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Research Article

Improving Cassification Engine in Content based Image Retrieval by Multi-point Queries via Pareto Approach

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Abstract. Machine learning methods have demonstrated promising performance for Content Based Image Retrieval (CBIR) using Relevance Feedback (RF). However, a very limited number of feedback images can significantly degrade the performance of these techniques. In this work, each image is represented by a vector of multiple distance measures corresponding to multiple features. Each feature is considered a sub-query for RF process. In RF process, we propose to use Pareto method to get Pareto points (also called trade-off points) according to different depths. These points are used as relevant queries for the next RF round. Experimental results show that our proposed approach is very effective to improve the performance of the classification engine.

Keywords. Content based image retrieval; Relevance feedback; Classification; Pareto point

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1. Introduction

With the recent interest in multimedia systems (e.g. Flickr[1], YouTube[2], Facebook[3], Twitter[4]) Content Based Image Retrieval (CBIR) has attracted the attention of reseachers. The traditional

[1] http://www.flickr.com [2] http://www.youtube.com [3] http://www.facebook.com [4] http://www.twitter.com

image retrieval systems search for a queried image in large database [16, 21] by analyzing the contents of the image based on multiple features such as colors, shapes, textures, or any other information that can be derived from the image itself. Combining multiple features has been effectively used in many CBIR systems such as [1, 7, 25, 27]. Image distance measure is the most common method for comparing two images in CBIR. A distance measure (also called partial distance measure) is used to evaluate the similarity of two images with respect to the dimensions that were considered and associated with a weight. The overall distance between the two images is computed by linearly combining partial distance measures. Search results then can be sorted based on their distance to the queried image. CBIR systems can make use of Relevance Feedback (RF) in which the user refines the search results by marking resulting images as relevant or irrelevant (also called positive/negative) in his/her conceptual image, then repeating the search with the new information [12, 28]. This process is repeated until the user is satisfied with the retrieval results or it reaches to a predefined number of iterations. However, image characterization and similarity measure may not follow perceptual characteristic. This explain why in some cases images with similar semantics could be scattered in distinct neighborhoods in the feature space. In order to reduce the semantic gap between low-level visual features and the high-level concepts conveyed by the query images, the Relevance Feedback (RF) technique proposed in [5, 12, 26] represent the query image as a single point in the feature space. During every round of RF, the centroid of the relevant images is used as the new query point in the next round.

Remark 1. In CBIR using multiple features or multiple queries, they consider each feature or query as a sub-query. Thus, in order to get appropriate resulting images, they need merge results from these sub-queries. In the case that the importance of each sub-query is unknown in advance, an issue is how to rank these resulting images where each image is a multi-dimensional distance measure vector obtained from the sub-queries. Using weights with partial distance measures to estimate the overall distance measure is fair only when the images are linearly ordered based on partial distance measures. However, this strategy is likely to miss some relevant images.

For query expansion, Porkaew *et al*. [23] represent a new query as multiple points to determine the shape of the contour which bounds query results (relevant images) based on the user's relevance judgement. However, the relevance images can be mapped to disjoint clusters because the feature space and distance measure of the user's intention may different from those of the systems. That discrepancy would lead to poor results. Kim *et al*. [18] improve this idea by using adaptive classification and cluster-merging. Their method uses aggregate distance function to rank returned results that would lead to missing some images which have minimal distance in one or several individual low-level feature spaces. Another disadvantage of both methods is that if the user do not select any relevant image then there will not exist the bound of the relevant images. In recent years, machine learning techniques are widely used to significantly improve classification engines in the CBIR [9, 13, 31–34, 36, 37]. These methods are proven to give better performance on classification than the standard Rocchio [24] that Rui *et al*. [26] used before. CBIR systems use machine learning methods to accomplish a learning task which classifies the images in the database as possitive or negative images. However, this learning task has some difficulties, i.e. small sample, real-time requirement due to the fact that very few users will be willing to provide sufficient images in the RF process. This means that most popular machine learning methods can hardly be applied to CBIR. In addition, these methods only consider the query as a single point, then compute the overall distance measures by linearly combining the partial measures between the query image with each image in database. This approach can face the problem mentioned in Remark 1 above. One of the keys to CBIR systems is how to measure the similarity of the two images in their low-level visual features. In practice, the images that are semantically related to a subject can be scattered in the visual feature space. In other words, it is possible that two images that are far away separated are more similar in their semantic content than two images that are close to each other. We argue that, the images that are semantically related to each other are likely to have a minimal distance measure in at least one feature space. The idea of using the Pareto approach in this paper is mainly based on this observation. In our method, we do not find optimal queries or modify the weights of the distance functions of the current relevant images which are used as next queries (as in [23]). All relevant results are used as query points for next round. Futhermore, to make sure that there exists relevant images for the next round, Pareto approach is proposed to create a bound on the set of possible relevant points or trade-off points. More intuitively, let us consider a detailed example as follows:

Example 1. Given a query image Q and three images o_1, o_2, o_3. The distances of the query image Q from three images according to features of color and texture are shown in Table 1. It is obviously easy to rank the order of images o_1, o_2, o_3 based on the overall distance measures (sum in this case). However, we cannot rank objects only based on partial distance measures, i.e. o_1 can be compared with o_2 whereas o_3 cannot be compared with the rest.

Table 1. The distances of Q from o_1, o_2, and o_3 in Color and Texture features

Image	Color (C)	Texture (T)	Sum
o_1	0.6	0.3	0.9
o_2	0.5	0.2	0.7
o_3	0.45	0.35	0.8

In this article, we propose to use the Pareto approach to collect all Pareto optimal points which minimize at least one partial distance measure (e.g., o_1 is the minimizer of color based partial distance measure) and optimal points which achieve a minimum score on overall distance measure (e.g., linear combination of distance measure). In Example 1, the trade-off curve illustrates the boundary which contains Pareto points of Pareto front or trade-off points (see Figure 1). The rest of this paper is organized as follows. Section 2 surveys related works using the multiple queries approach in CBIR. In Section 3, we formulate mathematical propositions of the Pareto set to get trade-off points from a large database and their proofs as well. In Section 4 we show the main experiments. Finally, the conclusion and future works are given in Section 5.

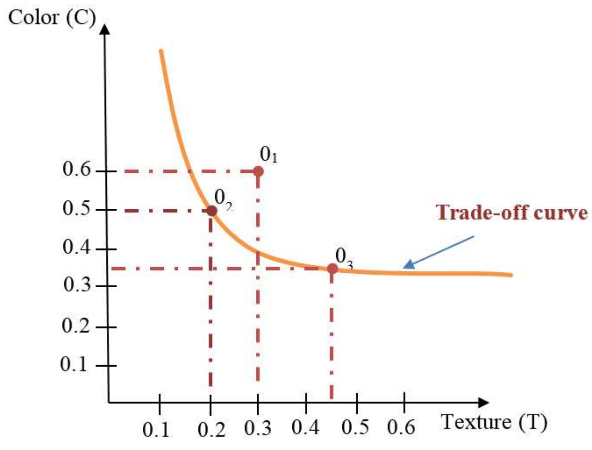

Figure 1. Description trade-off points from Example 1

2. Related Works

There are two main approaches to use multiple queries in traditional CBIR systems. One way is to consider a single query image with multiple features and to treat each feature as an independent sub-query. The other uses multiple query images as multiple queries. The first approach used in MARS [28] and MindReader [12] represents a query by a single point in each feature space and tries to move this point toward points marked relevant by the user whereas MARS [23], FALCON [35], QCluster [17] and [3,14] use the second one which partitions the relevant points according to user interaction into clusters and makes up centroids as a new query. However, to improve retrieved results for next iterations, MARS, FALCON and QCluster often require the user to determine a number of relevant images. In extreme cases, the CBIR engines have to work in a narrow search space that leads to poor retrieved results. In order to overcome this issue, Jin *et al*. [14] expand the search space to get more relevant images by combining multiple systems and representations. The recent works in CBIR use multiple input queries such as [2, 10, 15]. Joseph *et al*. [15] proposed to use multiple input queries and then logical operations for the query images to produce aggregated query. Hsiao *et al*. [10] proposed a multiple queries information retrieval algorithm that combines the Pareto front method (PFM) with efficient manifold ranking (EMR). To choose the final results, they use the Pareto depth approach. This is an effective approach to compare the partial distances measure in multi-objective optimization problem but just on small-scale database. In recent years, machine learning and ensemble methods have been widely used [6, 31, 32, 34, 36, 37]. These methods have demonstrated promising performance for CBIR using RF, when a sufficient number of labeled images are marked by the users. However, users typically mark a very limited number of feedback images during the RF process, and this issues can significantly degrade the performance of these techniques. Furthermore, the ensemble methods to learn user's intention in both negative and positive images often slow down speed of classifiers or even need many feedback rounds. Unlike previous approaches, our proposed framework use multiple representations (multiple point query [23]) to expand queries on visual perception and make effectively use of relevant images from the feedback results for the next round. We consider each feature as a sub query to use effectively Pareto approach.

3. The Proposed Method

3.1 Problem Statement

First, we formalize the problem as follows: suppose $\{\mathbb{E}_i^T \mid i = \overline{1,M}\}$ is a feature database extracted from M images and consists of color, texture and shape features. Thus, each image is represented by a tuple of T features, i.e., $I = (I^1, \dots, I^t, \dots, I^T)$. A query image Q is processed in the same way as the images of the database, i.e., $Q = (Q^1, \dots, Q^t, \dots, Q^T)$. We consider Q as multiple queries in which each Q^t is an individual query. The distance measure between the query image Q and image I is a vector $D_Q(I)$, defined by

$$(D_Q^1(I), \dots, D_Q^t(I), \dots, D_Q^T(I)), \tag{1}$$

where $D_Q^t(I) = D(Q^t, I^t)$ is corresponding distance of tth feature (also called partial distance measure in this article). The search space of multiple queries which is given by:

$$\mathbb{S}_Q = \{(I, D_Q(I))/I \in \mathbb{E}\}, \tag{2}$$

There exists a map π_Q, that is bijective in the search space $\mathbb{S}_Q$, that is

$$\begin{aligned} \pi_Q : \mathbb{S}_Q &\to \mathbb{E} \\ (I, D_Q(I)) &\mapsto I \end{aligned} \tag{3}$$

For simplicity, when Q is fixed we assume that $I \equiv \pi_Q(I) \in \mathbb{E}$ and $A \equiv \{\pi_Q(I)/\ \forall\ I \in A\} \subset \mathbb{E}$, $\forall\ I \in \mathbb{S}_Q$, $\forall\ A \subset \mathbb{S}_Q$.

3.2 The Pareto concept in multiple queries search space

In image retrieval by sample, given a query image, the image in the database which is identical or semantically similar to it is called *ideal point*. Relevance feedback in CBIR is a powerful tool to fill the gap between low level features and high level concepts, and also to use subjectivity of the human perception of images (or concept image), against a limited number of retrieved images.

Definition 1 (Ideal Point)**.** *Let I be a point in $\mathbb{S}_Q$ and its distance measure $D_Q(I)$. The point $I_{ideal} \in \mathbb{S}_Q$ is ideal iff*

$$\forall\ t = \overline{1,T} : D_Q^t(I_{ideal}) \approx \min_{I \in \mathbb{S}_Q} D_Q^t(I), \tag{4}$$

Multi-objective optimization approaches require all the objectives to be simultaneously optimized (minimum) for each criteria $D_Q^t(I)$ in a solution $D_Q(I)$. The multi-objective problem in the search space is defined as follows:

$$\begin{cases} D_Q(I) = (D_Q^t(I))_{t=1}^T \to D_Q(I_{ideal}) \\ \text{subject to } D_Q^t(I) \in \mathbb{S}_Q \end{cases}, \tag{5}$$

However, an ideal point I_{ideal} simultaneously optimizing all criteria usually may not exist. We intend to find the set of tradeoff solutions that offer different compromises among criteria, instead. A solution $D_Q(I)$ is optimal if there is no other solution in the search space that achieves partial distance measures smaller than $D_Q(I)$ on every criterion $D_Q^t(I)$, which implies that we should try to determine points belonging to the extracted Pareto front. As multi-objective optimization problem is based on finding a set of tradeoffs among potential solutions. We need a relaxed definition of dominance. An efficient solution does not dominate others in every criteria. On the other hand, it is not possible to find another solution that simultaneously improves the whole set of criteria regarding an existing efficient solution.

3.3 The formal definitions and properties of Pareto approach in multiple queries search space

As defined above, the query image is represented as $Q = (Q^1,\dots,Q^t,\dots,Q^T)$, where each Q^t is considered as a sub query. We adopt Pareto approach in search space $\mathbb{S}_Q = \{(I, D_Q(I))/I \in \mathbb{E}\}$, and then we formalize the problem by definitions and properties in this the space.

Definition 2 (Pareto dominance)**.** *Let I_1 and I_2 be two points of the search space $\mathbb{S}_Q$, I_2 is Pareto dominated by I_1 (noted $I_1 \prec_Q I_2$) iff*

$$\begin{cases} \forall\ t = \overline{1,T}, D_Q^t(I_1) \le D_Q^t(I_2), \\ \exists\ t_0 \in [1,T] : D_Q^{t_0}(I_1) < D_Q^{t_0}(I_2). \end{cases} \tag{6}$$

According to this definition, it is clear that if two points I_1 and I_2 in the search space $\mathbb{S}_Q$, satisfy this property $I_1 \prec_Q I_2$, then I_1 is more relevant than I_2 with respect to Q.

Example 2. Consider again the relation of Example 1, $o_2 \prec_Q o_1$ because $0.5 < 0.6$ and $0.2 < 0.3$.

Proposition 1. *Given I_1, I_2 and I_3 in $\mathbb{S}_Q$. We have:*

(1.1) $I_1 \prec_Q I_2 \Rightarrow I_2 \not\prec_Q I_1$.

(1.2) $I_1 \prec_Q I_2, I_2 \prec_Q I_3 \Rightarrow I_1 \prec_Q I_3$.

(1.3) $I_1 \prec_Q I_2 \Rightarrow Agg(D_Q(I_1)) < Agg(D_Q(I_2))$ *, where Agg is an aggregation operator.*

Proof. (1.1) $I_1 \prec_Q I_2 \Rightarrow\ \exists\ t_0 \in [1,T] : D_Q^{t_0}(I_2) < D_Q^{t_0}(I_1) \Rightarrow I_2 \not\prec_Q I_1$

(1.2) $I_1 \prec_Q I_2 \Rightarrow (D_Q^t(I_1) \le D_Q^t(I_2)) \wedge (\exists\ t_0 \in [1,T] : D_Q^{t_0}(I_1) < D_Q^{t_0}(I_2))$.

$I_2 \prec_Q I_3 \Rightarrow (D_Q(I_2) \le D_Q(I_3)) \wedge (\exists\ t_0 \in [1,T] : D_Q^{t_0}(I_2) < D_Q^{t_0}(I_3))$.

Therefore, $(D_Q(I_1) \le D_Q(I_3)) \wedge (D_Q^{t_0}(I_1) < D_Q^{t_0}(I_3)) \Rightarrow I_1 \prec_Q I_3$.

(1.3) According to the definition of the aggregation operation (see [4]),
if $I_1 \prec_Q I_2 \Rightarrow Agg(D_Q(I_1)) < Agg(D_Q(I_2))$. □

Definition 3 (Pareto front)**.** Given $A \subset \mathbb{S}_Q$, the Pareto front of A (noted $PF_Q(A)$) is defined as:

$$PF_Q(A) \stackrel{def}{=} \{I \in A \mid \nexists\ I' \in A : I' \prec_Q I\} \subset A\,. \tag{7}$$

The Pareto front or Pareto set is the set containing all points having at least one minimal distance. These points are called Pareto optimal points or trade-off points.

Example 3. (3.1) Consider again the relation of Example 1, $PF_Q(\mathbb{S}_Q) = \{o_2, o_3\}$, because they are not dominated by any point.

(3.2) $A \subset \mathbb{S}_Q, D_Q(I_1) = D_Q(I_2), \forall I_1, I_2 \in A \Rightarrow PF_Q(A) \equiv A$.

Proposition 2. (2.1) $\forall I \in \mathbb{S}_Q$ *if* $\exists t_0 \in [1,T], D_Q^{t_0}(I) < D_Q^{t_0}(I'), \forall I' \neq I, I \in PF_Q(\mathbb{S}_Q)$.

(2.2) $\forall A \subset \mathbb{S}_Q, w_1, w_2, \dots, w_T \in (0,1), \sum_{t=1}^{T} w_t = 1,$ *if* $I_0 = \arg\min_{I \in A} \sum_{t=1}^{T} w_t D_Q^t(I)$ *then* $I_0 \in PF_Q(A)$.

(2.3) $\forall A \subset \mathbb{S}_Q, A \neq \emptyset \Rightarrow PF_Q(A) \neq \emptyset$.

(2.4) $\forall A \subset \mathbb{S}_Q, \forall I \in A \setminus PF_Q(A) \Rightarrow \exists J \in PF_Q(A) : J \prec_Q I$.

Proof. (2.1) We prove by contradiction.

Let $I \notin PF_Q(\mathbb{S}_Q) \Rightarrow \exists I' \in \mathbb{S}_Q, D_Q^t(I') < D_Q^t(I) \Rightarrow D_Q^{t_0}(I') < D_Q^{t_0}(I)$ it is a contradiction, because $D_Q^{t_0}(I) = \min_{I' \in \mathbb{S}_Q} D_Q^{t_0}(I')$.

(2.2) $Agg : [0,1]^T \to [0,1]$

$(d_1, d_2, \dots, d_T) \mapsto \sum_{t=1}^{T} w_t d_t$.

This is an aggregation operator, thus if $I_0 \notin PF_Q(A)$, $\exists\ I \in A$, $I \neq I_0$, $I \prec_Q I_0 \Rightarrow Agg(I) < Agg(I_0)$ it is a contradiction, because $I_0 = \arg\min_{I \in A} Agg(I)$, so that $I_0 \in PF_Q(A)$.

(2.3) Put $I_0 = \arg\min_{I \in A} \sum_{t=1}^{T} \frac{1}{T} D_Q^t(I) \Rightarrow I_0 \in PF_Q(A) \Rightarrow PF_Q(A) \neq \emptyset$.

This result is a condition on (2.2).

(2.4) $N_I = \{k \in N^+ / \exists \{I_1, I_2, \dots, I_k\} \subset A, I_k \prec_Q I_{k-1} \prec_Q \dots \prec_Q I_0 = I, I_{k-1} \notin A\}$, therefore $I_0 \in A \setminus PF_Q(A) \Rightarrow \exists I_1 \in A : I_1 \prec_Q I_0 \Rightarrow 1 \in N_I \Rightarrow N_I \neq \emptyset, N_I \subset \{1, 2, \dots, \#A\} \Rightarrow \exists k_0 = \max N_I$. If $I_{k_0} \notin PF_Q(A)$ then $I_{k_0} \in A \wedge I_{k_0} \notin PF_Q(A) = \{I \in A / \nexists I' \in A \wedge I' \prec_Q I\} \Rightarrow \exists I' \in A \wedge I' \prec_Q I_{k_0} \Rightarrow \{I_1, I_2, \dots, I_{k_0}, I'\} \subset A, I' \prec_Q I_{k_0} \prec_Q I_{k_0 - 1} \prec_Q \dots \prec_Q I_1 = I_0 \Rightarrow k_0 + 1 \in N_I$. This is a contradiction, because $k_0 = \max N_I$. Put $J = I_{k_0} \in PF_Q(A), J \prec_Q I$ (by (1.3)). □

Example 4. In Example 1, $D_Q^{Texture}(o_2) = 0.2 = \min\{D_Q^{Texture}(o_1), D_Q^{Texture}(o_3)\} \Rightarrow o_2 \in PF_Q(\mathbb{S}_Q)$.

Definition 4 (Pareto depth)**.** (4.1) The l^{th} Pareto depth is defined as:

(i) $PFD_Q^0 = \emptyset$,

(ii) $PFD_Q^l \stackrel{def}{=} PF_Q(\mathbb{S}_Q \setminus \cup_{j=1}^{l-1} PFD_Q^j)$.

(4.2) Depth value: $\forall I \in \mathbb{S}_Q$, $depth_Q(I) \stackrel{def}{=} l \in N^+ \wedge l \leq \#\mathbb{S}_Q : I \in PFD_Q^l$.

Remark 2. $PFD_Q^1 = PF_Q(\mathbb{S}_Q)$.

Example 5. Consider again the relation of Example 1: $PFD_Q^1 = PF_Q(\mathbb{S}_Q) = \{o_2, o_3\}$.
$PFD_Q^2 = PF_Q(\mathbb{S}_Q \setminus PFD_Q^1) = PF_Q(\mathbb{S}_Q \setminus \{o_2, o_3\}) = PF_Q(\{o_1\}) = \{o_1\}$.

Example 6. Consider again the relation of Example 1, $\mathbb{S}_Q = \{o_1, o_2, o_3\}, o_2 \prec_Q o_1 \Rightarrow PFD_Q^2 = \{o_1\}$. If $\mathbb{S}_Q = \{I_1, I_2, \dots, I_k\}$, $I_1 \prec_Q I_2 \prec_Q I_3 \prec_Q \dots \prec_Q I_k$ then which mean that $PFD_Q^l = \{I_l\}$, $\forall\ l = \overline{1,k}$.

There are some other important properties of the Pareto front according to different depths which are described as follows:

Proposition 3. (3.1) $\forall l \neq k, PFD_Q^l \cap PFD_Q^k = \emptyset$.

(3.2) $\exists l \in N^+, l \leq \#\mathbb{S}_Q : PFD_Q^k = \emptyset \ \ \forall k > l$ *and* $\bigcup_{j=1}^{l} PFD_Q^j = \mathbb{S}_Q$.

(3.3) $l \geq 1, \forall I_1, I_2 \in PFD_Q^l \Rightarrow I_1 \not\prec_Q I_2 \wedge I_2 \not\prec_Q I_1$.

(3.4) *If* $\forall I \in PFD_Q^{l+1}, l \geq 1$ *then there exists* $J \in PFD_Q^l : J \prec_Q I$.

(3.5) *The definition 4.2 is valid. If* $I \in \mathbb{S}_Q$ *then there exists a unique* l, $1 \leq l \leq \#\mathbb{S}_Q$ *such that* $I \in PFD_Q^l$.

(3.6) $I_1 \prec_Q I_2 \Rightarrow depth_Q(I_1) < depth_Q(I_2)$.

(3.7) $\forall I \in \mathbb{S}_Q, depth_Q(I) = k \Rightarrow \exists I_1, \ldots, I_k \in \mathbb{S}_Q : I_1 \prec_Q I_2 \prec_Q \ldots \prec_Q I_{k-1} \prec_Q I_k = I$.

(3.8) $\forall I \in \mathbb{S}_Q, depth_Q(I) = \max\{p \in / \exists I_1, \ldots, I_p \in \mathbb{S}_Q : I_1 \prec_Q \ldots \prec_Q I_p = I\}$.

Proof. (3.1) Assume $l > k, PFD_Q^l = PF_Q(\mathbb{S}_Q \setminus \bigcup_{j=1}^{l-1} PFD_Q^j) \subset (\mathbb{S}_Q \setminus \bigcup_{j=1}^{l-1} PFD_Q^j) =$ $(\mathbb{S}_Q \setminus (PFD_Q^k \cup \bigcup_{1\leq j \leq l-1, j\neq k} PFD_Q^j)) \subset (\mathbb{S}_Q \setminus PFD_Q^k)$, which mean that $PFD_Q^l \cap PFD_Q^k = \emptyset$.

(3.2) Put $M = \#\mathbb{S}_Q$, we have $M+1$ subsets of $\mathbb{S}_Q$: $\{PFD_Q^l\}_{l=1}^{M+1}, \forall 1 \leq l < k \leq M+1$ imply $PFD_Q^l \cap PFD_Q^k = \emptyset$ (By (3.1)), therefore $PFD_Q^1 = PF_Q(\mathbb{S}_Q) \neq \emptyset$ so that $\exists l : 1 \leq l \leq M \wedge PFD_Q^{l+1} = \emptyset \wedge PFD_Q^l \neq \emptyset$, $PFD_Q^{l+1} = PF_Q(\mathbb{S}_Q \setminus \bigcup_{j=1}^{l} PFD_Q^j) = \emptyset \Rightarrow (\mathbb{S}_Q \setminus \bigcup_{j=1}^{l} PFD_Q^j) = \emptyset$ by (2.3) $\Rightarrow \bigcup_{j=1}^{l} PFD_Q^j = \mathbb{S}_Q$. On the one hand $\forall k > l, PFD_Q^k = PF_Q(\mathbb{S}_Q \setminus \bigcup_{j=1}^{k-1} PFD_Q^j) = PF_Q(\mathbb{S}_Q \setminus \mathbb{S}_Q) = PF_Q(\emptyset) = \emptyset$.

(3.3) Put $A = \mathbb{S}_Q \setminus \cup_{j=1}^{l-1} PFD_Q^j$ therefore $I_1, I_2 \in PF_Q(A) = \{I \in A / \nexists I' \in A : I' \prec_Q I\}$ so that $I_1 \not\prec_Q I_2$ and $I_2 \not\prec_Q I_1$.

(3.4) Put $A = (\mathbb{S}_Q \setminus \cup_{j=1}^{l-1} PFD_Q^j) \Rightarrow PFD_Q^l = PF_Q(A)$, therefore $(\mathbb{S}_Q \setminus \cup_{j=1}^{l} PFD_Q^j) = A \cap (\mathbb{S}_Q \setminus PFD_Q^l), I \in PFD_Q^{l+1} \Rightarrow (I \in A \wedge I \notin PF_Q(A))$, so that $\exists J : J \in PFD_Q^l \wedge J \prec_Q I$.

(3.5) We deduce from (3.1) and (3.2).

(3.6) Assume that $k = depth_Q(I_1) \geq l = depth_Q(I_2)$. Put $A = (\mathbb{S}_Q \setminus \cup_{j=1}^{l-1} PFD_Q^j) \Rightarrow I_2 \in PFD_Q^l = PF_Q(A)$. On the other hand $I_1 \in PF_Q(\mathbb{S}_Q \setminus \cup_{j=1}^{k-1} PFD_Q^j) \subset (\mathbb{S}_Q \setminus \cup_{j=1}^{k-1} PFD_Q^j) \subset (\mathbb{S}_Q \setminus \bigcup_{j=1}^{l-1} PFD_Q^j) = A (k \geq l)$. Therefore $I_2 \in PF_Q(A) \wedge (I_1 \in A) \wedge (I_1 \prec_Q I_2)$, it is a contradiction.

(3.7) By proposition (3.6), $\exists I_{k-1} \in PFD_Q^{k-1}$: $I_{k-1} \prec_Q I_k = I$. Applying similar process to I_{k-1}, $\exists I_{k-2} \in PFD_Q^{k-2} : I_{k-2} \prec_Q I_{k-1}, \ldots, \exists I_1 \in PFD_Q^1 : I_2 \prec_Q I_1$, so that $I_1 \prec_Q I_2 \ldots \prec_Q I_{k-1} \prec_Q I_k = I$.

(3.8) By proposition (3.4) $\Rightarrow depth_Q(I) = k \leq \max\{p \in N^+ / \exists \{I_1, \ldots, I_p\} \subset \mathbb{S}_Q : I_1 \prec_Q I_2 \prec_Q \ldots \prec_Q \ldots \prec_Q I_p = I\} = p_0$. On the other hand $\exists \{I_1, \ldots, I_{p_0}\} \subset \mathbb{S}_Q : I_1 \prec_Q I_2 \prec_Q \ldots. \prec_Q \ldots \prec_Q I_{p_0} = I$. Because of $\forall l = \overline{1, p_0 - 1}$, $depth_Q(I_l) < depth_Q(I_{l+1})$, so $k = depth_Q(I)$ $= depth_Q(I_1) + \sum_{l=1}^{p_0-1} \big(depth_Q(I_{l+1}) - depth_Q(I_l)\big) \geq 1 + \sum_{l=1}^{p_0-1} 1 = p_0$. Conclusion $k = p_0$. □

By propositions (3.7) and (3.8), we prove that for any point in the search space $\mathbb{S}_Q$ there always exists a target point stated in Theorem 1.

Theorem 1 (Dominant path)**.** *For all point I in the search space $\mathbb{S}_Q$, there are always at least $depth_Q(I)-1$ other points in $\mathbb{S}_Q$ which are more relevant than I with respect to the query point Q.*

Proof. Put $k = depth_Q(I)$ by proposition (3.7) $\Rightarrow \exists I_1,\ldots,I_k \in \mathbb{S}_Q : I_1 \prec_Q I_2 \prec_Q \ldots \prec_Q I_{k-1} \prec_Q I_k = I$. □

According to this theorem, for any point in the search space there always exists a dominant path. It also shows that the depth of point I in the search space $\mathbb{S}_Q$ is the length of the longest dominant path started from I.

Definition 5 (Pareto Union)**.** Given $\mathbb{E}_A \subset \mathbb{E}$ and L is the depth of the Pareto front, the Pareto Union of the subset $\mathbb{E}_A$ (denoted $PFU_L(\mathbb{E}_A)$) is defined as

$$PFU_L(\mathbb{E}_A) \stackrel{def}{=} \bigcup_{a\in\mathbb{E}_A,1\le l\le L} PFD_a^l, \tag{8}$$

Proposition 4. $\forall \mathbb{E}_A \subset \mathbb{E}, \forall L \in N^+ : \mathbb{E}_A \subset PFU_L(\mathbb{E}_A)$.

Proof. $\forall a \in \mathbb{E}_A, L \in N^+, a \in PFD_a^1 \subset \bigcup_{a\in\mathbb{E}_A,1\le l\le L} PFD_a^l \Rightarrow \mathbb{E}_A \subset \bigcup_{a\in\mathbb{E}_A,1\le l\le L} PFD_a^l = PFU_L(\mathbb{E}_A)$. □

Proposition 5. $\forall \mathbb{E}_A, \exists L \in N^+ : PFU_L(\mathbb{E}_A) \subseteq \mathbb{E}$.

Proof. Based on the Definition 5, $PFU_L(\mathbb{E}_A) = \bigcup_{a\in\mathbb{E}_A,1\le l\le L} PFD_a^l \subseteq \mathbb{E}$ holds unconditionally. □

3.4 Improvement of classification engines based on Pareto front

It is worth noting that the trade-off points which are at the same depth do not dominate each other in the search space $\mathbb{S}_Q$. Moreover, it consists of not only points minimizing linearly combined distance measure, but also points minimizing at least one distance measure with respect to a certain feature. As previously mentioned, each feature Q^t in multiple queries image $Q = \{Q^t\}_{t=1}^T$ is considered as a sub-query. Initially, each image in the database has a distance measure with each sub-query of the query image (i.e. the distance between this sub-query and corresponding feature of the image). All Pareto points (trade-off points) are extracted at different depths and then given scores by a classifier [37]. Top k results with best scores from this set are then used to make a training dataset NB, shown to the user and marked "-1" or "$+1$" corresponding to irrelevant or relevant by his or her perception. The training dataset is therefore, divided into NB^+ and NB^-, i.e., $T = \{NB^+, NB^-\} = \{(x_1,y_1),\ldots,(x_l,y_l)\} \in (R^n \times Y)^l$, where $x_i \in R^n$, class labels $y_i \in Y = \{-1,1\}$, and l is the training dataset size. Our contributions are focused on two algorithms. The first algorithm *PFDA* aims at obtaining a set of the trade-off points at L different depths and its implementation is based on the Definitions 3 and 4. The second algorithm *CUPF* works on the search space of trade-off points returned by *PFDA*. After each RF round, the set of relevance images NB^+ is expanded by considering each point in this set as a sub-query. Then, more Pareto points obtained by *PFDA* are included to the set NB^+. As proved in the Proposition 5, the size of this union set is smaller than or equal to that of the original database. After this step, a new SVM is trained for the next RF round (see Algorithms 1 and 2).

Algorithm 1 PFDA (Pareto Front Depth Algorithm)

Input: $\{\mathbb{S}_{Q_t}/t = \overline{1,T}\}$ ▷ $\mathbb{S}_{Q_t}$ contains M points, each point has T dimensions

L ; ▷ depth of the Pareto front

K ; ▷ The number of points in the Pareto depth set

Variables: $PF = PF\_Next = \emptyset; (threshold_t)_{t=1}^{T}; aMax = 0; depth = 0;$

while $depth < L \wedge \#PointSet < K$ **do**

  while $\nexists I_i \in PF$ that $I_i \prec_Q threshold$ **do**

    for each sub-query $Q_t, t = \overline{1,T}$ **do**

      Get I_i from top ranked list $\mathbb{S}_{Q_t}$ that is not marked;

      if $aMax < D_Q^t(I_i)$ **then** $aMax = D_Q^t(I_i)$;

      end if

      $isDominated = false$;

      while not(isDominated)$\wedge(\exists I_j \in PF)$ unmatched with I_i **do**

        if $I_i \prec_Q I_j$ **then** Move I_j from PF to PF_Next;

        end if

        if $I_j \prec_Q I_i$ **then**

          $isDominated = true$; Insert I_i into PF_Next;

        end if

      end while

      if not(isDominated) **then** Insert I_i into PF;

      end if

      $threshold_t = aMax$; ▷ reset threshold at t

      Select $I_i \in PF$ that $aTupleMax \nprec_Q I_i$ and insert it into $PointSet$

      update $depth$;

    end for

  end while

  if $\#PointSet < K$ **then**

    $PF = PF\_Next; PF\_Next = \emptyset$;

    for all $I_i, I_j \in PF$ that $I_i \prec_Q I_j$ **do**

      Move I_j from PF to PF_Next;

    end for

    Select images $I_i \in PF$ that $threshold \nprec I_i$ and Insert it into $PointSet$;

    update $depth$;

  end if

end while

Output: $PointSet$ ▷ Pareto depth set containing trade-off points according to front's depths

Algorithm 2 CUPF -Classification Union Pareto Front

Input: $\mathbb{E}$; ▷ Feature database
L ; ▷ depth of the Pareto front.
K ; ▷ The number of points in the Pareto front set
1: **Initialize:**
$listNB^+ \leftarrow Q$; ▷ Assign Q as a positive image
$listNB^- \leftarrow I_j$; ▷ Assign a random image as negative
Train initial SVM
2: **while** User is not satisfied **do**
3: $PFU_L(NB^+) = \emptyset$;
4: **for each** $Q_j \in NB^+$ **do**
5: Construct the search space $\mathbb{S}_{Q_j}$;
Construct Union Pareto set (see Algorithm 1 and Definition 5)
$PFU_L(NB^+) = PFU_L(NB^+) \cup PFDA(\{\mathbb{S}_{Q_j}\}, L, K)$;
6: **end for**
7: Calculate score for each image I_i in $PFU_L(NB^+)$ by trained SVM;
Rank the images in $PFU_L(NB^+)$ according to their scores;
S_k = Top k images in $PFU_L(NB^+)$;
Ask the user to mark images based on his/her perception as relevant or irrelevant.
$listNB^+ = listNB^+ \cup NB^+$;
$listNB^- = listNB^- \cup NB^-$;
Train new SVM [32].
8: **end while**
Output: Desired images;

4. Experiments and Results

For evaluating the performance of the proposed method, we carry out several experiments to make comparisons with a system using SVM [37] and MARS [23] using RF.

4.1 Image characterization

In our experiments, we extract overall six low-level features (see Table 2) to represent images.

Table 2. Image characterizations used in the experiment

Discription	Type	Dimension	Distance function
HSV Histogram [30]	Color	32	$L1$
Color moments [29]	Color	6	$L2$
Color auto correlogram [11]	Color	64	$L1$
Gabor filters [19]	Texture	48	$L2$
Wavelet moments [8]	Texture	40	$L2$
Gist [22]	Shape	512	$L2$

The database we use consists of 10000 images and can be downloaded from here[5]. Images in this database are organized according to semantic categories (The images of the same category are considered relevant and irrelevant otherwise). After feature extraction, each dimension is normalized into interval $[0,1]$ by normalization methods in [28].

4.2 Performance measures

We use two measures Precision vs. Recall and retrieved relevant images vs. number of iterations (retrieval efficiency) [20] to evaluate the effectiveness of the proposed method. Precision vs. Recall curve is a general evaluation criterion for information retrieval systems. Precision $Pr(q)$ can be defined as the number of retrieved relevant images Relevant(q) over the total number of retrieved images $N(q)$ for a given query q, namely: $Pr(q) = \frac{\mathrm{R}el(q)}{N(q)}$. Recall $Re(q)$ is the number of retrieved relevant images $Rel(q)$ over the total number of relevant images $C(q)$ present in the database for a given query q, namely:$Re(q) = \frac{\mathrm{R}el(q)}{C(q)}$. Retrieved relevant images vs. number of iterations curves are used to show the percentage of relevant images retrieved to the user given a number of RF iterations. This curve allows to evaluate how the number of retrieved relevant images grows over iterations. For iteration zero, we consider the number of relevant images retrieved in the initial set. The average Pr vs. Re and Rel vs. iterations curves, the results for all query images are used to compare the RF approaches.

4.3 Experimental results

A retrieved image is considered relevant (irrelevant) if it belongs to the same (different) category as (than) the initial query. In this simulation scheme, random 10% images in each category are used once as initial queries. For each initial query, we simulate 5 rounds of RF. In each round, the first 20 images of the ranking are shown to users and marked "-1" or "$+1$" corresponding to "irrelevant" or "relevant" by his or her subjective perception. These labeled images are then used to train a new SVM classifier for the next RF round. To evaluate the effectiveness of our proposed method, we compare it with SVM [37] and MARS [23] under the same setting. Tables 3,4 and 5 show the results of the RF simulations, i.e. the average Precision vs. Recall, relevant images during different RF rounds. Figure 2 and Figure 3 illustrate Pr vs. Re and Rel vs. Iters curves for our proposed method, SVM, and MARS.

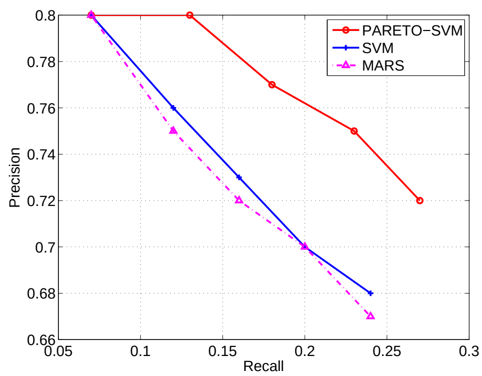

Figure 2. Precision vs. Recall

[5] http://wang.ist.psu.edu/docs/related/

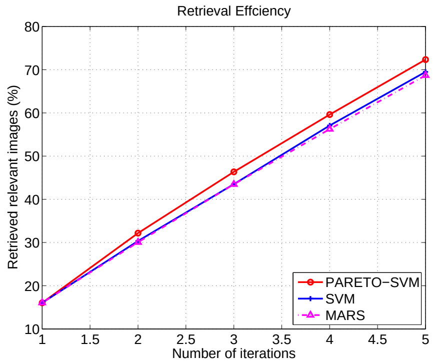

Figure 3. Retrieved relevant images vs. number of iterations

Table 3. Precision vs. Recall and Retrieved relevant images vs. number of iterations of the proposed method

(a) Precision vs. Recall

Feedback Loop	0	1	2	3	4
Precision	0.8	0.8	0.77	0.75	0.72
Recall	0.07	0.13	0.18	0.23	0.27

(b) Retrieved relevant images vs. number of iterations

Feedback Loop	0	1	2	3	4
Average of relevant images	16.04	32.18	46.36	59.61	72.33
Number of retrieval images	20	40	60	80	100

Table 4. Precision vs. Recall and Retrieved relevant images vs. number of iterations of SVM method [37]

(a) Precision vs. Recall

Feedback Loop	0	1	2	3	4
Precsion	0.8	0.76	0.73	0.7	0.68
Recall	0.07	0.12	0.16	0.2	0.24

(b) Retrieved relevant images vs. number of iterations

Feedback Loop	0	1	2	3	4
Average of relevant images	16.04	30.36	43.51	57.07	69.48
Number of retrieval images	20	40	60	80	100

Table 5. Precision vs. Recall and Retrieved relevant images vs. number of iterations of MARS method [23]

(a) Precision vs. Recall

Feedback Loop	0	1	2	3	4
Precsion	0.8	0.75	0.72	0.7	0.67
Recall	0.07	0.12	0.16	0.2	0.24

(b) Retrieved relevant images vs. number of iterations

Feedback Loop	0	1	2	3	4
Average of relevant images	16.07	30.06	43.47	56.25	68.69
Number of retrieval images	20	40	60	80	100

5. Conclusion and Future Work

This article formalizes properties of Pareto fronts in the search space and its application in CBIR systems using multiple queries. In summary, the main feature of the Pareto method is to efficiently optimize the retrieval quality of interactive CBIR. On one hand, the Pareto method can be used to help the classification engine of CBIR systems handle issues of dealing with insufficient labeled samples and real-time learning requirement from user interaction during information retrieval. On the other hand, it also overcomes difficulties of query movement or expansion techniques in MARS which appear to be properly seeded, bootstrapped. For evaluating the performance of the proposed method, the experimental data came from a subset of the Corel image database. Experimental results reveal that our proposed approach is effective in terms of precision and recall, compared to Boosting SVM and RF techniques used in MARS. In the future, there are some remaining issues to consider. First, we will continue to expand the Pareto method for reducing set of the search space. Second, in face of very large-scale datasets, we will scale our proposed method.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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