarly, $d(e_2) = d_2(e_2) = d(e_4) = d_2(e_4) = \ldots = d(e_{2n}) = k_2$ (say).

Since cycle is of even length thus, there must be n number of e_i's having absolute membership k_1 and k_2.

Also, we know that for a cycle absolute degree of any vertex u is

$$d(u) = \left| \sum_{u \neq v, v \in V} M_{BU}(u,v) - \sum_{u \neq v, v \in V} N_{BU}(u,v) \right| = d(e_i) + d(e_{i+1}) = k_1 + k_2 .$$

Therefore, $d(u) = k\ \forall\ u \in G$ so G is regular. Hence the theorem. □

Corollary 2.10. *An odd length interval-valued intuitionistic fuzzy cycle is regular iff* $d(e) = d_2(e) = k\ \forall\ e \in G$, *where* k *is any real number.*

Theorem 2.11. *Cartesian product of two regular interval-valued intuitionistic fuzzy graphs* G_1 *and* G_2 *is regular iff* G_1 *is a week regular interval-valued intuitionistic fuzzy subgraph of* G_2 *or vice versa.*

Proof. Let G_1 and G_2 be the two regular interval-valued intuitionistic fuzzy graph then the Cartesian product of G_1 and G_2 is regular if the absolute membership of each arc e of $G_1 \times G_2$ is equal and this is possible if $d(e) = \min\{d(e_i), d(e_j)\}$, where $e_i \in G_1$ and $e_j \in G_2$ for all i and j thus the condition is necessary for regularity of $G_1 \times G_2$ is either of G_1 or G_2 be a week regular subgraph of each other. Now, let G_1 is week regular subgraph of G_2 then we know that each edge of $G_1 \times G_2$ get interval-valued membership and non-membership as minimum of M_1 and M_2 and maximum of N_1 and N_2 thus, if G_1 is week then M_1 and N_1 dominates all the arc of $G_1 \times G_2$. So all the arc receive same absolute membership which imply $G_1 \times G_2$ is regular. Hence the theorem. □

Theorem 2.12. *Any interval-valued intuitionistic fuzzy path graph of length* l *is never an regular interval-valued intuitionistic fuzzy graph for* $l > 1$.

Proof. For any interval-valued intuitionistic fuzzy path graph $G = (A,B)$ either every edge have same absolute membership or some edges have distinct absolute membership. Thus when all edges receive same absolute membership then at least both the end vertices of the path graph G get different absolute degree than in-vertices of the path graph hence G is not regular. Similarly if some edges have distinct absolute membership, let $d(e_1) \neq d(e_2)$ and both e_1 and e_2 are adjacent let u be the common vertex of e_1 and $e_2 \Rightarrow d(u)$ is always greater than other end vertices of e_1 and e_2 which imply G is not regular. For $l = 1$ graph is always regular because in this case absolute membership of an edge become the absolute degree of the vertices. Hence the theorem. □

2.1 $(2,k)$-Regular and Totally $(2,k)$-Regular IVIFG

Definition 2.13. Let $G = (A,B)$ be an interval-valued intuitionistic fuzzy graph on $G^*(V,E)$. If $d_2(v) = 2$, $\forall\ v \in V$ then G is said to be $(2,k)$-regular interval-valued intuitionistic fuzzy graph.

Example 2.14. Consider $G^* = (V,E)$ where $V = \{u,v,w,x\}$ and $E = \{uv,vw,wx,xu\}$. Define $G = (A,B)$ by

$$\begin{aligned}
M_A(u) &= [.3,.6], & N_A(u) &= [.2,.4];\\
M_A(v) &= [.4,.7], & N_A(v) &= [.1,.3];\\
M_A(w) &= [.3,.7], & N_A(w) &= [0,.2];\\
M_A(x) &= [.2,.5], & N_A(x) &= [.2,.4];\\
M_B(uv) &= [.3,.5], & N_B(uv) &= [.2,.4];\\
M_B(vw) &= [.2,.4], & N_B(vw) &= [.1,.3];\\
M_B(wx) &= [.2,.5], & N_B(wx) &= [.2,.4];\\
M_B(xu) &= [.2,.4], & N_B(xu) &= [.3,.6].
\end{aligned}$$

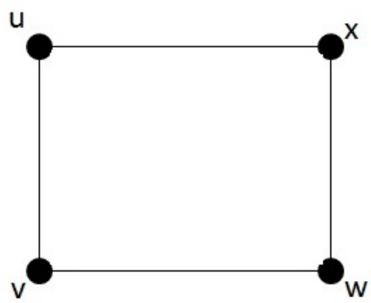

Figure 3. $(2,k)$-Regular interval-valued intuitionistic fuzzy graph

Now,

$$d_2(u) = |\inf\{.5,\ .4\} + \inf\{.4,\ .5\} - \sup\{.4,\ .3\} - \sup\{.6,\ .4\}| = .2\ ,$$

$$d_2(v) = |\inf\{.4,\ .5\} + \inf\{.5,\ .4\} - \sup\{.3,\ .4\} - \sup\{.4,\ .6\}| = .2\ ,$$

$$d_2(w) = |\inf\{.5,\ .4\} + \inf\{.4,\ .5\}\} - \sup\{.4,\ .6\} - \sup\{.3,\ .4\}| = .2\ ,$$

$$d_2(x) = |\inf\{.4,\ .5\} + \inf\{.5,\ .4\}\} - \sup\{.6,\ .4\} - \sup\{.4,\ .3\}| = .2\ .$$

Here $d_2(u) = d_2(v) = d_2(w) = d_2(x) = .2$ thus the graph G is $(2,.2)$-regular interval-valued intuitionistic fuzzy graph.

Theorem 2.15. *Let $G(A,B)$ be an strong interval-valued intuitionistic fuzzy graph on $G^*(V,E)$ Then $M_{AU}(u) = c_1$ and $N_{AU}(u) = c_2$; for all $u \in V$ if and only if the following conditions are equivalent.*

(i) *$G(A,B)$ is a $(2,k)$-regular interval-valued intuitionistic fuzzy graph.*

(ii) *$G(A,B)$ is a totally $(2,k+c)$-regular interval-valued intuitionistic fuzzy graph where $c = |c_1 - c_2|$.*

Proof. Let $M_{AU}(u)=c_1$ and $N_{AU}(u)=c_2$ for all $u\in V$. Thus $|M_{AU}(u)-N_{AU}(u)|=|c_1-c_2|=c$ for all $u\in V$. Suppose that $G:(A,B)$ is a $(2,k)$-regular interval-valued intuitionistic fuzzy graph then $d_2(u)=k$, for all $u\in V$. Hence, $td_2(u)=d_2(u)+|M_{AU}(u)-N_{AU}(u)|\Rightarrow td_2(u)=k+c$, $\forall\ u\in V$. Hence,$G:(A,B)$ is a totally $(2,k+c)$-regular interval-valued intuitionistic fuzzy graph. Thus (i)⇒(ii) is proved.

Suppose, $G(A,B)$ is a totally $(2,k+c)$-regular interval-valued intuitionistic fuzzy graph therefore,

$$\begin{aligned}
& td_2(u)=k+c, \quad \forall\ u\in V\\
\Rightarrow\ & d_2(u)+|M_{AU}(u)-N_{AU}(u)|=k+c, \quad \forall\ u\in V\\
\Rightarrow\ & d_2(u)+|c_1-c_2|=k+c, \quad \forall\ u\in V\\
\Rightarrow\ & d_2(u)+c=k+c, \quad \forall\ u\in V\\
\Rightarrow\ & d_2(u)=k, \quad \forall\ u\in V.
\end{aligned}$$

Hence, $G(A,B)$ is a $(2,k)$-regular interval-valued intuitionistic fuzzy graph. Hence (i) and (ii) are equivalent. Conversely assume that (i) and (ii) are equivalent i.e., suppose $G(A,B)$ is $(2,k)$-regular interval-valued intuitionistic fuzzy graph and also a totally $(2,k+c)$-regular interval-valued intuitionistic fuzzy graph where $c=|c_1-c_2|$.

Thus,

$$\begin{aligned}
& td_2(u)=k+c \ \text{ and } \ d_2(u)=k, \quad \forall\ u\in V\\
\Rightarrow\ & d_2(u)+|M_{AU}(u)-N_{AU}(u)|=k+c \ \text{ and } \ d_2(u)=k, \quad \forall\ u\in V\\
\Rightarrow\ & |M_{AU}(u)-N_{AU}(u)|=c=|c_1-c_2|, \quad \forall\ u\in V\\
\Rightarrow\ & M_{AU}(u)=c_1 \ \text{ and } \ N_{AU}(u)=c_2, \quad \forall\ u\in V.
\end{aligned}$$

□

Corollary 2.16. *$(2,k)$-regular interval-valued intuitionistic fuzzy graph is always totally $(2,k+c)$-regular interval-valued intuitionistic fuzzy graph where $c=|c_1-c_2|$ if interval-valued intuitionistic fuzzy graph $G(A,B)$ is strong.*

3. Regularity on Isomorphic IVIFG

Definition 3.1. Let $G_1=(A_1,B_1)$ and $G_2=(A_2,B_2)$ be two interval-valued intuitionistic fuzzy graphs on (V_1,E_1) and (V_2,E_2), respectively.

A bijective function $f:A_1\to A_2$ is called interval-valued intuitionistic fuzzymorphism or f-morphism of interval-valued intuitionistic fuzzy graph if there exists some positive real number k_1 and k_2 such that

(i) $M_{A_2}(f(u))=k_1M_{A_1}u$ and $N_{A_2}(f(u))=k_1N_{A_1}u$, $\forall\ u\in V_1$

(ii) $M_{B_2}(f(u),f(v))=k_2M_{B_1}(u,v)$ and $N_{B_2}(f(u),f(v))=k_2N_{B_1}(u,v)$, $\forall\ u,v\in V_1$. In these cases f is called $(k_1,k_2)f$-interval-valued intuitionistic morphism on G_1 over G_2. When $k_1=k_2=k$ then we say it is $k-f$-interval-valued intuitionistic morphism on G_1 over G_2.

Definition 3.2. A co-week isomorphism from G_1 to G_2 is a map $h : A_1 \to A_2$ which is bijective homomorphism that satisfies $M_{B_1}(u,v) = M_{B_2}(h(u),h(v))$ and $N_{B_1}(u,v) = N_{B_2}(h(u),h(v))$, $\forall\ u,v \in A$.

A week isomorphism from G_1 to G_2 is a map $h : A_1 \to A_2$ which is bijective homomorphism that satisfies $M_{A_1}(u) = M_{A_2}(h(u))$ and $N_{A_1}(u) = N_{A_2}(h(u))$, $\forall\ u,v \in A$.

Theorem 3.3. *The relation f-interval-valued intuitionistic fuzzy morphic is an equivalence relation in the collection of all interval-valued intuitionistic fuzzy graphs.*

Proof. Let S be the set of all interval-valued intuitionistic fuzzy graphs. Now, define the relation $G_1 \approx G_2$ when G_1 is $(k_1,k_2)f$-interval-valued intuitionistic morphism on G_2 where k_1,k_2 are any non zero real numbers and $G_1,G_2 \in S$.

Now for any identity morphism G_1 over G_1 is an one-one mapping and hence $'\approx'$ is reflexive.

Let $G_1 \approx G_2$, then there exists a (k_1,k_2)-interval-valued intuitionistic fuzzy morphism from G_1 to G_2 for some non zero k_1 and k_2.

$$M_{A_2}(f(u)) = k_1 M_{A_1} u \quad \text{and} \quad N_{A_2}(f(u)) = k_1 N_{A_1} u, \quad \forall\ u \in V_1$$

$$M_{B_2}(f(u),f(v)) = k_2 M_{B_1}(u,v) \quad \text{and} \quad N_{B_2}(f(u),f(v)) = k_2 N_{B_1}(u,v), \quad \forall\ u,v \in V_1$$

Consider $f^{-1} : G_1 \to G_2$. Let $x,y \in V_2$.

As f^{-1} is bijective, $x = f(u)$, $y = f(v)$, for some $u,v \in V_1$.

Now,

$$M_{A_1}(f^{-1}(x)) = M_{A_1}(f^{-1}(f(u)) = M_{A_1}(u) = \frac{1}{k_1} M_{A_2} f(u) = \frac{1}{k_1} M_{A_2}(x);$$

$$N_{A_1}(f^{-1}(x)) = N_{A_1}(f^{-1}(f(u)) = N_{A_1}(u) = \frac{1}{k_1} N_{A_2} f(u) = \frac{1}{k_1} N_{A_2}(x).$$

$$M_{B_1}(f^{-1}(x),f^{-1}(y)) = M_{B_1}(f^{-1}(f(u),f^{-1}(f(v)) = M_{B_1}(u,v) = \frac{1}{k_2} M_{B_2}(f(u),f(v))$$
$$= \frac{1}{k_2} M_{B_2}(x,y);$$

$$N_{B_1}(f^{-1}(x),f^{-1}(y)) = N_{B_1}(f^{-1}(f(u),f^{-1}(f(v)) = N_{B_1}(u,v) = \frac{1}{k_2} N_{B_2}(f(u),f(v))$$
$$= \frac{1}{k_2} N_{B_2}(x,y).$$

Thus there exists $(\frac{1}{k_1},\frac{1}{k_2})f$-interval-valued intuitionistic morphism from G_2 to G_1.

Therefore $G_2 \approx G_1$ and hence '$\approx$' is symmetric.

Let $G_1 \approx G_2$ and $G_2 \approx G_3$.

Thus there exist two interval-valued intuitionistic morphism say $(k_1,k_2) - f$ and $(k_2,k_3) - g$ such that f is interval-valued intuitionistic morphism from G_1 to G_2 and g is interval-valued intuitionistic morphism from G_2 to G_3 for non-zero k_1,k_2,k_3,k_4.

So,

$$M_{A_3}(g(x)) = k_3 M_{A_2}(x) \text{ and } N_{A_3}(g(x)) = k_3 N_{A_2}(x), \ \forall \ x \in V_2$$

and

$$M_{B_3}(g(x), g(y)) = k_4 M_{B_2}(x, y) \text{ and } N_{B_3}(g(x), g(y)) = k_4 N_{B_2}(x, y), \ \forall \ (x, y) \in E_2.$$

Let $h = g \circ f : G_1 \to G_3$. Now,

$$M_{A_3}(h(u)) = M_{A_3}((g \circ f)(u)) = M_{A_3}(g(f(u)) = k_3 M_{A_3}(f(u)) = k_3 k_1 M_{A_1}(u),$$

$$N_{A_3}(h(u)) = N_{A_3}((g \circ f)(u)) = N_{A_3}(g(f(u)) = k_3 N_{A_3}(f(u)) = k_3 k_1 N_{A_1}(u),$$

$$\begin{aligned} M_{B_3}(h(u), h(v)) &= M_{B_3}((g \circ f)(u), (g \circ f)(v)) = M_{B_3}(g(f(u), g(f(v)) = k_4 M_{B_2}(f(u), f(v)) \\ &= k_4 k_2 M_{B_1}(u, v), \end{aligned}$$

$$\begin{aligned} N_{B_3}(h(u), h(v)) &= N_{B_3}((g \circ f)(u), (g \circ f)(v)) = N_{B_3}(g(f(u), g(f(v)) = k_4 N_{B_2}(f(u), f(v)) \\ &= k_4 k_2 N_{B_1}(u, v). \end{aligned}$$

Thus, there exists $(k_3k_1, k_4k_2)h$-interval-valued intuitionistic fuzzy morphism from G_1 over G_3. Therefore, $G_1 \approx G_3$ hence, '$\approx$' is transitive.

So, the relation f-interval-valued intuitionistic fuzzy morphic is an equivalence relation in the collection of all interval-valued intuitionistic fuzzy graphs. □

Theorem 3.4. *Let G_1 and G_2 be two IVIFG's such that G_1 is (k_1, k_2) interval-valued intuitionistic fuzzy morphic to G_2 for some non-zero k_1 and k_2. The image of strong edge in G_1 is strong edge in G_2 if and only if $k_1 = k_2$.*

Proof. Let (u, v) be strong edge in G_1 such that $(f(u), f(v))$ is also strong edge in G_2.

Now, as $G_1 \approx G_2$

$$\begin{aligned} k_2 M_{B_1}(u, v) &= M_{B_2}(f(u), f(v)) = M_{A_2} f(u) \wedge M_{A_2}(f(v) = k_1\{M_{A_1}(u) \wedge M_{A_1}(v)\} \\ &= k_1 M_{B_1}(u, v), \ \ \forall \ u \in V_1. \end{aligned}$$

Hence,

$$k_2 M_{B_1}(u, v) = k_1 M_{B_1}(u, v), \quad \forall \ u \in V_1. \tag{3.1}$$

Similarly,

$$\begin{aligned} k_2 N_{B_1}(u, v) &= N_{B_2}(f(u), f(v)) = N_{A_2} f(u) \vee N_{A_2}(f(v) = k_1\{N_{A_1}(u) \vee N_{A_1}(v)\} \\ &= k_1 N_{B_1}(u, v), \ \ \forall \ u \in V_1. \end{aligned}$$

Hence,

$$k_2 N_{B_1}(u, v) = k_1 N_{B_1}(u, v), \ \ \forall \ u \in V_1. \tag{3.2}$$

Equations (3.1) and (3.2) holds if and only if $k_1 = k_2$. Hence the theorem. □

Theorem 3.5. *If an IVIFG G_1 is coweak isomorphic to G_2 and if G_1 is regular then G_2 is regular.*

Proof. As IVIFG G_1 is coweak isomorphic to IVIFG G_2, there exists a coweak isomorphism $h : G_1 \to G_2$ which is bijective that satisfies

$$M_{A_1}(u) \le M_{A_2}(h(u)) \text{ and } N_{A_1}(u) \ge N_{A_2}(h(u)).$$

It also satisfies,

$$M_{B_1}(u,v) = M_{B_2}(h(u),h(v)) \text{ and } N_{B_1}(u,v) = N_{B_2}(h(u),h(v)), \ \forall\ u,v \in V_1.$$

As G_1 is regular, for $u \in V$,

$$\sum_{u \neq v, v \in V_1} M_{BU}(u,v) = \text{constant}$$

and

$$\sum_{u \neq v, v \in V_1} N_{BU}(u,v) = \text{constant}.$$

Now

$$\sum_{h(u) \neq h(v)} M_{B_2}(h(u),h(v)) = \sum_{u \neq v, v \in V_1} M_{BU}(u,v)$$
$$= \text{constant}$$

and

$$\sum_{h(u) \neq h(v)} N_{B_2}(h(u),h(v)) = \sum_{u \neq v, v \in V_1} N_{BU}(u,v)$$
$$= \text{constant}$$

Therefore G_2 is regular. □

Theorem 3.6. *Let G_1 and G_2 be two IVIFG's. If G_1 is weak isomorphic to G_2 and if G_1 is strong then G_2 is strong.*

Proof. As an IVIFG G_1 be weak isomorphic with an IVIFG $G2$, there exists a weak isomorphism $h : G_1 \to G_2$ which is bijective that satisfies

$$M_{A_1}(u) = M_{A_2}(h(u)) \text{ and } N_{A_1}(u) = N_{A_2}(h(u)),$$

$$M_{B_1}(u,v) \le M_{B_2}(h(u),h(v)) \text{ and } N_{B_1}(u,v) \ge N_{B_2}(h(u),h(v)), \quad \forall\ u,v \in V_1.$$

As G_1 is strong,

$$M_{B_1}(u,v) = \min M_{A_1}(u), M_{A_1}(v) \text{ and } N_{B_1}(u,v) = \max N_{A_1}(u), N_{A_1}(v).$$

Now,

$$M_{B_2}(h(u),h(v)) \le M_{B_1}(u,v) = \min\{M_{A_1}(u), M_{A_1}(v)\}$$
$$= \min\{M_{A_2}h(u), M_{A_2}h(v)\}.$$

By definition,

$$M_{B_2}(h(u),h(v)) \le \min\{M_{A_2}h(u), M_{A_2}h(v)\}.$$

Therefore,

$$M_{B_2}(h(u),h(v)) = \min\{M_{A_2}h(u), M_{A_2}h(v)\}.$$

Similarly,

$$N_{B_2}(h(u),h(v)) \geq N_{B_1}(u,v) = \max\{N_{A_1}(u), N_{A_1}(v)\}$$
$$= \max\{N_{A_2}h(u), N_{A_2}h(v)\}.$$

And by definition,

$$N_{B_2}(h(u),h(v)) \geq \max\{N_{A_2}h(u), N_{A_2}h(v)\}.$$

Therefore,

$$N_{B_2}(h(u),h(v)) = \max\{N_{A_2}h(u), N_{A_2}h(v)\}.$$

Thus G_2 is strong. □

4. Conclusion

A regular interval-valued intuitionistic fuzzy graph has numerous applications in the modeling of real life systems where the level of information inherited in the system varies with respect to time and have a different level of precision and hesitation. Most of the actions in real life are time dependent, symbolic models used in the expert system are more effective than traditional one. In this paper, we introduced the concept of a regular interval-valued intuitionistic fuzzy graph and obtained some properties over it. In future, we can extend this concept to bipolar fuzzy graphs, hypergraphs and in some more areas of graph theory.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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