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Research Article

A Robust Ranking Technique to Solve Fuzzy Transportation Problem of Triangular Fuzzy Number

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Abstract. Transportation problems are widely used in supply chain management and logistics to reduce costs.When the costs of unit transportation, demand, and supply are all provided precisely, then efficient algorithms have been presented to solve the transportation problem. In this research, we transform the fuzzy transportation problem into a crisp valued transportation problem by using a ranking method. Here, we first simplify the fuzzy transportation problem using the robust ranking approach to obtain an initial basic feasible solution. Next, we apply the MODI method to obtain the fuzzy optimal solution. Ranking method is an appropriate method in ranking fuzzy numbers which is demonstrated with numerical example.

Keywords. Fuzzy transportation problem, Fuzzy number, Triangular fuzzy number and robust ranking technique

Mathematics Subject Classification (2020). 90xx

1. Introduction

A particular kind of linear programming problem known as a “transportation problem” is related to daily real-world activities and mostly addresses logistics. It assists with problems related to resource distribution and movement from one location to another. To satisfy the particular needs, the commodities are carried from a collection of sources, like a factory, to a collection of destinations, like a warehouse. In other words, transporting a single product made at several factories (supply–origin) to several warehouses (demand–destination) is the subject of transportation problems. The goal is to minimize transportation costs which fulfill the demand from the supply constraints at the destinations. We need to know how much quantity is needed in terms of demand and supply in order to achieve this goal. To determine how much it costs to move one unit of a commodity from its point of origin to its final destination, we also need to know the location. The model can be used to make strategic decisions about which transportation routes are best to allocate production from different plants to different warehouses or distribution centers.

Assume that the costs associated with transporting goods from each origin to each destination are distinct and well-known. The problem is Transporting goods from multiple origins to different destinations while keeping shipping or transportation costs is minimum. Sometimes uncontrollable factors lead to uncertainty in the supply and demand quantities as well as the cost coefficients of a transportation related problem.

In 1941, Hitchcock [3] introduced and developed the transportation problem, wherein characteristics like supply, demand, and transportation costs are distinct values (Chauhan and Joshi [2]). Shugani *et al*. [9] discussed about the ranking technique and fuzzy transportation problem of triangular numbers with α-cut. Chauhan and Joshi *et al*. [2] proposed fuzzy transportation problem solved with an enhanced VAM and a robust ranking technique (Roy *et al*. [8]). Zadeh [12] was the one who first introduced the concept of fuzzy sets. Since the beginning many ranking methods have been developed. There onwards multiple authors introduced several methods for solving the FTP problems (Samuel and Venkatachalapathy [10]).

This study examines the problem of fuzzy transportation. Triangular fuzzy numbers are used to represent the cost, supply, and demand variables in the fuzzy transportation problem. The *Triangular Fuzzy Numbers* (TFN) are transformed into crisp values. using the proposed robust ranking technique. In order to obtain the initial basic feasible solution and the optimal solution, the problem is then solved using the standard VAM (Vogel's Approximation Method) approach and the MODI (Modified Distribution Method) method.

2. Literature Review

Maliniad and Ananthanarayanan [6] explained that the ranking of fuzzy number is important in domains such as data analysis, socioeconomic systems, and decision-making.Several mathematical models need the ranking of fuzzy numbers. There are not many discriminating strategies that have been proposed thus far. In this work, the author applies a new ranking technique to transform the fuzzy transportation problem into a crisp valued transportation problem. Then they applied MODI method to solve the problem and identify the fuzzy optimal solution. The fuzzy ranking method is demonstrated using numerical examples to provide an effective tool for handling fuzzy transportation challenges.

Kumar and Subramanian [4] explained that the optimization approaches are used in real-world situations to solve issues like network flow analysis, project scheduling, and assignment challenges. This paper's main goal is to determine the lowest transportation cost of a given set of commodities via capacitated network, where the nodes' fuzzy integers are used to indicate supply and demand, as well as the cost and capacity of the edges. The author used the robust ranking technique to solve the transportation problem in this case, where the supply and demand are all represented by fuzzy trapezoidal numbers. Therefore, decision makers will find this technique to be very easy to comprehend and apply to actual transportation challenges.

Malini and Ananthanarayanan [5] proposed a fuzzy valued transportation problem which is ranked on Octagonal fuzzy number. The author used this ranking method to convert the cost, supply, and demand fuzzy transportation problem (represented as octagonal fuzzy numbers) into a critical valued transportation problem, which can then be solved using the MODI method. For Octagonal fuzzy numbers, a numerical example is given to show how well the proposed technique works.

Akila and Raveena [1] explained one of the first applications of linear programming, the transportation problem is a significant one that has been thoroughly examined in operational research. In the method's suggested technique, a trapezoidal fuzzy number represents the product's availability, demand, and transportation costs. The author extended and developed Vogel's fuzzy strategy to obtain the fuzzy optimal solution. A numerical example is provided to illustrate the effectiveness of the transportation problem solution approach.

According to Roy *et al*. [8], the fuzzy transportation problem is represented by triangular fuzzy numbers for the origin (as supply), destination (as demand), and unit transporter cost, is examined in this work with the goal of identifying the least expensive way to move certain goods from origin to destination. Additionally, we compare the zero-point method with the least-cost, VAM, and North-West corner methods using a comprehensive ranking technique. Using a numerical example, the study's viability is ultimately verified in order to calculate the fuzzy transportation problem.

Samuel and Venkatachalapathy [10] proposed the solution to the fuzzy transportation problem is offered by a new method called the *Modified Vogel's Approximation Method* (MVAM). There are other algorithms out there that are less efficient than this one. Author illustrated a numerical example that shows how the solution is arrived at. Additionally, a sample problem is used to establish a comparative analysis between the new algorithm and the other existing algorithms.

Purushothkumar *et al*. [7] applied a novel ranking method for sorting fuzzy integers which is presented in this study. It is based on the centroid ranking method. First, author use the suggested ranking method to convert the fuzzy quantities—cost, coefficients, supply, and demands—into crisp quantities. Next, they use the VAM algorithm to solve the problem and get the first basic feasible solution. Finally, they use the Modified Distribution Method to get the optimal solution. To validate the approach, examples are provided.

Chauhan and Joshi [2] explained the objective of this study that is to find the least expensive way to carry certain commodities from origin to destination by analyzing the fuzzy transportation problem, where origin (as supply), destination (as demand), and unit transporter cost are all represented in triangular fuzzy integers. Using a complete ranking system, they compared the zero-point method with the least-cost, VAM, and North-West corner methods. The study's feasibility is finally confirmed using a numerical example to compute the fuzzy transportation problem.

Uthra *et al*. [11] explained that this work uses symmetric triangular fuzzy numbers to find the optimal solution for the fuzzy transportation problem. In the *Fuzzy Transportation Problem* (FTP), the supply, demand, and cost values are represented as symmetric triangular fuzzy numbers. Using the suggested ranking, the symmetric triangular fuzzy numbers are transformed into crisp values. *Vogel's Approximation Method* (VAM) is then used to obtain the Initial Solution, and the *Modified Distribution Method* (MODI) is used to produce the Optimal Solution. A set of numerical examples is provided to demonstrate the suggested approach.

3. Definitions

3.1 Fuzzy Set

Let $\mu_A(x)$ be a function from A to $[0,1]$, and let A be a crisp set. Given a fuzzy set $\widetilde{A}$, its membership function $\mu_A(x)$ is

$$\widetilde{A} = \{(x, \mu_A(x)); x \in A \text{and } \mu_A(x) \in [0,1]\}.$$

3.2 Fuzzy Numbers

A fuzzy number is an extension of a regular real number in that it refers to a connected collection of alternative values rather than a single one, with each possible value having a weight between 0 and 1. This weight is called the membership function. Thus, a convex, normalized fuzzy set of real lie is a specific example of a fuzzy number. Fuzzy numbers are an extension of real numbers, much as fuzzy logic is an extension of Boolean logic. It is possible to include uncertainty about parameters, geometry, characteristics, initial circumstances, etc. in calculations involving fuzzy numbers. There are two ways to use fuzzy arithmetic operators to perform arithmetic calculations on fuzzy numbers:

(i) Interval arithmetic approach.

(ii) The extension principle approach.

3.3 Fuzzy Transportation Problem

Using m fuzzy sources and n fuzzy destinations, let's examine a fuzzy number-based transportation problem. Assume further that C_{ij} represents the transportation cost of a single unit of product from ith fuzzy source to jth fuzzy destination. Let the amount of fuzzy supply of the ith source and the amount of fuzzy demand of the jth destination be $\widetilde{a}_i$ and $\widetilde{b}_j$, respectively. Assume that the amount transmitted from fuzzy source i to fuzzy destination j is x_{ij}.

Minimize $Z = \sum\sum \widetilde{C}_{ij}\widetilde{x}_{ij}$

subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = \widetilde{a}_i, \quad i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} x_{ij} = \widetilde{b}_j, \quad j = 1,2,\dots,m$$

$$x_{ij} \geq 0, \quad i = 1,2,\dots,m;\ j = 1,2,\dots,n.$$

If the total fuzzy supply is equal to total fuzzy demand

$$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_i.$$

Then, it is called balanced fuzzy transportation problem.

3.4 Triangular Fuzzy Number

A membership function used to define a fuzzy number. If the membership function $\mu_{\tilde{A}}(x)$ of a number $\tilde{A} = (a_1, a_2, a_3)$ satisfies the criteria, then the number is said to be triangular

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases}$$

3.5 Arithmetic Operation

Let $\tilde{A}_1$ and $\tilde{A}_2$ be two triangular fuzzy numbers, i.e., $\tilde{A}_1 = (a_1, b_1, c_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2)$, then

(1) $\tilde{A}_1 + \tilde{A}_2 = (a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$,

(2) $\tilde{A}_1 - \tilde{A}_2 = (a_1, b_1, c_1) - (a_2, b_2, c_2) = (a_1 - a_2, b_1 - b_2, c_1 - c_2)$,

(3) $\tilde{A}_1 \times \tilde{A}_2 = (a_1, b_1, c_1) \times (a_2, b_2, c_2) = (m_1, m_2, m_3)$,

$$m_1 = \min\{a_1a_2, a_1c_2, c_1a_2, c_1c_2\},$$

$$m_2 = b_1b_2,$$

$$m_3 = \max\{a_1a_2, a_1c_2, c_1a_2, c_1c_2\}.$$

4. Robust Ranking Technique

Robust ranking technique that complies with linearity, compensation, and additive properties. If $\tilde{a}$ is a fuzzy number, then robust ranking is defined by

$$R(\tilde{a}) = 0.5 \int_0^1 (a_\alpha^L, a_\alpha^U) d\alpha,$$

where (a_α^L, a_α^U) is the α-level cut of fuzzy number $\tilde{a}$ and

$$(a_\alpha^L, a_\alpha^U) = \{((a_2 - a_1)\alpha + a_1), (-(a_3 - a_2)\alpha + a_3)\}.$$

For the transportation problem, Fuzzy objective function

$$\text{Mininimize } Z = \sum\sum \tilde{C}_{ij}\tilde{x}_{ij}.$$

To obtain the minimum objective value Z^* from the formulation, we utilize the robust ranking technique,

$$R(Z^*) = \sum\sum R(\tilde{a}_{ij})x_{ij}.$$

4.1 Numerical Example

We will use triangular fuzzy numbers to give a solution to a transportation conundrum involving product availability, consumer demand, and delivery costs. Consider the following fuzzy transportation problem

	D_1	D_2	D_3	Supply
S_1	(1,4,9)	(16,25,36)	(9,36,49)	(4,25,36)
S_2	(16,25,64)	(36,64,81)	(16,36,49)	(16,36,49)
S_3	(4,25,81)	(25,36,64)	(49,64,81)	(25,49,81)
Demand	(16,25,36)	(4,49,81)	(25,36,49)	

Step 1: Now by using the robust ranking technique, above problem can be reduced as follows:

	D_1	D_2	D_3	Supply
S_1	4.5	25.5	32.5	22.5
S_2	32.5	61.25	41.5	34.25
S_3	33.75	38.25	64.5	51
Demand	25.5	45.75	36.5	

Step 2: By using the VAM method, we can get the initial solution as

22.5		
		34.25
3	45.75	2.25

which is not an optimal solution.

Step 3: Hence by using MODI method we shall get the optimal solution as

20.25		2.25
		34.25
5.25	45.75	

The crisp value of the optimum fuzzy transportation for the given problem is Rs. 3511.4.

4.2 Numerical Example

The fuzzy transportation is given as follows:

	D_1	D_2	D_3	D_4	Supply
S_1	(−2,3,8)	(−2,3,8)	(−2,3,8)	(−1,1,4)	(0,3,6)
S_2	(4,9,16)	(4,8,12)	(2,5,8)	(1,4,7)	(2,7,13)
S_3	(2,7,13)	(0,5,10)	(0,5,10)	(4,8,12)	(2,5,8)
Demand	(1,4,7)	(0,3,5)	(1,4,7)	(2,4,8)	(4,15,27)

Step 1: Now by using the robust ranking technique, above problem can be reduced as follows:

	D_1	D_2	D_3	D_4	Supply
S_1	3	3	3	1.5	3
S_2	10	8	5	4	7.5
S_3	7.5	5	5	8	5
Demand	4	2.5	4	5	

Step 2: By using the VAM method, we can get initial solution as

3			
		2.5	5
1	2.5	1.5	

Step 3: Hence by using MODI method we shall get the optimal solution as

3			
		2.5	5
1	2.5	1.5	

The crisp value of the optimum fuzzy transportation for the given problem is Rs. 69.

5. Conclusions

The transportation expenses are viewed in this research as imprecise figures that are more realistic and all-encompassing than fuzzy numbers. Furthermore transportation problem of triangular fuzzy number has been converted into fuzzy transportation problem by use of robust ranking algorithm indices. Both the optimal solution and the crisp and fuzzy optimal total cost may be obtained using this method, as demonstrated by numerical examples. We have demonstrated that the total cost found is ideal by employing the robust ranking technique. Aside from project timelines, assignments, and network flow issues, other problems can also be solved with this method.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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