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Research Article

Common Fixed Points of Generalized Interpolative Kannan-Meir-Keeler Pair Contraction

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Abstract. In this paper, we introduce a generalized version of Meir-Keeler type contraction by incorporating interpolative conditions for a pair of mappings in metric spaces. We establish new common fixed point theorems that extend as well as unify several well-known results in the literature. Our findings generalize and enhance the recent results of Noorwali *et al*. [10].

Keywords. Kannan type contraction, Interpolation, Fixed point, Meir-Keeler contraction, Kannan-Meir-Keeler pair contraction

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1. Introduction

Fixed point theory has emerged as a powerful and indispensable tool in nonlinear functional analysis, topology and various branches of applied mathematics. Rather than reiterating its foundational importance, it is more appropriate to highlight the vast spectrum of practical applications that fixed point theory offers across diverse disciplines, including economics, computer science, optimization and qualitative analysis of differential and integral equations.

The evolution of fixed point theory has been marked by several landmark contributions. Following Banach's celebrated fixed point theorem [2], Kannan [5, 6] introduced a distinct contraction condition that significantly enriched the field. Subsequent research revealed that the Banach and Kannan contractions are, in fact, independent of each other, each capturing different nuances of contractive behavior.

Another influential advancement came from Meir and Keeler [9], who proposed a refined contraction principle based on a uniform contraction inequality, known today as the Meir-Keeler contraction [9]. This elegant generalization provided a broader framework for establishing the existence of fixed points under conditions weaker than those of classical contractions.

Motivated by these seminal works, the present paper investigates a generalized form of the Meir-Keeler contraction that elegantly combines the features of Kannan-type contractions with interpolative conditions. Specifically, we establish common fixed point theorems for a pair of mappings within this enriched framework, thereby extending and generalizing the results of Noorwali *et al*. [10].

2. Preliminaries

In this section, we recall some fundamental definitions and results that will be used throughout this paper.

Definition 2.1 ([9])**.** Let (M,d) be a complete metric space. A mapping $\mathcal{T}\colon M \to M$ is said to be a *Meir-Keeler contraction* on M if for every $\epsilon > 0$, there exists $\delta > 0$ such that

$$\epsilon \le d(p,q) < \epsilon + \delta \implies d(\mathcal{T}p,\mathcal{T}q) < \epsilon, \quad \forall\ p,q \in M. \tag{1}$$

We call the inequality (1) the *MK-contraction condition*.

Theorem 2.1 ([9])**.** *Let (M,d) be a complete metric space. Then, every MK-contraction $\mathcal{T}\colon M \to M$ has a unique fixed point.*

Kannan [5, 6] established notable fixed point results for a different class of contractions. Building on this, Karapınar [8] introduced the notion of interpolative Kannan type contractions.

Definition 2.2 ([7])**.** Let (M,d) be a complete metric space. A mapping $\mathcal{T}\colon M \to M$ is said to be an *interpolative Kannan type contraction* on M if there exist constants $\mu \in [0,1)$ and $\alpha \in (0,1)$ such that

$$d(\mathcal{T}p,\mathcal{T}q) \le \mu[d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{T}q)]^{1-\alpha}, \tag{2}$$

for every $p,q \in M \setminus \mathrm{Fix}(\mathcal{T})$, where $\mathrm{Fix}(\mathcal{T}) = \{p \in M \mid \mathcal{T}p = p\}$.

Theorem 2.2 ([7])**.** *Let (M,d) be a complete metric space. Then every interpolative Kannan contraction $\mathcal{T}\colon M \to M$ has a fixed point.*

Karapınar [8] further introduced a hybrid contraction combining features of both Kannan and Meir-Keeler contractions, known as the interpolative Kannan-Meir-Keeler type contraction.

Definition 2.3 ([8])**.** Let (M,d) be a complete metric space. A mapping $\mathcal{T}\colon M \to M$ is called an *interpolative Kannan-Meir-Keeler type contraction* if there exists $\mu \in [0,1)$ such that for all

$p,q \in M \setminus \mathrm{Fix}(\mathcal{T})$ the following conditions hold:

(i) Given $\epsilon > 0$, there exists $\delta > 0$ such that

$$\epsilon < [d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{T}q)]^{1-\alpha} < \epsilon + \delta \implies d(\mathcal{T}p,\mathcal{T}q) \le \epsilon. \tag{3}$$

(ii) For all $p,q \in M$,

$$d(\mathcal{T}p,\mathcal{T}q) \le \mu[d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{T}q)]^{1-\alpha}. \tag{4}$$

We now introduce a further generalization by allowing different exponents for the two distance terms.

Definition 2.4. Let (M,d) be a complete metric space. A mapping $\mathcal{T}: M \to M$ is said to be a *generalized interpolative type contraction* on M if there exist constants $\mu \in [0,1)$ and $\alpha,\beta \in (0,1)$ such that

$$d(\mathcal{T}p,\mathcal{T}q) \le \mu[d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{T}q)]^{\beta}, \tag{5}$$

for all $p,q \in M \setminus \mathrm{Fix}(\mathcal{T})$.

This leads to the following notion of a generalized interpolative Meir-Keeler type contraction, which merges the Meir-Keeler condition with the above generalized structure.

Definition 2.5. Let (M,d) be a complete metric space. A mapping $\mathcal{T}: M \to M$ is said to be a *generalized interpolative Meir-Keeler type contraction* on M if there exists $\mu \in [0,1)$ and parameters $\alpha,\beta \in (0,1)$ with $\alpha + \beta < 1$, such that for all $p,q \in M \setminus \mathrm{Fix}(\mathcal{T})$ the following hold:

(i) For every $\epsilon > 0$, there exists $\delta > 0$ such that

$$\epsilon < [d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{T}q)]^{\beta} < \epsilon + \delta \implies d(\mathcal{T}p,\mathcal{T}q) \le \epsilon. \tag{6}$$

(ii) For all $p,q \in M$,

$$d(\mathcal{T}p,\mathcal{T}q) \le \mu[d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{T}q)]^{\beta}. \tag{7}$$

Theorem 2.3. *Let (M,d) be a complete metric space. Then, every generalized interpolative Meir-Keeler type contraction $\mathcal{T}: M \to M$ has a fixed point.*

Finally, we recall a related result that involves a pair of mappings.

Theorem 2.4 ([10])**.** *Let (M,d) be a complete metric space and let $\mathcal{T},\mathcal{S}: M \to M$ be two mappings. Suppose there exist $\mu \in [0,1)$ and $\alpha \in (0,1)$ such that*

$$d(\mathcal{T}p,\mathcal{S}q) \le \mu[d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{S}q)]^{1-\alpha}, \tag{8}$$

for all $p,q \in M$ such that $\mathcal{T}p \ne p$ and $\mathcal{S}q \ne q$. Then $\mathcal{T}$ and $\mathcal{S}$ have a unique common fixed point.

3. Main Results

We start this section with the definition of generalized interpolative Meir-Keeler type contraction for pair of mapping.

Definition 3.1. Let (M,d) be a complete metric space. $\mathcal{T},\mathcal{S}: M \to M$ be a self-mappings. Assume that there are some $\mu \in [0,1), \alpha,\beta \in (0,1)$ such that the condition

$$d(\mathcal{T}p,\mathcal{S}q) \le \mu[d^{\alpha}(p,\mathcal{T}p)\cdot d^{\beta}(q,\mathcal{S}q)] \tag{9}$$

is satisfied $\forall\ p,q \in M$ such that $\mathcal{T}p \neq p$ whenever $\mathcal{S}q \neq q$. Then $\mathcal{S}$ and $\mathcal{T}$ have a unique common fixed point.

Theorem 3.1. *Let (M,d) be a complete metric space. $\mathcal{T},\mathcal{S}: M \to M$ be two self-mappings. Assume that there are some $\mu \in [0,1), \alpha,\beta \in (0,1)$ such that the condition*

$$d(\mathcal{T}p,\mathcal{S}q) \le \mu[d^{\alpha}(p,\mathcal{T}p).d^{\beta}(q,\mathcal{S}q)]$$

is satisfied $\forall\ p,q \in M$ such that $\mathcal{T}p \neq p$ whenever $\mathcal{S}q \neq q$. Then $\mathcal{S}$ and $\mathcal{T}$ have a unique common fixed point.

Proof. Let $p_o \in M$ and define the sequence $\{p_n\}_{n=0}^{\infty}$ by

$$p_{2n+1} = \mathcal{T}p_{2n}, p_{2n+2} = \mathcal{S}p_{2n+1}, \quad \forall\ n = \{0,1,2,3,\dots\}.$$

If $\exists\ n \in \{0,1,2,3,\dots\}$ such that $p_n = p_{n+1} = p_{n+2}$ then p_n is a common fixed point of $\mathcal{S}$ and $\mathcal{T}$. Suppose that there are three consecutive identical terms in the sequence $\{p_n\}_{n=0}^{\infty}$ and that $p_0 \neq p_1$.

Now, using (9), we deduce for $p = p_{2n}, q = p_{2n+1}$ that

$$\begin{aligned} d(p_{2n+1},p_{2n+2}) &= d(\mathcal{T}p_{2n},\mathcal{S}p_{2n+1}) \\ &\le \mu\cdot d^{\alpha}(p_{2n},p_{2n+1})\cdot d^{\beta}(p_{2n+1},p_{2n+2}). \end{aligned}$$

Thus,

$$d^{1-\beta}(p_{2n+1},p_{2n+2}) \le \mu d^{\alpha}(p_{2n},p_{2n+1})$$

or,

$$\begin{aligned} d(p_{2n+1},p_{2n+2}) &\le \mu^{\frac{1}{1-\beta}}\cdot d^{\frac{\alpha}{1-\beta}}(p_{2n},p_{2n+1}) \\ &\le \mu^{\frac{1}{1-\beta}}\cdot d^{\frac{\alpha}{1-\beta}}(p_{2n},p_{2n+1}) \\ &\le \mu d(p_{2n},p_{2n+1}), \quad \text{as } \frac{\alpha}{1-\beta} < 1,\ \alpha,\beta \in (0,1). \end{aligned} \tag{10}$$

Hence,

$$\begin{aligned} d(p_{2n+1},p_{2n+2}) &\le \mu d(p_{2n},p_{2n+1}) \\ &\le \mu^2 d(p_{2n-1},p_{2n}) \\ &\ \vdots \\ &\le \mu^k d(p_{2n-2},p_{2n-1}) \\ &\ \vdots \\ &\le \mu^{2n+1} d(p_0,p_1) \end{aligned}$$

or

$$d(p_{2n+1},p_{2n+2}) \le \mu^{2n+1} d(p_0,p_1). \tag{11}$$

Similarly, on putting $p = p_{2n}$ and $q = p_{2n-1}$, we have

$$\begin{aligned} d(p_{2n+1}, p_{2n}) &= d(\mathcal{T}p_{2n}, \mathcal{S}p_{2n-1}) \\ &\le \mu d^{\alpha}(p_{2n}, \mathcal{T}p_{2n}) \cdot d^{\beta}(p_{2n-1}, \mathcal{S}p_{2n-1}) \\ &\le \mu d^{\alpha}(p_{2n}, p_{2n+1}) \cdot d^{\beta}(p_{2n-1}, p_{2n}). \end{aligned}$$

Thus,

$$d^{1-\alpha}(p_{2n}, p_{2n+1}) \le \mu d^{\beta}(p_{2n-1}, p_{2n})$$

or

$$\begin{aligned} d(p_{2n}, p_{2n+1}) &\le \mu^{\frac{1}{1-\alpha}} d^{\frac{\beta}{1-\alpha}}(p_{2n-1}, p_{2n}) \\ &\le \mu d^{\frac{\beta}{1-\alpha}}(p_{2n-1}, p_{2n}) \quad \text{as } \frac{\alpha}{1-\beta} < 1,\ \alpha, \beta \in (0,1) \\ &\le \mu(p_{2n-1}, p_{2n}). \end{aligned}$$

Hence,

$$\begin{aligned} d(p_{2n+1}, p_{2n}) &\le \mu \cdot d(p_{2n-1}, p_{2n}) \\ &\le \mu^2 \cdot d(p_{2n-2}, p_{2n-1}) \\ &\le \mu^3 d(p_{2n-3}, p_{2n-2}) \\ &\vdots \\ &\le \mu^{2n} d(p_0, p_1). \end{aligned}$$

Thus,

$$d(p_{2n+1}, p_{2n}) \le \mu^{2n} d(p_0, p_1). \tag{12}$$

Unifying (11) and (12), we can deduce that

$$d(p_{2n+1}, p_{2n}) \le \mu^{n} d(p_0, p_1). \tag{13}$$

Now, using (13), we can prove that the sequence $\{p_n\}_{n=0}^{\infty}$ is a Cauchy sequence.
Let $m, r \in \{0, 1, 2, 3, \dots\}$,

$$\begin{aligned} d(p_m, p_{m+r}) &\le d(p_m, p_{m+1}) + d(p_{m+1}, p_{m+2}) + \dots + (p_{m+r-1}, p_{m+r}) \\ &\le \mu^m + \mu^{m+1} + \dots + \mu^{m+r-1} d(p_0, p_1) \\ &\le (\mu^m + \mu^{m+1} + \dots + \mu^{m+r-1} + \dots) d(p_0, p_1) \\ &= \frac{\mu^m}{1-\mu} d(p_0, p_1). \end{aligned}$$

Letting $m \to \infty$ we deduce that $\{p_n\}_{n=0}^{\infty}$ is a Cauchy sequence.
As M is complete, so $\exists\ u \in M$ such that $\lim_{n\to\infty} p_n = u$. Using the contrary of the metric in its both variables we may prove that u is a fixed point of $\mathcal{T}$, as follows:

$$\begin{aligned} d(\mathcal{T}u, p_{2n+2}) &= d(\mathcal{T}u, \mathcal{S}p_{2n+1}) \\ &\le \mu \cdot d^{\mu}(u, \mathcal{T}u) \cdot d^{\beta}(p_{2n+1}, p_{2n+2}). \end{aligned}$$

Letting $n \to \infty$, we get $d(\mathcal{T}u, u) = 0$ so $\mathcal{T}u = u$.

Similarly,

$$d(p_{2n+1},\mathcal{S}u)=d(\mathcal{T}p_{2n},\mathcal{S}u)$$
$$\leq \mu\cdot d^{\mu}(p_{2n},p_{2n+1})\cdot d^{\beta}(u,\mathcal{S}u).$$

Letting $n\to\infty$, we get $u=\mathcal{S}u$.

Thus, u is a common fixed point of $\mathcal{S}$ and $\mathcal{T}$. To prove that u is a unique common fixed point of $\mathcal{S}$ and $\mathcal{T}$, suppose that $v\in M$ is another common fixed point of $\mathcal{S}$ and $\mathcal{T}$.

Then

$$d(u,v)=d(\mathcal{T}u,\mathcal{S}v)\leq \mu d^{\mu}(u,\mathcal{T}u)\cdot d^{\beta}(v,\mathcal{S}v)=0.$$

Hence $u=v$. Thus, $\mathcal{S},\mathcal{T}:X\to X$ has a unique common fixed point in M. □

Definition 3.2. Let (X,d) be a complete metric space. A mapping $\mathcal{T},\mathcal{S}: M\to M$ is said to be a generalized interpolative Meir-Keeler type contraction on M, if there exist $\mu\in[0,1)$ such that for every $p,q\in M\setminus Fix(\mathcal{T})$, we have

(1) given $\epsilon>0$, there exists $\delta>0$ so that

$$\epsilon<[d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{S}q)]^{\beta}<\epsilon+\delta \implies d(\mathcal{T}p,\mathcal{S}p)\leq\epsilon, \tag{14}$$

(2) for $\alpha,\beta\in(0,1)$ with $\alpha+\beta<1$;

$$d(\mathcal{T}p,\mathcal{S}q)\leq\mu[d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{S}q)]^{\beta}. \tag{15}$$

Now we prove our second main theorem for generalized interpolative Meir-Keeler type contraction:

Theorem 3.2. *Let (M,d) be a complete metric space and let $\mathcal{T},\mathcal{S}: M\to M$ be two self-mappings satisfying the following conditions:*

(i) *Given $\epsilon>0$, there exists $\delta>0$ such that for every $p,q\in M$,*

$$\epsilon<[d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{S}q)]^{\beta}<\epsilon+\delta \implies d(\mathcal{T}p,\mathcal{S}q)\leq\epsilon;$$

(ii) *There exist constants $\mu\in[0,1)$ and $\alpha,\beta\in(0,1)$ with $\alpha+\beta<1$ such that for all $p,q\in M$,*

$$d(\mathcal{T}p,\mathcal{S}q)\leq\mu[d(p,\mathcal{T}p)]^{\alpha}[d(q,\mathcal{S}q)]^{\beta}.$$

Then $\mathcal{T}$ and $\mathcal{S}$ have a unique common fixed point in M.

Proof. Let $p_0\in M$ be arbitrary. Define the sequence $\{p_n\}$ by:

$$p_{2n+1}=\mathcal{T}p_{2n},\quad p_{2n+2}=\mathcal{S}p_{2n+1},\qquad \forall\ n\geq 0.$$

If for some n we have $p_n=p_{n+1}=p_{n+2}$, then clearly this point is a common fixed point. Otherwise, assume that no such repetition occurs.

Applying condition (ii) to $p=p_{2n}$ and $q=p_{2n+1}$:

$$d(p_{2n+1},p_{2n+2})=d(\mathcal{T}p_{2n},\mathcal{S}p_{2n+1})\leq\mu[d(p_{2n},p_{2n+1})]^{\alpha}[d(p_{2n+1},p_{2n+2})]^{\beta}.$$

Rearranging,

$$d^{1-\beta}(p_{2n+1},p_{2n+2})\leq\mu[d(p_{2n},p_{2n+1})]^{\alpha}.$$

Thus,

$$d(p_{2n+1},p_{2n+2})\leq\mu^{\frac{1}{1-\beta}}[d(p_{2n},p_{2n+1})]^{\frac{\alpha}{1-\beta}}.$$

Since $\frac{\alpha}{1-\beta} < 1$, we obtain

$$d(p_{2n+1}, p_{2n+2}) < d(p_{2n}, p_{2n+1}).$$

Similarly, using condition (ii) with $p = p_{2n}$ and $q = p_{2n-1}$,

$$d(p_{2n+1}, p_{2n}) = d(\mathcal{T}p_{2n}, \mathcal{S}p_{2n-1}) \le \mu[d(p_{2n}, p_{2n+1})]^{\alpha}[d(p_{2n-1}, p_{2n})]^{\beta}.$$

This gives

$$d^{1-\alpha}(p_{2n}, p_{2n+1}) \le \mu[d(p_{2n-1}, p_{2n})]^{\beta}.$$

Therefore,

$$d(p_{2n}, p_{2n+1}) < d(p_{2n-1}, p_{2n}).$$

Combining both results, the sequence $\{d(p_n, p_{n+1})\}$ is strictly decreasing and bounded below by zero. Thus, $\lim_{n\to\infty} d(p_n, p_{n+1}) = w \ge 0$.

To prove $w = 0$, suppose by contradiction that $w > 0$. By condition (i) (the Meir-Keeler property), there exists $\delta > 0$ such that whenever the product $[d(p, \mathcal{T}p)]^{\alpha}[d(q, \mathcal{S}q)]^{\beta}$ is sufficiently close to w, the distance between images is less than any given ϵ. This leads to a contradiction because the strictly decreasing sequence cannot stabilize at a positive value. Thus, $w = 0$.

Now we show that $\{p_n\}$ is Cauchy. Let $\epsilon > 0$ be given. Since

$$\lim_{n\to\infty} d(p_n, p_{n+1}) = 0,$$

there exists N such that for all $n \ge N$, $d(p_n, p_{n+1}) < \epsilon/2$. Using the triangle inequality recursively, we have:

$$d(p_m, p_{m+r}) \le d(p_m, p_{m+1}) + d(p_{m+1}, p_{m+2}) + \cdots + d(p_{m+r-1}, p_{m+r}),$$

which can be made smaller than ϵ for large m. Thus, $\{p_n\}$ is Cauchy and converges to some $u \in M$ since M is complete.

To show that u is a common fixed point, we use continuity of the metric:

$$d(u, \mathcal{T}u) = \lim_{n\to\infty} d(p_{2n}, \mathcal{T}p_{2n}) = 0,$$

so $\mathcal{T}u = u$. Similarly, $\mathcal{S}u = u$. Hence u is a common fixed point.

To prove uniqueness, suppose v is another common fixed point. Then:

$$d(u, v) = d(\mathcal{T}u, \mathcal{S}v) \le \mu[d(u, \mathcal{T}u)]^{\alpha}[d(v, \mathcal{S}v)]^{\beta} = 0,$$

so $u = v$.

Thus, $\mathcal{T}$ and $\mathcal{S}$ have a unique common fixed point. □

4. Numerical Example

Example 4.1. Let $\mathcal{T}, \mathcal{S}: M \to M$ be two mappings on the finite set $M = \{p, q, z, w\}$. The mappings are defined by the following tables:

$\mathcal{T}$:

Input	Output
p	p
q	w
z	z
w	w

$\mathcal{S}$:

Input	Output
p	p
q	q
z	w
w	z

Define a metric $d: M \times M \to \mathbb{R}$ as follows:

$$d(p,q) = d(q,p) = 3, \quad d(p,z) = d(z,p) = 4, \quad d(p,w) = d(w,p) = \frac{5}{2},$$

$$d(w,z) = d(z,w) = \frac{3}{2}, \quad d(a,a) = 0, \quad \text{for all } a \in M.$$

We compute the values of $d(\mathcal{T}x, \mathcal{S}y)$, $d(x, \mathcal{T}x)$ and $d(y, \mathcal{S}y)$ in the following table whenever $x \neq \mathcal{T}x$ and $y \neq \mathcal{S}y$.

Consider the inequality used in Definition 3.2:

$$d(\mathcal{T}x, \mathcal{S}y) \leq \mu[d(x, \mathcal{T}x)]^{\alpha}[d(y, \mathcal{S}y)]^{\beta},$$

where $\mu \in (0,1)$ and $\alpha, \beta > 0$ with $\alpha + \beta < 1$.

Case 1: Take $x = q$ and $y = w$. Then, $\mathcal{T}q = w$, $\mathcal{S}w = z$,

$$d(\mathcal{T}q, \mathcal{S}w) = d(w,z) = \frac{3}{2}, \quad d(q, \mathcal{T}q) = d(q,w) = 2, \quad d(w, \mathcal{S}w) = d(w,z) = \frac{3}{2}.$$

Thus, the inequality becomes:

$$\frac{3}{2} \leq \mu \cdot 2^{\alpha} \left(\frac{3}{2}\right)^{\beta}.$$

Solving for μ,

$$\mu \geq \frac{\frac{3}{2}}{2^{\alpha}\left(\frac{3}{2}\right)^{\beta}} = 3^{1-\beta} \cdot 2^{\beta-\alpha-1}.$$

For example, choosing $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ with $\alpha + \beta = \frac{5}{6} < 1$, we get the following

$$\mu \geq 3^{1-\frac{1}{3}} \cdot 2^{\frac{1}{3}-\frac{1}{2}-1} = 3^{\frac{2}{3}} \cdot 2^{-\frac{7}{6}} \approx 2.08 \times 0.44 = 0.91.$$

Thus, such a $\mu \in (0,1)$ exists and the inequality holds for this case.

Note. For other cases, either $d(x, \mathcal{T}x) = 0$ or $d(y, \mathcal{S}y) = 0$, the inequality is trivially satisfied.

Table 1. Values of $d(\mathcal{T}x, \mathcal{S}y)$, $d(x, \mathcal{T}x)$ and $d(y, \mathcal{S}y)$

x	y	$d(x,y)$	$\mathcal{T}x$	$\mathcal{T}y$	$\mathcal{S}x$	$\mathcal{S}y$	$d(\mathcal{T}x,\mathcal{S}y)$	$d(x,\mathcal{T}x)$	$d(y,\mathcal{S}y)$
p	q	3	p	w	p	q	$d(p,q)=3$	0	2
q	p	3	w	p	q	p	$d(w,p)=\frac{5}{2}$	2	0
z	p	4	z	p	w	p	$d(z,p)=4$	0	0
p	z	4	p	z	p	w	$d(p,w)=\frac{5}{2}$	0	0
q	z	$\frac{3}{2}$	w	z	q	w	$d(w,w)=0$	2	0
z	q	$\frac{3}{2}$	z	w	w	q	$d(z,q)=\frac{3}{2}$	0	2
w	p	$\frac{5}{2}$	w	p	z	q	$d(w,q)=2$	0	0
p	w	$\frac{5}{2}$	p	w	p	z	$d(p,z)=4$	0	0
w	q	2	w	w	z	q	$d(w,q)=2$	0	2
q	w	2	w	w	q	z	$d(w,z)=\frac{3}{2}$	2	0
w	z	$\frac{3}{2}$	w	z	z	w	$d(w,w)=0$	0	0
z	w	$\frac{3}{2}$	z	w	w	z	$d(z,z)=0$	0	0

This validates the conditions of Theorem 3.1.

We show that the above example also satisfies the Meir-Keeler type condition.

Consider the mappings $\mathcal{T},\mathcal{S}: M \to M$ on the set $M = \{p,q,z,w\}$ as defined earlier, along with the metric d on M.

Take the case where $x = q$ and $y = w$. From the previous calculations, we have the following

$$d(\mathcal{T}q,\mathcal{S}w) = d(w,z) = \frac{3}{2}, \quad d(q,\mathcal{T}q) = d(q,w) = 2, \quad d(w,\mathcal{S}w) = d(w,z) = \frac{3}{2}.$$

We now check the Meir-Keeler condition: For any given $\epsilon > 0$, there exists $\delta > 0$ such that

$$\epsilon < [d(q,\mathcal{T}q)]^{\alpha}[d(w,\mathcal{S}w)]^{\beta} < \epsilon + \delta \implies d(\mathcal{T}q,\mathcal{S}w) \le \epsilon.$$

Let us choose $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{3}$ satisfying $\alpha + \beta < 1$. Then, we compute

$$[d(q,\mathcal{T}q)]^{\alpha} = 2^{1/2} = \sqrt{2} \approx 1.414,$$

$$[d(w,\mathcal{S}w)]^{\beta} = \left(\frac{3}{2}\right)^{1/3} \approx 1.187.$$

Thus,

$$[d(q,\mathcal{T}q)]^{\alpha}[d(w,\mathcal{S}w)]^{\beta} \approx 1.414 \times 1.187 = 1.679.$$

Take $\epsilon = 1.6$ and choose $\delta = 0.1$, so that

$$1.6 < 1.679 < 1.6 + 0.1 = 1.7.$$

We see that the product falls within $(\epsilon, \epsilon + \delta)$.

Now, we check the corresponding distance:

$$d(\mathcal{T}q,\mathcal{S}w) = \frac{3}{2} = 1.5 < \epsilon = 1.6,$$

which satisfies the conclusion of the Meir-Keeler type condition.

5. Conclusion

This paper introduces a generalized Meir-Keeler-type contraction incorporating interpolative conditions for a pair of mappings in metric spaces. The resulting common fixed point theorems extend, unify and strengthen several existing results, including those of Noorwali *et al*. [10], thereby contributing significantly to the advancement of fixed point theory.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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