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Research Article

Minimum Equitable Dominating Partition Energy of a Graph

Sashi Kanth Reddy Avula[1], P. Siva Kota Reddy[2,*] and K.N. Prakasha[3]

[1] *Department of Computer Science, JAIN University, Bangalore, India*
[2] *Department of Mathematics, Siddaganga Institute of Technology, Tumkur 572103, India*
[3] *Department of Mathematics, Vidyavardhaka College of Engineering, Mysuru 570002, India*
***Corresponding author:** pskreddy@sit.ac.in

Abstract. The partition energy of a graph was introduced by Sampathkumar et al. [12]. Motivated by this, we introduce the concept of minimum equitable dominating partition energy of a graph, $E_p^E(G)$ and compute the minimum equitable dominating partition energy $E_p^E(G)$ of few families of graphs. Also, we establish the bounds for minimum equitable dominating partition energy.

Keywords. Minimum equitable dominating set; Minimum equitable dominating k-partition eigenvalues; Minimum equitable dominating k-partition energy; k-Complement; $k(i)$-Complement

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1. Introduction

The energy of graph originates from chemistry to estimate the total π-electron energy of a molecule. The conjugated hydrocarbons can be represented by a graph called molecular graph. Every carbon atom is represented by a vertex and every carbon-carbon bond by an edge and we ignore hydrogen atoms. The eigenvalues of the molecular graph represent the energy level of the electron in the molecule. In spectral graph theory, the eigenvalues of several matrices like adjacency matrix, Laplacian matrix, distance matrix, maximum degree matrix, minimum degree matrix are studied in [1], [2], [4], [5], [9].

In the subsequent sections, we have defined the minimum equitable dominating partition energy of a graph with respect to given partition of a graph. Further, we determine minimum equitable dominating partition energy of two types of complements of a partition graph called k-complement and $k(i)$-complement of a graph.

2. Minimum Equitable Dominating Partition Energy of a Graph

Let G be a simple graph of order n with vertex set $V(G) = \{v_1, v_2, v_3, \dots, v_n\}$ and edge set E. A subset U of $V(G)$ is an equitable dominating set, if for every $v \in V(G) - U$ there exists a vertex $u \in U$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \le 1$, where $\deg(x)$ denotes the degree of vertex x in $V(G)$. Any equitable dominating set with minimum cardinality is called a minimum equitable dominating set. Let E be a minimum equitable dominating set of a graph G. The minimum equitable dominating partition matrix is given by

$$a_{ij} = \begin{cases} 2 & \text{if } v_i \text{ and } v_j \text{ are adjacent where } v_i, v_j \in V_r, \\ -1 & \text{if } v_i \text{ and } v_j \text{ are non-adjacent where } v_i, v_j \in V_r, \\ 1 & \text{if } i = j \text{ and } v_i \in E, \\ 1 & \text{if } v_i \text{ and } v_j \text{ are adjacent between the sets} \\ & V_r \text{ and } V_s \text{ for } r \neq s, \text{ where } v_i \in V_r \text{ and } v_j \in V_s, \\ 0 & \text{otherwise.} \end{cases}$$

In this paper, we study minimum equitable dominating partition energy of a graph with respect to given partition of a graph. Further, we determine minimum equitable dominating partition energy of two types of complements of a partition graph called k-complement and $k(i)$-complement of a graph introduced by Sampathkumar [12].

Definition 2.1. The complement of a graph G is a graph $\overline{G}$ on the same vertices such that two distinct vertices of $\overline{G}$ are adjacent if and only if they are not adjacent in G.

Definition 2.2 ([12])**.** Let G be a graph and $P_k = \{V_1, V_2, \dots, V_k\}$ be a partition of its vertex set V. Then the k-complement of G is obtained as follows: For all V_i and V_j in P_k, $i \neq j$ remove the edges between V_i and V_j and add the edges between the vertices of V_i and V_j which are not in G and is denoted by $\overline{(G)_k}$.

Definition 2.3 ([12])**.** Let G be a graph and $P_k = \{V_1, V_2, \dots, V_k\}$ be a partition of its vertex set V. Then the $k(i)$-complement of G is obtained as follows: For each set V_r in P_k, remove the edges of G joining the vertices within V_r and add the edges of $\overline{G}$ (complement of G) joining the vertices of V_r, and is denoted by $\overline{(G)_{k(i)}}$.

3. Some Basic Properties of Minimum Equitable Dominating Partition Energy of a Graph

Let $G=(V,E)$ be a graph with n vertices and $P_k=\{V_1,V_2,\ldots,V_k\}$ be a partition of V. For $1\le i\le k$, let b_i denote the total number of edges joining the vertices of V_i and c_i be the total number of edges joining the vertices from V_i to V_j for $i\ne j, 1\le j\le k$ and d_i be the number of non-adjacent pairs of vertices within V_i. Let $m_1=\sum_{i=1}^{k} b_i$, $m_2=\sum_{i=1}^{k} c_i$ and $m_3=\sum_{i=1}^{k} d_i$. Let $P_k^E(G)$ be the minimum equitable dominating partition matrix. If the characteristic polynomial of $P_k^E(G)$ denoted by $\Phi_k^E(G,\lambda)$ is $a_0\lambda^n+a_1\lambda^{n-1}+a_2\lambda^{n-2}+\cdots+a_n$, then the coefficient a_i can be interpreted using the principal minors of $P_k^E(G)$.

The following proposition determines the first three coefficients of the characteristic polynomial of $P_k^E(G)$.

Proposition 3.1. *The first three coefficients of $\phi_k^E(G,\lambda)$ are given as follows:*

(i) $a_0=1$,

(ii) $a_1=-|E|$,

(iii) $a_2=|E|C_2-[4m_1+m_2+m_3]$.

Proof. (i) From the definition $\Phi_k(G,\lambda)=\det[\lambda I-P_k^E(G)]$, we get $a_0=1$.

(ii) The sum of determinants of all 1×1 principal submatrices of $P_k^E(G)$ is equal to the trace of $P_k^E(G)$.

Implies $a_1=(-1)^1$ trace of $[P_k^E(G)]=-|E|$.

(iii)
$$\begin{aligned}(-1)^2a_2&=\sum_{1\le i<j\le n}\begin{vmatrix}a_{ii} & a_{ij}\\ a_{ji} & a_{jj}\end{vmatrix}\\
&=\sum_{1\le i<j\le n} a_{ii}a_{jj}-a_{ji}a_{ij}\\
&=\sum_{1\le i<j\le n} a_{ii}a_{jj}-\sum_{1\le i<j\le n} a_{ji}a_{ij}\\
&=|E|C_2-[(2)^2m_1+(1)^2m_2+(-1)^2m_3]\\
&=|E|C_2-[4m_1+m_2+m_3].\end{aligned}$$
□

Proposition 3.2. *If $\lambda_1,\lambda_2,\ldots,\lambda_n$ are partition eigenvalues of $P_k^E(G)$, then*

$$\sum_{i=1}^{n}{\lambda_i}^2=|E|+2[4m_1+m_2+m_3].$$

Proof. We know that

$$\begin{aligned}\sum_{i=1}^{n}\lambda_i^2&=\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}a_{ji}\\
&=2\sum_{i<j}(a_{ij})^2+\sum_{i=1}^{n}(a_{ii})^2\end{aligned}$$

$$= 2\sum_{i<j}(a_{ij})^2 + |D|$$
$$= |E| + 2[4m_1 + m_2 + m_3].$$ □

Theorem 3.3. *Let G be a graph with n vertices and P_k be a partition of G. Then*

$$E^D_{P_k}(G) \le \sqrt{n(|E| + 2[4m_1 + m_2 + m_3])},$$

where m_1, m_2, m_3 are as defined above for G.

Proof. Let $\lambda_1, \lambda_2, \dots, \lambda_n$ be the eigenvalues of $P_k(G)$.

Now by Cauchy-Schwartz inequality, we have

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \le \left(\sum_{i=1}^{n} {a_i}^2\right)\left(\sum_{i=1}^{n} {b_i}^2\right).$$

Let $a_i = 1$, $b_i = |\lambda_i|$. Then,

$$\left(\sum_{i=1}^{n} |\lambda_i|\right)^2 \le \left(\sum_{i=1}^{n} 1\right)\left(\sum_{i=1}^{n} |\lambda_i|^2\right)$$
$$[E^E_{P_k}]^2 \le n(|E| + 2[4m_1 + m_2 + m_3])$$
$$[E^D_{P_k}] \le \sqrt{n(|D| + 2([4m_1 + m_2 + m_3])},$$

which is upper bound.

Theorem 3.4. *Let G be a partition graph with n vertices. If $R =$ det $P^E_k(G)$, then*

$$E^E_{P_k}(G) \ge \sqrt{(|D| + 2[4m_1 + m_2 + m_3]) + n(n-1)R^{\frac{2}{n}}}.$$

Proof. By definition,

$$\left(E^E_{P_k}(G)\right)^2 = \left(\sum_{i=1}^{n} |\lambda_i|\right)^2$$
$$= \sum_{i=1}^{n} |\lambda_i| \sum_{j=1}^{n} |\lambda_j|$$
$$= \left(\sum_{i=1}^{n} |\lambda_i|^2\right) + \sum_{i\ne j} |\lambda_i||\lambda_j|.$$

Using arithmetic mean and geometric mean inequality, we have

$$\frac{1}{n(n-1)} \sum_{i\ne j} |\lambda_i|\,|\lambda_j| \ge \left(\prod_{i\ne j} |\lambda_i|\,|\lambda_j|\right)^{\frac{1}{n(n-1)}}.$$

Therefore,

$$[E^E_{P_k}(G)]^2 \ge \sum_{i=1}^{n} |\lambda_i|^2 + n(n-1)\left(\prod_{i\ne j} |\lambda_i|\,|\lambda_j|\right)^{\frac{1}{n(n-1)}}$$

$$\geq \sum_{i=1}^{n} |\lambda_i|^2 + n(n-1)\left(\prod_{i=1}^{n} |\lambda_i|^{2(n-1)}\right)^{\frac{1}{n(n-1)}}$$
$$= \sum_{i=1}^{n} |\lambda_i|^2 + n(n-1)R^{\frac{2}{n}}$$
$$= (|D| + 2[4m_1 + m_2 + m_3]) + n(n-1)R^{\frac{2}{n}}.$$

Thus,

$$E^E_{P_k}(G) \geq \sqrt{(|D| + 2[4m_1 + m_2 + m_3]) + n(n-1)R^{\frac{2}{n}}}. \qquad \square$$

Theorem 3.5. *If the minimum equitable dominating partition energy of a graph is a rational number, then it must be a positive even number.*

Proof of this theorem is similar to the proof of Theorem 2.12 in [3].

4. Energy of Some Partition Graphs and Their Complements

Theorem 4.1. *The minimum equitable dominating* 1*-partition energy of a complete graph* K_n *is* $E^E_{P_1}(K_n) = 2(n-2) + \sqrt{4n^2 - 4n + 9}$.

Proof. Consider all the vertices is in one partition. The minimum equitable dominating set $= E = \{v_1\}$. The minimum equitable dominating 1-partition matrix is

$$P^E_1(K_n) = \begin{bmatrix} 1 & 2 & 2 & \dots & 2 & 2 \\ 2 & 0 & 2 & \dots & 2 & 2 \\ 2 & 2 & 0 & \dots & 2 & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & 2 & \dots & 0 & 2 \\ 2 & 2 & 2 & \dots & 2 & 0 \end{bmatrix}.$$

Characteristic equation is

$$(\lambda + 2)^{n-2}(\lambda^2 - (2n-3)\lambda - 2n) = 0$$

and the spectrum is $\text{Spec}^E_{P_1}(K_n) = \begin{pmatrix} -2 & \frac{(2n-3)+\sqrt{4n^2-4n+9}}{2} & \frac{(2n-3)-\sqrt{4n^2-4n+9}}{2} \\ n-2 & 1 & 1 \end{pmatrix}$.

Therefore, $E^E_{P_1}(K_n) = 2(n-2) + \sqrt{4n^2 - 4n + 9}$. $\square$

Theorem 4.2. *The minimum equitable dominating* 1*-partition energy of star graph* $K_{1,n-1}$ *is*

$$E^E_{P_1}(K_{1,n-1}) = (n-2) + \sqrt{n^2 + 14n - 15}.$$

Proof. Consider all the vertices is in one partition. The minimum equitable dominating set

$= E = \{v_1, v_2, \ldots, v_n\}$. The minimum equitable dominating 1-partition matrix is

$$P_1^E(K_{1,n-1}) = \begin{bmatrix} 1 & 2 & 2 & \ldots & 2 & 2 \\ 2 & 1 & -1 & \ldots & -1 & -1 \\ 2 & -1 & 1 & \ldots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & -1 & -1 & \ldots & 1 & -1 \\ 2 & -1 & -1 & \ldots & -1 & 1 \end{bmatrix}.$$

Characteristic equation is

$$(\lambda - 2)^{n-2}[\lambda^2 + (n-4)\lambda - (5n-7)] = 0$$

spectrum is $\mathrm{Spec}_{P_1}^E(K_{1,n-1}) = \begin{pmatrix} 2 & \frac{-(n-4)+\sqrt{n^2+12n-12}}{2} & \frac{-(n-4)-\sqrt{n^2+12n-12}}{2} \\ n-2 & 1 & 1 \end{pmatrix}.$

Therefore, $E_{P_1}^E(K_{1,n-1}) = 2(n-2) + \sqrt{n^2+12n-12}$.

Definition 4.3. The Crown graph S_n^0 for an integer $n \geq 3$ is the graph with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$ and edge set $\{u_i v_i : 1 \leq i, j \leq n, i \neq j\}$. S_n^0 is therefore equivalent to the complete bipartite graph $K_{n,n}$ with horizontal edges removed.

Theorem 4.4. *The minimum equitable dominating* 1*-partition energy of Crown graph* S_n^0 *is*

$$E_{P_1}^E(S_n^0) = 6(n-2) + \sqrt{n^2-2n+5} + \sqrt{9n^2+6n-11}.$$

Proof. Consider all the vertices is in one partition. Let S_n^0 be a crown graph of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$ and minimum equitable dominating set $= E = \{u_1, v_1\}$.The minimum equitable dominating 1-partition matrix is

$$P_1^E(S_n^0) = \begin{bmatrix} 1 & -1 & -1 & \ldots & -1 & -1 & 2 & \ldots & 2 & 2 \\ -1 & 0 & -1 & \ldots & -1 & 2 & -1 & \ldots & 2 & 2 \\ -1 & -1 & 0 & \ldots & -1 & 2 & 2 & \ldots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & -1 & \ldots & 0 & 2 & 2 & \ldots & 2 & -1 \\ -1 & 2 & 2 & \ldots & 2 & 1 & -1 & \ldots & -1 & -1 \\ 2 & -1 & 2 & \ldots & 2 & -1 & 0 & \ldots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 2 & 2 & -1 & \ldots & 2 & -1 & -1 & \ldots & 0 & -1 \\ 2 & 2 & 2 & \ldots & -1 & -1 & -1 & \ldots & -1 & 0 \end{bmatrix}.$$

Characteristic equation is

$$(\lambda+2)^{n-2}(\lambda-4)^{n-2}[\lambda^2 + (3n-9)\lambda - (15n-23)][\lambda^2 - (n-3)\lambda - (n-1)] = 0$$

spectrum is $\mathrm{Spec}_{P_1}^E(S_n^0) =$

$$\begin{pmatrix} -2 & 4 & \frac{-(3n-9)+\sqrt{9n^2+6n-11}}{2} & \frac{-(3n-9)-\sqrt{9n^2+6n-11}}{2} & \frac{(n-3)+\sqrt{n^2-2n+5}}{2} & \frac{(n-3)-\sqrt{n^2-2n+5}}{2} \\ (n-2) & (n-2) & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Therefore, $E^E_{P_1}(S^0_n) = 6(n-2) + \sqrt{n^2-2n+5} + \sqrt{9n^2+6n-11}$.

Theorem 4.5. *The minimum equitable dominating* 1*-partition energy of Cocktail party graph* $K_{n\times 2}$ *is*

$$E^E_{P_1}(K_{n\times 2}) = (6n-9) + \sqrt{16n^2-8n+17}.$$

Proof. Consider all the vertices in the one partition. The minimum equitable dominating set $= D = \{u_1, v_1\}$. The minimum equitable dominating 1-partition matrix is

$$P_1^E(K_{n\times 2}) = \begin{bmatrix} 1 & -1 & 2 & 2 & \dots & 2 & 2 & 2 & 2 \\ -1 & 1 & 2 & 2 & \dots & 2 & 2 & 2 & 2 \\ 2 & 2 & 0 & -1 & \dots & 2 & 2 & 2 & 2 \\ 2 & 2 & -1 & 0 & \dots & 2 & 2 & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 2 & 2 & 2 & 2 & \dots & 0 & -1 & 2 & 2 \\ 2 & 2 & 2 & 2 & \dots & -1 & 0 & 2 & 2 \\ 2 & 2 & 2 & 2 & \dots & 2 & 2 & 0 & -1 \\ 2 & 2 & 2 & 2 & \dots & 2 & 2 & -1 & 0 \end{bmatrix}.$$

Characteristic equation is

$$(\lambda+5)^{n-2}(\lambda-1)^{n-1}(\lambda-2)[\lambda^2-(4n-9)\lambda-(16n-16)] = 0.$$

Hence, spectrum is $\operatorname{Spec}_{P_1}^E(K_{n\times 2}) = \begin{pmatrix} -5 & 1 & 2 & \frac{(4n-9)+\sqrt{16n^2-8n+17}}{2} & \frac{(4n-9)-\sqrt{16n^2-8n+17}}{2} \\ (n-2) & (n-1) & 1 & 1 & 1 \end{pmatrix}$.

Therefore, $E_{P_1}^E(K_{n\times 2}) = (6n-9)+\sqrt{16n^2-8n+17}$. □

Theorem 4.6. *The minimum equitable dominating* 1*-partition energy of complete bipartite graph* $K_{n,n}$ *is*

$$E_{P_1}^E(K_{n,n}) = (3n-1)+\sqrt{9n^2+6n-11}.$$

Proof. Consider all the vertices in the one partition. The minimum equitable dominating set $= E = \{u_1, v_1\}$. The minimum equitable dominating 1-partition matrix is

$$P_1^E(K_{n,n}) = \begin{bmatrix} 1 & -1 & -1 & -1 & \dots & 2 & 2 & 2 & 2 \\ -1 & 0 & -1 & -1 & \dots & 2 & 2 & 2 & 2 \\ -1 & -1 & 0 & -1 & \dots & 2 & 2 & 2 & 2 \\ -1 & -1 & -1 & 0 & \dots & 2 & 2 & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 2 & 2 & 2 & 2 & \dots & 1 & -1 & -1 & -1 \\ 2 & 2 & 2 & 2 & \dots & -1 & 0 & -1 & -1 \\ 2 & 2 & 2 & 2 & \dots & -1 & -1 & 0 & -1 \\ 2 & 2 & 2 & 2 & \dots & -1 & -1 & -1 & 0 \end{bmatrix}.$$

Characteristic equation is

$$(\lambda-1)^{2n-4}[\lambda^2+(3n-3)\lambda-(6n-5)][\lambda^2-(n+3)\lambda+(2n+1)] = 0.$$

Hence, spectrum is

$$\operatorname{Spec}_{P_1}^E(K_{n,n}) = \begin{pmatrix} 1 & \frac{-(3n-3)+\sqrt{9n^2+6n-11}}{2} & \frac{-(3n-3)-\sqrt{9n^2+6n-11}}{2} & \frac{(n+3)+\sqrt{n^2-2n+5}}{2} & \frac{(n+3)-\sqrt{n^2-2n+5}}{2} \\ (2n-4) & 1 & 1 & 1 & 1 \end{pmatrix}.$$

Therefore, $E_{P_1}^E(K_{n,n}) = (3n-1)+\sqrt{9n^2+6n-11}$. □

Theorem 4.7. *The minimum equitable dominating 2-partition energy of star graph $K_{1,n-1}$ in which the vertex of degree $n-1$ is in one partition and vertices of degree 1 are in another partition is $E_{P_2}(K_{1,n-1}) = 3n-4$.*

Proof. The 2-partition of star graph $K_{1,n-1}$ in which the vertex of degree $n-1$ is in one partition and vertices of degree 1 are in another partition The minimum equitable dominating set $= E = \{v_0\}$. The minimum equitable dominating 1-partition matrix is

$$P_2^E(K_{1,n-1}) = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 & 1 \\ 1 & 1 & -1 & \dots & -1 & -1 \\ 1 & -1 & 1 & \dots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & -1 & -1 & \dots & 1 & -1 \\ 1 & -1 & -1 & \dots & -1 & 1 \end{bmatrix}.$$

Hence, its characteristic equation is $(\lambda+(n-2))(\lambda-2)^{n-1} = 0$ Hence, spectrum is $\mathrm{Spec}_{P_2}^E(K_{1,n-1}) = \begin{pmatrix} -(n-2) & 2 \\ 1 & (n-1) \end{pmatrix}$. Therefore, $E_{P_2}^E(K_{1,n-1}) = 3n-4$.

Theorem 4.8. *The minimum equitable dominating 2-partition energy of complete bipartite graph $K_{n,n}$ is*

$$E_{P_2}^E(K_{n,n}) = (2n-1) + \sqrt{4n^2+4n-7}.$$

Proof. Consider the complete bipartite graph $K_{n,n}$ whose vertex set is partitioned into $U_n = \{u_1, u_2, \dots, u_n\}$, $V_n = \{v_1, v_2, \dots, v_n\}$. The minimum equitable ominating set $= E = \{u_1, v_1\}$. The minimum equitable dominating 2-partition matrix is

$$P_2^E(K_{n,n}) = \begin{bmatrix} 1 & -1 & -1 & -1 & \dots & 1 & 1 & 1 & 1 \\ -1 & 0 & -1 & -1 & \dots & 1 & 1 & 1 & 1 \\ -1 & -1 & 0 & -1 & \dots & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 0 & \dots & 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 & \dots & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & \dots & -1 & 0 & -1 & -1 \\ 1 & 1 & 1 & 1 & \dots & -1 & -1 & 0 & -1 \\ 1 & 1 & 1 & 1 & \dots & -1 & -1 & -1 & 0 \end{bmatrix}.$$

Characteristic equation is

$$(\lambda-1)^{2n-3}(\lambda-2)[\lambda^2+(2n-3)\lambda-(4n-4)] = 0.$$

Hence, spectrum is $\mathrm{Spec}_{P_2}^E(K_{n,n}) = \begin{pmatrix} 1 & 2 & \frac{-(2n-3)+\sqrt{4n^2+4n-7}}{2} & \frac{-(2n-3)-\sqrt{4n^2+4n-7}}{2} \\ (2n-3) & 1 & 1 & 1 \end{pmatrix}$.

Therefore, $E_{P_2}^E(K_{n,n}) = (2n-1) + \sqrt{4n^2+4n-7}$. □

Theorem 4.9. *The minimum equitable dominating* 2*-partition energy of complete bipartite graph* $K_{m,n}$ *is*

$$E^E_{P_2}(K_{m,n}) = 3(n+m)-4.$$

Proof. Consider the complete bipartite graph $K_{m,n}$ whose vertex set is partitioned into $U_m = \{u_1, u_2, \dots, u_m\}$, $V_n = \{v_1, v_2, \dots, v_n\}$. The minimum equitable dominating set $= E = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n,\}$. The minimum equitable dominating 2-partition matrix is

$$P^E_2(K_{m,n}) = \begin{bmatrix} 1 & -1 & -1 & -1 & \dots & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & -1 & \dots & 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & -1 & \dots & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 1 & \dots & 1 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 & \dots & 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & 1 & \dots & -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & \dots & -1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & \dots & -1 & -1 & -1 & 1 \end{bmatrix}.$$

Characteristic equation is

$$(\lambda+n+m-2)(\lambda-2)^{n+m-1} = 0.$$

Hence, spectrum is $\text{Spec}^E_{P_2}(K_{n,n}) = \begin{pmatrix} -(n+m-2) & 2 \\ 1 & n+m-1 \end{pmatrix}$.

Therefore, $E^E_{P_2}(K_{m,n}) = 3(n+m)-4$. □

5. Conclusion

In this paper, we tried to connect the three important aspects of graph theory i.e. partitioning, equitable dominating set and spectral characterization. There is plenty of research work to be done by connecting these aspects. Further any graph theorist can enhance this Energy for real world problems. We can connect the spectral aspect with other dominating sets.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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