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Research Article

Effects of Triaxiality of Primaries on Existence and Stability of Collinear Equilibrium Points in Elliptical Restricted Three Body Problem

Shilpi Dewangan*[1], Poonam Duggad[2] and A. Narayan[3]

[1] *Department of Mathematics, Shri Shankaracharya Professional University, Bhilai, Durg, India*
[2] *Shri Shankaracharya Technical Campus, Bhilai, Durg, India*
[3] *Department of Science and Technology, Darbhanga College of Engineering (Government of Bihar), Darbhanga, India*
***Corresponding author:** shilpi.mahesh2003@gmail.com

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Abstract. This paper deals with the motion of oblate infinitesimal mass around the triaxial primaries in the framework of elliptical restricted three body problem. The triaxial primaries are moving around each other in elliptic orbits about the common barycentre and the oblate infinitesimal is moving in the neighbourhood of collinear equilibrium points. It is observed that location and stability of the oblate infinitesimal around the collinear points are affected by the triaxiality of primaries. Furthermore, the results shown that the behaviour of infinitesimal mass around the collinear points L_1 and L_2 are unstable, while the behaviour of infinitesimal mass around L_3 is shown stable for some parameters of triaxiality of primaries.

Keywords. ER3BP, Lagrangian points, Triaxiality, Dynamical system, Collinear points

Mathematics Subject Classification (2020). 70F07, 70F15, 70H03

1. Introduction

The restricted three body problem has been studied widely by many researchers because of its applications in the real world problems as compared to the general three body problem in space dynamics, Celestial mechanics and analytic dynamics. The *Elliptical Restricted Three Body*

Problem (ER3BP) model comprises the system consist of two finite bodies, known as primaries, which moves around their common centre of mass being attracted by the gravitational attraction to each other. The primaries describe either a circular motion or elliptical motion. If the system performs motion in a circular path, then it is called a circular restricted three body problem otherwise it is an elliptical restricted three body problem. The circular restricted three body problem has been generalized by the introduction of the eccentricity of the orbit, thus improving its applicability and retaining some useful properties of the circular model suitable to the elliptical case. Ammar [1], Grebenikov [5], Gyrögyey [6], Kumar and Ishwar [7], Markellos *et al*. [8], Moulton [9], Narayan and Kumar [11], Narayan *et al*. [15, 16], Narayan and Singh [12, 13], Singh and Umar [18, 19], Szebehely [20, 21], Usha and Narayan [22], Zimvoschikov and Tihai [23] have studied the effect radiation pressure on the motion of the infinitesimal body by taking one or both the primaries as a source of radiation.

The bodies in celestial model of the problem were considered as spherical, but many celestial bodies are either oblate spheroids or triaxial or both, and not spherically. For instance, the Mars, Jupiter, Saturn, Neurons stares, Regulus and white dwarfs are oblate spheroids, whereas the Moon and Pluto and its Moon Charon are triaxial. This oblateness and triaxiality of primaries causes perturbation on the system. That is why many researchers have included these characterisations in their study of ER3BP. The Earth is also considered oblate triaxial as well.

This inspired many authors to include these characterizations in their study of ER3BP. The stability of the collinear equilibrium points in the photogravitational/Generalised photogravitational ER3BP was studied Sahoo and Ishwar [17], Kumar and Ishwar [7], and Markellos *et al*. [8]. The linear stability of periodic orbits of the Lagrangian equilibrium points of the ERTBP, was studied in Moulton [9] based on some value of the mass ratio. The dynamical properties of the radiation pressure in ER3BP were also analysed and studied Narayan and Kumar [11], and Narayan *et al*. [15, 16].

There are five equilibrium points in the ER3BP. The equilibrium points are the points at which the particle has zero velocity and zero acceleration and are very important for astronautical applications. Three of the equilibrium points are called collinear points as they lie on the x-axis (axis joining the two primaries) and are denoted by L_1, L_2, L_3. The remaining two are called Lagrangian points and are denoted by L_4, L_5.

In the present work, the motion of the oblate infinitesimal mass around the collinear points under the triaxial primaries has been studied in the frame work of the ER3BP. The study of motion of infinitesimal around the collinear point is useful for spacecraft mission. These are the suitable to set permanent observatories of the Sun, the magnetosphere of the Earth links with the hidden part of the Moon and others (Gomez and Mondelo [4]). In this paper, we have derived location of collinear points and their stability, when the primaries are oblate triaxial. The study of the stability of infinitesimal around the collinear points is important as these points can serve as a possible fuel depot for future space probe in the lunar mission. The numerical calculations and the graphs have been plotted using MATLAB software, respectively. The location of the collinear points and their stability has been analysed for the Jupiter-Earth system.

The present paper is organised as follows: Section 1, which is introduction; Section 2 provides the equation of motion; Section 3 gives the location of collinear points; Section 4 focuses on the stability of the different collinear points. The conclusion of the work is drawn in Section 5.

2. Equation of Motion

The differential equation of motion of the oblate infinitesimal mass in the ER3BP under triaxial primaries in the barycentric, pulsating and rotating, non-dimensional coordinates are derived in equation. The notations in principle follow Szebehely [20] with some minor modifications in the notation being done for adapting to the present problem, which is given by Duggad *et al*. [3]:

$$x'' - 2y' = \frac{1}{1+e\cos v}\left(\frac{\partial\Omega}{\partial x}\right), \quad y'' + 2x' = \frac{1}{1+e\cos v}\left(\frac{\partial\Omega}{\partial y}\right), \tag{2.1}$$

where ($'$) denotes differentiation with respect to v and Ω is defined as

$$\Omega = \left(\frac{x^2+y^2}{2}\right) + \frac{1}{n^2}\left\{\frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1^3} - \frac{3(1-\mu)[(\sigma_1-\sigma_2)y^2+A_4]}{2r_1^5}\right.$$
$$\left. + \frac{\mu[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^3} - \frac{3\mu[(\sigma_1'-\sigma_2')y^2+A_4]}{2r_2^5}\right\}. \tag{2.2}$$

Let

$$K = \frac{1}{n^2}\left\{\frac{1-\mu}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1^5} - \frac{15(1-\mu)[(\sigma_1-\sigma_2)y^2+A_4]}{2r_1^7}\right.$$
$$\left. + \frac{3\mu[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^5} - \frac{15\mu[(\sigma_1'-\sigma_2')y^2+A_4]}{2r_2^7}\right\}. \tag{2.3}$$

Then, equation (2.1) can be written as,

$$x'' - 2y' = \frac{1}{1+e\cos v}\left\{x\left[1-K+\frac{3(1-\mu)(\sigma_1-\sigma_2)}{n^2r_1^5}+\frac{3\mu(\sigma_1'-\sigma_2')}{n^2r_2^5}\right]\right.$$
$$-\frac{\mu(1-\mu)}{n^2}\left[\frac{1}{r_1^3}-\frac{1}{r_2^3}+\frac{3[(2\sigma_1-\sigma_2)+A_4]}{2r_1^5}-\frac{3[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^5}\right.$$
$$\left.\left.-\frac{15[(\sigma_1-\sigma_2)y^2+A_4]}{2r_1^7}+\frac{15[(\sigma_1'-\sigma_2')y^2+A_4]}{2r_2^7}\right]\right\},$$
$$y'' + 2x' = \frac{1}{1+e\cos v}[1-K]y, \tag{2.4}$$

where

$$n^2 = 1 + \frac{3}{2}e^2 + \frac{3}{2}(2\sigma_1-\sigma_2) + \frac{3}{2}(2\sigma_1'-\sigma_2') \tag{2.5}$$

and

$$\sigma_1 = \frac{a^2+b^2}{5R^2}, \quad \sigma_2 = \frac{b^2-c^2}{5R^2},$$
$$r_1 = (x+\mu)^2+y^2, \quad r_2 = (x-1+\mu)^2+y^2. \tag{2.6}$$

3. Location of Collinear Equilibrium Points

The collinear equilibrium points of the system are the saddle points. The minima of the function $\Omega(x,y)$ occur at the collinear points. Hence, they are obtained as:

$$\frac{\partial\Omega}{\partial x}=0, \quad \frac{\partial\Omega}{\partial y}=0, \tag{3.1}$$

where Ω is given by equation (2.2). But, the collinear point lie on the x-axis; hence, are given by the conditions:

$$\frac{\partial\Omega}{\partial x}=0, \quad \frac{\partial\Omega}{\partial y}=0, \quad y=0. \tag{3.2}$$

Hence using equation (3.1) and (3.2), we get

$$f(x)=\left[x-\frac{1}{n^2}\left\{\frac{(1-\mu)(x+\mu)}{r_1^3}+\frac{\mu(x-1+\mu)}{r_2^3}+\frac{3(1-\mu)(x+\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1^5}\right.\right.$$
$$\left.\left.-\frac{3\mu(x-1+\mu)[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^5}\right\}\right]=0. \tag{3.3}$$

There are three collinear equilibrium points. These are denoted by L_1 lying between the bigger and smaller primary $(-\mu<x<1-\mu)$; L_2 lying to the right of smaller primary $(x>1-\mu)$ and L_3, lying to the left of the bigger primary $(x<-\mu)$.

3.1 Location of L_1

To find the solution for L_1, substituting $x=x_2-\rho=1-\mu-\rho$, such that $r_2=\rho$ and $r_1=1-\rho$ into the equation (3.3) we have:

$$\left[1-\mu-\rho-\frac{1}{n^2}\left\{\frac{(1-\mu)}{(1-\rho)^2}-\frac{\mu}{\rho^2}+\frac{3(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{2(1-\rho)^4}-\frac{3\mu[(2\sigma_1'-\sigma_2')+A_4]}{2\rho^4}\right\}\right]=0. \tag{3.4}$$

Now rearranging the terms, and simplifying, we have:

$$\rho^3\left\{1-\frac{\{3+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+3[(2\sigma_1-\sigma_2)+A_4]\}\rho}{\{1+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\right.$$
$$\left.+\frac{\{\frac{10}{3}+3e^2+3(2\sigma_1-\sigma_2)+3(2\sigma_1'-\sigma_2')+2[(2\sigma_1-\sigma_2)+A_4]\}\rho^2}{\{1+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\right\}$$
$$=\frac{\mu}{3(1-\mu)}\left[\frac{\{1+15[(2\sigma_1'-\sigma_2')+A_4]\}(1-\rho)^4}{\{1+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\right]$$
$$\cdot\left\{1-30[(2\sigma_1'-\sigma_2')+A_4]\rho+\frac{45}{2}[(2\sigma_1'-\sigma_2')+A_4]\rho^2-\{n^2+6[(2\sigma_1'-\sigma_2')+A_4]\rho^3\}\right\}. \tag{3.5}$$

Now, let

$$\frac{\mu}{3(1-\mu)}\left[\frac{\{1+15[(2\sigma_1'-\sigma_2')+A_4]\}}{\{1+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\right]^{\frac{1}{3}}=\lambda.$$

Then, we have:

$$\rho^3\left\{1-\frac{\{3+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+3[(2\sigma_1-\sigma_2)+A_4]\}\rho}{\{1+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\right.$$

$$+\frac{\{\frac{10}{3}+3e^2+3(2\sigma_1-\sigma_2)+3(2\sigma_1'-\sigma_2')+2[(2\sigma_1-\sigma_2)+A_4]\}\rho^2}{\{1+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\Bigg\}$$
$$=\lambda^3(1-\rho)^4\left\{1-30[(2\sigma_1'-\sigma_2')+A_4]\rho+\frac{45}{2}[(2\sigma_1'-\sigma_2')+A_4]\rho^2-\{n^2+6[(2\sigma_1'-\sigma_2')+A_4]\rho^3\}\right\}. \tag{3.6}$$

Thus, using the series expansion given as:

$$\rho=\lambda(1+c_1\lambda+c_2\lambda^2+\dots). \tag{3.7}$$

The simplified equation can be written as:

$$\rho=\lambda\Bigg[1-\frac{1}{3}\left\{\frac{\{1+6e^2+6(2\sigma_1-\sigma_2)+6(2\sigma_1'-\sigma_2')-\frac{5}{3}[(2\sigma_1-\sigma_2)+A_4]+90[(2\sigma_1'-\sigma_2')+A_4]\}}{\{1+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\right\}\lambda$$
$$-\frac{1}{9}\left\{\frac{\{10+9e^2+9(2\sigma_1-\sigma_2)+9(2\sigma_1'-\sigma_2')+6[(2\sigma_1-\sigma_2)+A_4]-\frac{855}{2}[(2\sigma_1'-\sigma_2')+A_4]\}}{\{1+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}^2}\right\}\lambda^2+\dots\Bigg]. \tag{3.8}$$

Hence the solution for L_1 is given by:

$$x=1-\mu-\lambda\Bigg[1-\frac{1}{3}\left\{\frac{\{1+6e^2+6(2\sigma_1-\sigma_2)+6(2\sigma_1'-\sigma_2')-\frac{5}{3}[(2\sigma_1-\sigma_2)+A_4]+90[(2\sigma_1'-\sigma_2')+A_4]\}}{\{1+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\right\}\lambda$$
$$-\frac{1}{9}\left\{\frac{\{10+9e^2+9(2\sigma_1-\sigma_2)+9(2\sigma_1'-\sigma_2')+6[(2\sigma_1-\sigma_2)+A_4]-\frac{855}{2}[(2\sigma_1'-\sigma_2')+A_4]\}}{\{1+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}^2}\right\}\lambda^2+\dots\Bigg]. \tag{3.9}$$

3.2 Location of L_2

For finding the location of L_2, substituting $x=x_2+\rho$ such that $r_2=\rho$, $r_1=1+\rho$. Then substituting, the values in equation (3.3), we have:

$$\frac{n^2(1+\rho)^5-(1+\rho)^2-\frac{3}{2}[(2\sigma_1-\sigma_2)+A_4]}{(1+\rho)^4}=\frac{\mu}{1-\mu}\left[\frac{1+\frac{\frac{3}{2}[(2\sigma_1'-\sigma_2')+A_4]}{\rho^2}-n^2\rho^3}{\rho^2}\right]. \tag{3.10}$$

On simplification, we have

$$\rho^3\Bigg\{1-\frac{\{3+2e^2+2(2\sigma_1-\sigma_2)+2(2\sigma_1'-\sigma_2')+3[(2\sigma_1-\sigma_2)+A_4]\}\rho}{\{1+3e^2+3(2\sigma_1-\sigma_2)+3(2\sigma_1'-\sigma_2')-\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}$$
$$+\frac{\{\frac{10}{3}+7e^2+7(2\sigma_1-\sigma_2)+7(2\sigma_1'-\sigma_2')-2[(2\sigma_1-\sigma_2)+A_4]\}\rho^2}{\{1+3e^2+3(2\sigma_1-\sigma_2)+3(2\sigma_1'-\sigma_2')-\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\Bigg\}$$
$$=\frac{\mu}{3(1-\mu)}\cdot\left[\frac{\{1+15[(2\sigma_1'-\sigma_2')+A_4]\}(1-\rho)^4}{\{1+3e^2+3(2\sigma_1-\sigma_2)+3(2\sigma_1'-\sigma_2')-\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\right]$$
$$\cdot\left\{1-30[(2\sigma_1'-\sigma_2')+A_4]\rho+\frac{45}{2}[(2\sigma_1'-\sigma_2')+A_4]\rho^2-\{n^2+6[(2\sigma_1'-\sigma_2')+A_4]\rho^3\}\right\}. \tag{3.11}$$

Using the series as in equation (3.7), we have

$$\rho=\lambda\left[1-\frac{1}{3}\left\{\frac{\{1+10e^2+10(2\sigma_1-\sigma_2)+10(2\sigma_1'-\sigma_2')-5[(2\sigma_1-\sigma_2)+A_4]+30[(2\sigma_1'-\sigma_2')+A_4]\}}{\{1+3e^2+3(2\sigma_1-\sigma_2)+3(2\sigma_1'-\sigma_2')-\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\right\}\lambda\right.$$
$$\left.-\frac{1}{9}\left\{\frac{\{1+\frac{83}{13}e^2+\frac{83}{13}(2\sigma_1-\sigma_2)+\frac{83}{13}(2\sigma_1'-\sigma_2')-\frac{23}{13}[(2\sigma_1-\sigma_2)+A_4]+\frac{225}{26}[(2\sigma_1'-\sigma_2')+A_4]\}}{\{1+3e^2+3(2\sigma_1-\sigma_2)+3(2\sigma_1'-\sigma_2')-\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}^2}\right\}\lambda^2+\dots\right. \tag{3.12}$$

Hence the solution for L_2 is given by:

$$x=1-\mu-\lambda\left[1-\frac{1}{3}\left\{\frac{\{1+10e^2+10(2\sigma_1-\sigma_2)+10(2\sigma_1'-\sigma_2')-5[(2\sigma_1-\sigma_2)+A_4]+30[(2\sigma_1'-\sigma_2')+A_4]\}}{\{1+3e^2+3(2\sigma_1-\sigma_2)+3(2\sigma_1'-\sigma_2')-\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}}\right\}\lambda\right.$$
$$\left.-\frac{1}{9}\left\{\frac{\{1+\frac{83}{13}e^2+\frac{83}{13}(2\sigma_1-\sigma_2)+\frac{83}{13}(2\sigma_1'-\sigma_2')-\frac{23}{13}[(2\sigma_1-\sigma_2)+A_4]+\frac{225}{26}[(2\sigma_1'-\sigma_2')+A_4]\}}{\{1+3e^2+3(2\sigma_1-\sigma_2)+3(2\sigma_1'-\sigma_2')-\frac{1}{2}[(2\sigma_1-\sigma_2)+A_4]\}^2}\right\}\lambda^2+\dots\right]. \tag{3.13}$$

3.3 Location of L_3

In order to find the solution for L_3, substituting $x=x_1-\rho$ such that $r_1=\rho$ and $r_2=1+\rho$, into equation (3.3), we have:

$$\frac{\mu}{1-\mu}=\frac{\left[n^2\rho^3-1-\frac{3[(2\sigma_1-\sigma_2)+A_4]}{\rho^2}\right]}{\left[1+\frac{3\left[(2\sigma_1'-\sigma_2')+A_4\right]}{2(1+\rho)^2}-n^2(1+\rho)^3\right]\rho^2}. \tag{3.14}$$

Let $\rho=1+\alpha$, and using the elementary algorithm for division up to $O(\alpha)^4$, we have

$$-\frac{\mu}{1-\mu}=\left\{-\frac{6e^2}{7}-\frac{6}{7}(2\sigma_1-\sigma_2)-\frac{6}{7}(2\sigma_1'-\sigma_2')+\frac{12}{7}[(2\sigma_1-\sigma_2)+A_4]\right.$$
$$+\left(-\frac{12\alpha}{7}\right)\left\{1-\frac{132e^2}{84}-\frac{132}{84}(2\sigma_1-\sigma_2)-\frac{132}{84}(2\sigma_1'-\sigma_2')+\frac{396}{84}[(2\sigma_1-\sigma_2)+A_4]\right.$$
$$\left.+\frac{36}{672}[(2\sigma_1'-\sigma_2')+A_4]\right\}+\left(-\frac{12\alpha}{7}\right)^2\left\{1+\frac{19257e^2}{2016}+\frac{19257}{2016}(2\sigma_1-\sigma_2)\right.$$
$$\left.+\frac{19257}{2016}(2\sigma_1'-\sigma_2')-\frac{237}{144}[(2\sigma_1-\sigma_2)+A_4]-\frac{756}{8064}[(2\sigma_1'-\sigma_2')+A_4]\right\}$$
$$+\left(-\frac{12\alpha}{7}\right)^3\left\{-\frac{935}{1728}-\frac{104538e^2}{24192}-\frac{66507}{24192}(2\sigma_1-\sigma_2)-\frac{66507}{24192}(2\sigma_1'-\sigma_2')\right.$$
$$\left.\left.+\frac{36}{1728}[(2\sigma_1-\sigma_2)+A_4]-\frac{105}{96768}[(2\sigma_1'-\sigma_2')+A_4]\right\}+O[\alpha]^4+\dots\right\}. \tag{3.15}$$

Now using the method of successive approximations and Lagrange inversion formula Murray and Dermott [10], and retaining only linear terms in δ,$\sigma_1$andσ_2, we get

$$\rho=1-\left\{-\frac{7e^2}{2}-\frac{1}{2}(2\sigma_1-\sigma_2)-\frac{1}{2}(2\sigma_1'-\sigma_2')-[(2\sigma_1-\sigma_2)+A_4]\right\}$$
$$-\frac{7}{12}\left\{1-\frac{219e^2}{21}-\frac{219}{21}(2\sigma_1-\sigma_2)-\frac{219}{21}(2\sigma_1'-\sigma_2')-\frac{171}{21}[(2\sigma_1-\sigma_2)+A_4]\right.$$

$$-\frac{36}{672}[(2\sigma_1'-\sigma_2')+A_4]\Big\}\left(\frac{\mu}{1-\mu}\right)-\frac{7}{12}\Big\{-1-\frac{9587e^2}{672}-\frac{9587}{672}(2\sigma_1-\sigma_2)$$
$$-\frac{9587}{672}(2\sigma_1'-\sigma_2')+\frac{5305}{336}[(2\sigma_1-\sigma_2)+A_4]+\frac{57}{224}[(2\sigma_1'-\sigma_2')+A_4]\Big\}\left(\frac{\mu}{1-\mu}\right)^2$$
$$-\frac{7}{12}\Big\{-\frac{2521}{1728}-\frac{290013e^2}{36288}+\frac{351321}{72576}(2\sigma_1-\sigma_2)+\frac{351321}{72576}(2\sigma_1'-\sigma_2')$$
$$+\frac{167463}{4536}[(2\sigma_1-\sigma_2)+A_4]+\frac{121545}{290304}[(2\sigma_1'-\sigma_2')+A_4]\Big\}\left(\frac{\mu}{1-\mu}\right)^3+O\left(\frac{\mu}{1-\mu}\right)^4+\dots\Big\}. \tag{3.16}$$

Hence the solution for $L_3$is given as:

$$x=-\mu-\Big[1-\Big\{-\frac{7e^2}{2}-\frac{1}{2}(2\sigma_1-\sigma_2)-\frac{1}{2}(2\sigma_1'-\sigma_2')-[(2\sigma_1-\sigma_2)+A_4]\Big\}$$
$$-\frac{7}{12}\Big\{1-\frac{219e^2}{21}-\frac{219}{21}(2\sigma_1-\sigma_2)-\frac{219}{21}(2\sigma_1'-\sigma_2')-\frac{171}{21}[(2\sigma_1-\sigma_2)+A_4]$$
$$-\frac{36}{672}[(2\sigma_1'-\sigma_2')+A_4]\Big\}\left(\frac{\mu}{1-\mu}\right)-\frac{7}{12}\Big\{-1-\frac{9587e^2}{672}-\frac{9587}{672}(2\sigma_1-\sigma_2)$$
$$-\frac{9587}{672}(2\sigma_1'-\sigma_2')+\frac{5305}{336}[(2\sigma_1-\sigma_2)+A_4]+\frac{57}{224}[(2\sigma_1'-\sigma_2')+A_4]\Big\}\left(\frac{\mu}{1-\mu}\right)^2$$
$$-\frac{7}{12}\Big\{-\frac{2521}{1728}-\frac{290013e^2}{36288}+\frac{351321}{72576}(2\sigma_1-\sigma_2)+\frac{351321}{72576}(2\sigma_1'-\sigma_2')$$
$$+\frac{167463}{4536}[(2\sigma_1-\sigma_2)+A_4]+\frac{121545}{290304}[(2\sigma_1'-\sigma_2')+A_4]\Big\}\left(\frac{\mu}{1-\mu}\right)^3+O\left(\frac{\mu}{1-\mu}\right)^4+\dots\Big\}\Big]. \tag{3.17}$$

4. Linear Stability of Collinear Points

The stability of motion of the infinitesimal mass near the collinear equilibrium point is analysed using following lemma (Bonavito *et al*. [23]):

Lemma 4.1. *At the collinear points:*

$$K\equiv\Big\{\frac{1-\mu}{r_1^3}+\frac{\mu}{r_2^3}+\frac{3(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1^5}-\frac{15(1-\mu)[(\sigma_1-\sigma_2)y^2+A_4]}{2r_1^7}$$
$$+\frac{3\mu[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^5}-\frac{15\mu[(\sigma_1'-\sigma_2')y^2+A_4]}{2r_2^7}\Big\}>1. \tag{4.1}$$

Proof. For an equilibrium points, we have the condition:

$$x-\frac{1}{n^2}\Big[\frac{(1-\mu)(x+\mu)}{r_1^3}+\frac{\mu(x-1+\mu)}{r_2^3}+\frac{3(1-\mu)(x+\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1^5}$$
$$+\frac{3\mu(x-1+\mu)[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^5}-\frac{15(1-\mu)(x+\mu)[(\sigma_1-\sigma_2)y^2+A_4]}{2r_1^7}$$
$$-\frac{15\mu(x-1+\mu)[(\sigma_1'-\sigma_2')y^2+A_4]}{2r_2^7}\Big]=0. \tag{4.2}$$

The condition for a collinear equilibrium points is$y = 0$, so the above equation can be written as:

$$x-\frac{1}{n^2}\left[\frac{(1-\mu)(x+\mu)}{r_1^3}+\frac{\mu(x-1+\mu)}{r_2^3}+\frac{3(1-\mu)(x+\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1^5}\right.$$
$$\left.+\frac{3\mu(x-1+\mu)[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^5}\right]=0. \tag{4.3}$$

Rearranging the terms, the above equation (4.3)can be given as:

$$\frac{1}{n^2}\left[\frac{(1-\mu)(x+\mu)}{r_1}(r_1-r_1^{-2})+\frac{\mu(x-1+\mu)}{r_2}(r_2-r_2^{-2})\right.$$
$$+\frac{3(1-\mu)(x+\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1}(r_1-r_1^{-4})$$
$$\left.+\frac{3\mu(x-1+\mu)[(2\sigma_1'-\sigma_2')+A_4]}{2r_2}(r_2-r_2^{-4})\right]=0. \tag{4.4}$$

Next, to prove equation (4.1) we analyse each collinear equilibrium point separately.

4.1 Stability at Collinear Point L_1

Now, at the point L_1, $r_1+r_2=1$, so $r_1=x+\mu$ and $r_2=x-1-\mu$, Substituting the values in equation (4.3) and simplifying using equation (4.3), we have:

$$\frac{1}{n^2}\left[\left\{1-k+\frac{3}{2}(1-\mu)[(2\sigma_1-\sigma_2)+A_4]+\frac{3}{2}\mu[(2\sigma_1'-\sigma_2')+A_4]\right\}r_1\right.$$
$$\left.-\mu\left\{1-\frac{1}{r_2^3}+\frac{3}{2}[(2\sigma_1'-\sigma_2')+A_4]-\frac{3}{2}\frac{[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}\right\}\right]=0.$$

Since $\frac{1}{n^2}\neq 0$ and $r_2<1$ we have,

$$k=1+\left[\frac{\mu}{r_1}\left\{\frac{1}{r_2^3}+\frac{3}{2}[(2\sigma_1'-\sigma_2')+A_4]+\frac{3}{2}\frac{[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}-1\right\}\right.$$
$$\left.+\frac{3}{2}(1-\mu)[(2\sigma_1-\sigma_2)+A_4]+\frac{3}{2}\mu[(2\sigma_1'-\sigma_2')+A_4]\right]. \tag{4.5}$$

Hence, we have $k>1$ for L_1, collinear point

4.2 Stability at Collinear Point L_2

At L_2, $r_1-r_2=1$, so $r_1=x+\mu$ and $r_2=x-1+\mu$. Inserting the values in equation (4.3), and using equation (2.3), and proceeding in the same way as for L_1, also for collinear point L_2, we have

$$k=1+\left[\frac{\mu}{r_1}\left\{\frac{1}{r_2^3}+\frac{3}{2}[(2\sigma_1'-\sigma_2')+A_4]+\frac{3}{2}\frac{[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}-1\right\}\right.$$
$$\left.+\frac{3}{2}(1-\mu)[(2\sigma_1-\sigma_2)+A_4]+\frac{3}{2}\mu[(2\sigma_1'-\sigma_2')+A_4]\right]. \tag{4.6}$$

4.3 Stability at Collinear Point L_3

At, L_3, $r_1 - r_2 = 1$, so $r_1 = -x - \mu$ and $r_2 = -x + 1 - \mu$. Proceeding in same manner as in L_1 and L_2, substituting values in equation (4.1) and using equation (2.3), we have:

$$k = 1 + \left[\frac{\mu}{r_1} \left\{ 1 - \frac{1}{r_2^3} + \frac{3}{2}[(2\sigma_1' - \sigma_2') + A_4] + \frac{3}{2} \frac{[(2\sigma_1' - \sigma_2') + A_4]}{r_2^5} \right\} + \frac{3}{2}(1-\mu)[(2\sigma_1 - \sigma_2) + A_4] + \frac{3}{2}\mu[(2\sigma_1' - \sigma_2') + A_4] \right]. \tag{4.7}$$

Hence, for collinear point L_3, also $k > 1$. Thus, for all collinear point L_1, L_2 and L_3, we have $k > 1$. This completes the proof of lemma.

Now, for investigating the roots of characteristics equation and analysing the stability of motion of the infinitesimal mass around the primaries near the collinear point, assuming that the particle receives a small displacement from the equilibrium position. Then finding the variational equations of motion by substituting the coordinates of displaced points in the equation of motion equation (2.1) and expanding by Taylor's series about the collinear point and taking only the linear terms, we get the equation as:

$$\xi'' - 2\eta' = \phi[\Omega_{xx}^0 \xi + \Omega_{xy}^0 \eta], \quad \eta'' + 2\xi' = \phi[\Omega_{yx}^0 \xi + \Omega_{yy}^0 \eta], \tag{4.8}$$

where $\phi = \frac{1}{1+e\cos v}$ and (x_0, y_0) are the coordinates of the collinear points respectively. The subscript of Ω denotes the second order partial derivatives of Ω with respect to x and y, as it appears, respectively.

Since, all the collinear points lie on the x-axis, hence $y = 0$, resulting, $\Omega_{xy} = 0$. Introducing new variables given by,

$$x_1 = \xi, \quad x_2 = \eta, \quad x_3 = \frac{d\xi}{dv}, \quad x_4 = \frac{d\eta}{dv}.$$

Substituting these values in equation (4.8), the system of equations can be written as:

$$\frac{dx_i}{dv} = P_{i1}x_1 + P_{i2}x_2 + P_{i3}x_3 + P_{i4}x_4, \quad i = 1,2,3,4 \tag{4.9}$$

where

$$P_{11} = P_{12} = P_{14} = P_{21} = P_{22} = P_{23} = P_{33} = P_{44} = 0,$$

$$P_{13} = P_{24} = 1, P_{34} = 2, P_{43} = -2$$

and

$$P_{31} = \phi\Omega_{xx}^0, \quad P_{32} = P_{42} = \phi\Omega_{xy}^0, \quad P_{42} = \phi\Omega_{yy}^0,$$

where subscript '0' indicates the value evaluated as respective collinear points. As $y = 0$, hence we have $P_{31} = P_{41} = 0$ and $\phi = \frac{1}{1+e\cos v}$. The coefficient of equation (4.9) is 2π periodic function of$v$. So, considering the averaged system, given by:

$$\frac{dx_i^{(0)}}{dv} = P_{i1}^{(0)}x_1^{(0)} + P_{i2}^{(0)}x_2^{(0)} + P_{i3}^{(0)}x_3^{(0)} + P_{i4}^{(0)}x_4^{(0)}, \quad i = 1,2,3,4, \tag{4.10}$$

where

$$P_{is}^{(0)} = \frac{1}{2\pi}\int_0^{2\pi} P_{is}(v)dv, \quad i,s = 1,2,3,4.$$

Thus, we get:

$$P_{31}^0 = \frac{1}{\sqrt{1-e^2}}\Omega_{xx}^0, \quad P_{42}^0 = \frac{1}{\sqrt{1-e^2}}\Omega_{yy}^0, \tag{4.11}$$

where subscript '0' where ever appears, indicates the value of the corresponding collinear point L_1, L_2, L_3. Thus, the characteristics equation for the system of equation (4.11) can be given as:

$$\lambda^4 + Q\lambda^2 + R = 0, \tag{4.12}$$

where

$$Q = -(4 - P_{31}^0 - P_{42}^0), \quad R = P_{31}^0 . P_{42}^0, \tag{4.13}$$

The motion of infinitesimal particle will be stable near the collinear point when given a small displacement and small velocity, the particle oscillates for a considerable time about the point (Sahoo and Ishwar [17]). That is, the system will be stable if the roots of the characteristics equation are purely imaginary.

The roots of the characteristics equation (4.12) are given by:

$$\begin{aligned}\lambda_{1,2}^2 = &\left[k - 2(1-e^2) - \frac{3(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{r_1^5} - \frac{3\mu[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}\right] \\ &\pm \left[9k^2 - 8k - \frac{15(1-\mu)k[(2\sigma_1-\sigma_2)+A_4]}{r_1^5} - \frac{15\mu k[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}\right. \\ &+ \frac{18(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{r_1^5} \\ &+ \frac{\frac{18\mu[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5} + 8e^2\{1+\frac{k}{2} - \frac{3(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1^5} - \frac{3\mu[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^5}\}]^{\frac{1}{2}}}{2\sqrt{1-e^2}}.\end{aligned} \tag{4.14}$$

Let $\lambda^2 = s$, $i = 1, 2$, so the above equation (4.14) can be written as:

$$\begin{aligned}s_1 = &\left[k - 2(1-e^2) - \frac{3(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{r_1^5} - \frac{3\mu[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}\right] \\ &+ \left[9k^2 - 8k - \frac{15(1-\mu)k[(2\sigma_1-\sigma_2)+A_4]}{r_1^5} - \frac{15\mu k[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}\right. \\ &+ \frac{18(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{r_1^5} \\ &+ \frac{\frac{18\mu[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5} + 8e^2\{1+\frac{k}{2} - \frac{3(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1^5} - \frac{3\mu[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^5}\}]^{\frac{1}{2}}}{2\sqrt{1-e^2}},\end{aligned}$$

$$\begin{aligned}s_2 = &\left[k - 2(1-e^2) - \frac{3(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{r_1^5} - \frac{3\mu[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}\right] \\ &- \left[9k^2 - 8k\frac{15(1-\mu)k[(2\sigma_1-\sigma_2)+A_4]}{r_1^5} - \frac{15\mu k[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}\right. \\ &+ \frac{18(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{r_1^5}\end{aligned}$$

$$+\frac{\frac{18\mu[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}-8e^2\left\{1+\frac{k}{2}-\frac{3(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1^5}-\frac{3\mu[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^5}\right\}\Big]^{\frac{1}{2}}}{2\sqrt{1-e^2}}. \tag{4.15}$$

As $k > 1$ for the collinear point we have,

$$\Big[9k^2-8k-\frac{15(1-\mu)k[(2\sigma_1-\sigma_2)+A_4]}{r_1^5}-\frac{15\mu k[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}$$
$$+\frac{18(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{r_1^5}+\frac{18\mu[(2\sigma_1'-\sigma_2')+A_4]}{r_2^5}$$
$$+8e^2\left\{1+\frac{k}{2}-\frac{3(1-\mu)[(2\sigma_1-\sigma_2)+A_4]}{2r_1^5}-\frac{3\mu[(2\sigma_1'-\sigma_2')+A_4]}{2r_2^5}\right\}\Big]^{\frac{1}{2}}>1. \tag{4.16}$$

Thus, from equation, equation (4.15) and equation (4.16), we have $s_1 > 0$ and $s_2 < 0$. As, $s = \lambda^2$, $s_1 > 0$, we have two real roots of opposite sign, and $s_2 < 0$ resulting into two imaginary roots. Hence, the solution to the equation (4.14) is of the form:

$$\lambda_i = C_{i1}e^{p_1 v}+C_{i2}e^{p_2 v}+C_{i3}\cos(p_3 v - C_{i4}), \quad i = 1,2, \tag{4.17}$$

where p_1, p_2 and p_3 are the roots of the equation (4.13). Since, C_{i1}, C_{i2} and C_{i3} are real constant. Since the equation (4.17) contains positive exponential function; a small change in the initial conditions makes the solution unbounded. Thus, the motion is unstable near a collinear equilibrium point.

5. Conclusion

The formula derived in the paper can be applied to the Jupiter and the Earth as primaries and the particle as space craft. So, the system, the mass parameter $\mu = \frac{m_2}{m_1+m_2} = 3.00317 \times 10^{-6}$, the eccentricity of the elliptical orbit of the primaries, $e = 0.0167$. The nature of motion around the collinear point can be analysed as:

(1) The motion around the point L_1 is unstable for all the values of σ_1, σ_2, σ_1', σ_1 and σ_1 as $\lambda_1^2 > 0$ and $\lambda_2^2 < 0$. This can be seen from Figure 1, Figure 2, Figure 3, Figure 4, Figure 5 and Figure 6.

(2) The point L_2 also exhibits the instability of motion in its vicinity as $k > 1$, $\lambda_1^2 > 0$ and $\lambda_2^2 < 0$ for all values of σ_1, σ_2, σ_1', σ_1 and σ_1. This is evident from Figure 7, Figure 8, Figure 9, Figure 10, Figure 11 and Figure 12.

(3) For L_3, the motion appears to be stable for some value of σ_1, σ_2, σ_1', σ_1 and σ_1 because the values of $\lambda_{1,2}^2 < 0$. That is, the roots will be imaginary implying the stability of the system which is evident from the Figure 13, Figure 14, Figure 15, Figure 16, Figure 17 and Figure 18.

For different values of σ_1, σ_2, σ_1', σ_1 and σ_1 our result is conformity of the result of Narayan and Usha [14], and Narayan and Singh [12]. The existence and Stability of collinear points of the ER3BP with different condition has been analysed. The figures are drawn using MATLAB.

Hence we arrived at the conclusion that motion around collinear point L_1 and L_2 are unstable, while motion around L_3 is conditionally stable for some values of σ_1, σ_2, σ_1', σ_1 and σ_1.

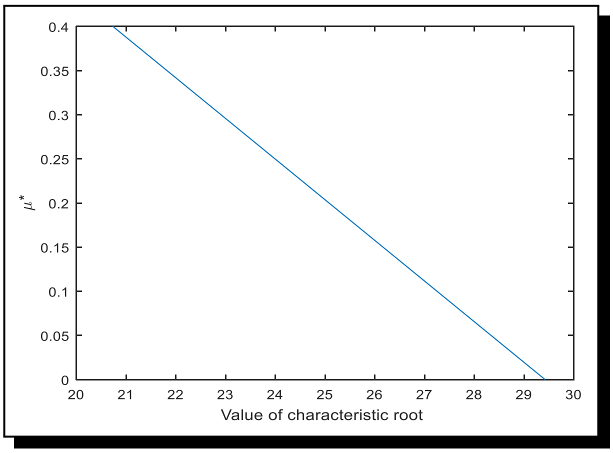

Figure 1. Correlation of characteristic root λ_1 and μ for L_1 ($A_4 = 0.0005$, $\sigma_1 = 0.0005$, $\sigma_2 = 0.0002$, $\sigma'_1 = 0.00005$, $\sigma'_2 = 0.00002$, $e = 0.0167$)

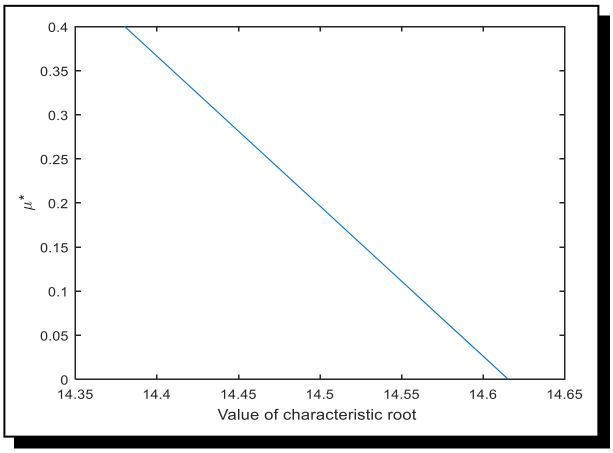

Figure 2. Correlation of characteristic root λ_1 and μ for L_1 ($A_4 = 0.0005$, $\sigma_1 = 0.0$, $\sigma_2 = 0.00$, $\sigma'_1 = 0.00$, $\sigma'_2 = 0.00$, $e = 0.0$)

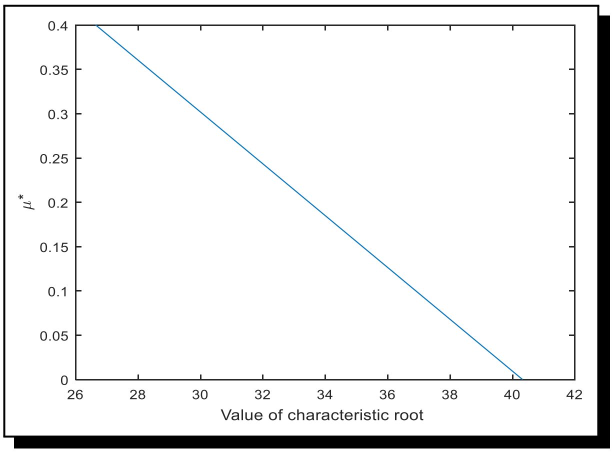

Figure 3. Correlation of characteristic root λ_1 and μ for L_1 ($A_4 = 0.0005$, $\sigma_1 = 0.0005$, $\sigma_2 = 0.0002$, $\sigma'_1 = 0.0007$, $\sigma'_2 = 0.0002$, $e = 0.0167$)

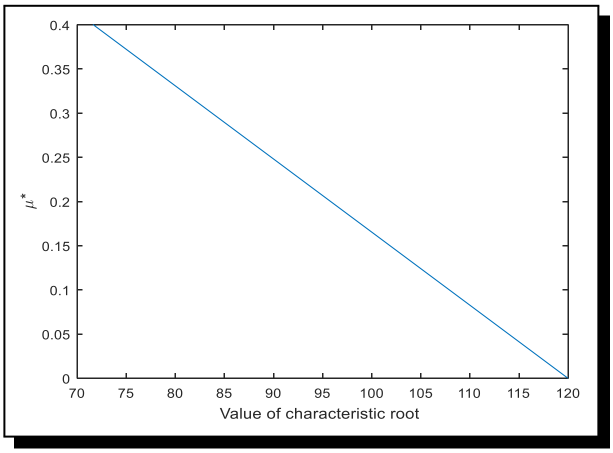

Figure 4. Correlation of characteristic root λ_1 and μ for L_1 ($A_4 = 0.005$, $\sigma_1 = 0.005$, $\sigma_2 = 0.002$, $\sigma'_1 = 0.007$, $\sigma'_2 = 0.002$, $e = 0.0167$)

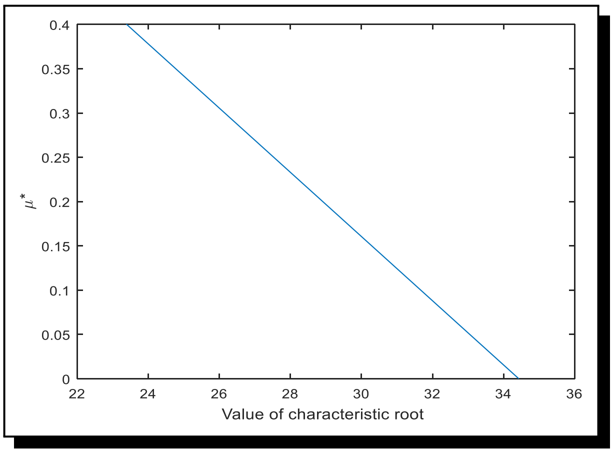

Figure 5. Correlation of characteristic root λ_1 and μ for L_1 ($A_4 = 0.0006$, $\sigma_1 = 0.00009$, $\sigma_2 = 0.00001$, $\sigma'_1 = 0.0005$, $\sigma'_2 = 0.0001$, $e = 0.0167$)

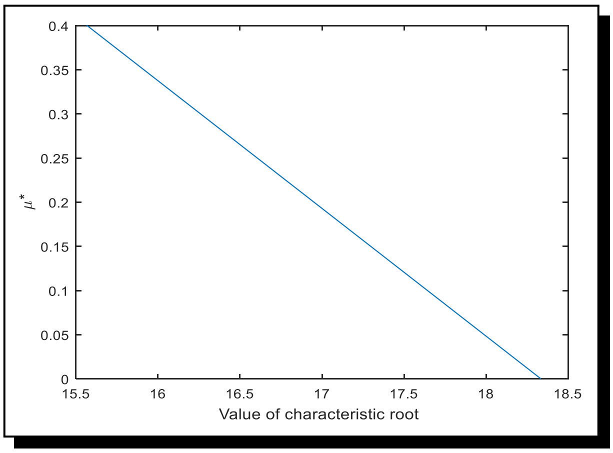

Figure 6. Correlation of characteristic root λ_1 and μ for L_1 ($A_4 = 0.0006$, $\sigma_1 = 0.0005$, $\sigma_2 = 0.0002$, $\sigma'_1 = 0.00008$, $\sigma'_2 = 0.00002$, $e = 0.0167$)

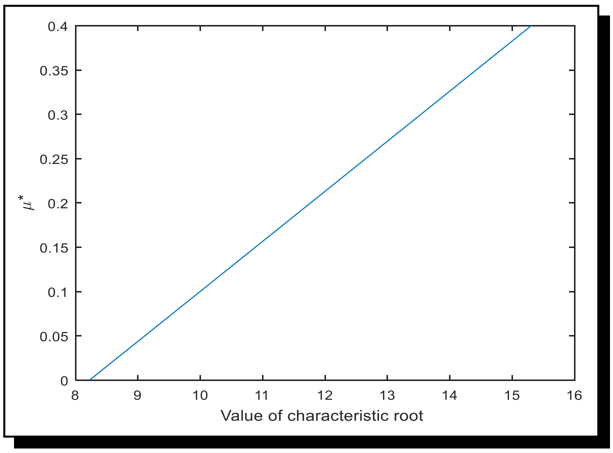

Figure 7. Correlation of characteristic root λ_1 and μ for L_2 ($A_4 = 0.0005$, $\sigma_1 = 0.0005$, $\sigma_2 = 0.0002$, $\sigma'_1 = 0.00005$, $\sigma'_2 = 0.00002$, $e = 0.0167$)

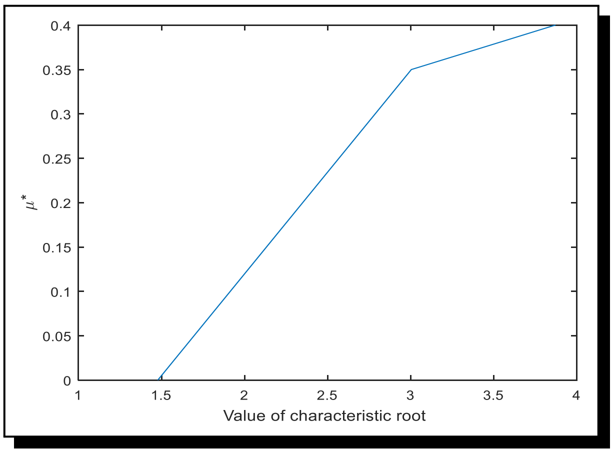

Figure 8. Correlation of characteristic root λ_1 and μ for L_2 ($A_4 = 0.0005$, $\sigma_1 = 0.0$, $\sigma_2 = 0.00$, $\sigma'_1 = 0.00$, $\sigma'_2 = 0.00$, $e = 0.0$)

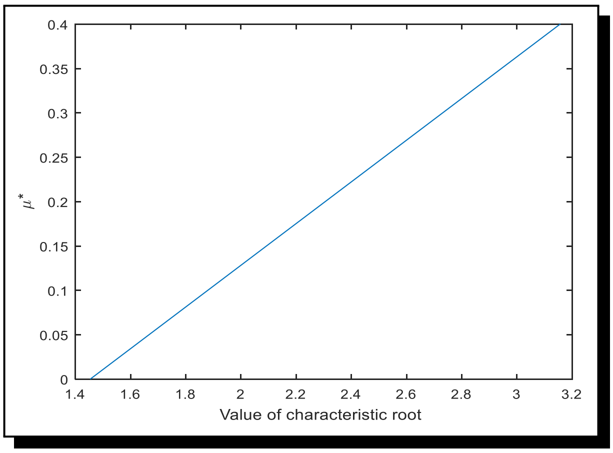

Figure 9. Correlation of characteristic root λ_1 and μ for L_2 ($A_4 = 0.0005$, $\sigma_1 = 0.0005$, $\sigma_2 = 0.0002$, $\sigma'_1 = 0.0007$, $\sigma'_2 = 0.0002$, $e = 0.0167$)

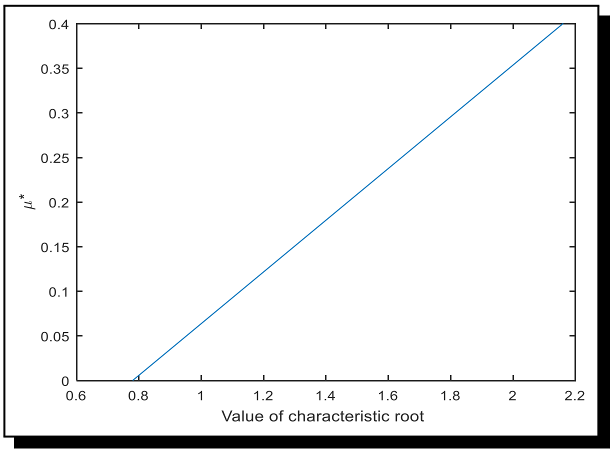

Figure 10. Correlation of characteristic root λ_1 and μ for L_2 ($A_4 = 0.005$, $\sigma_1 = 0.005$, $\sigma_2 = 0.002$, $\sigma'_1 = 0.007$, $\sigma'_2 = 0.002$, $e = 0.0167$)

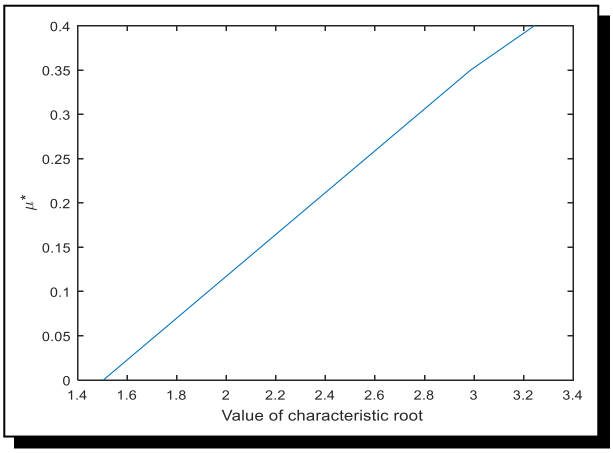

Figure 11. Correlation of characteristic root λ_1 and μ for L_2 ($A_4 = 0.0006$, $\sigma_1 = 0.00009$, $\sigma_2 = 0.00001$, $\sigma'_1 = 0.0005$, $\sigma'_2 = 0.0001$, $e = 0.0167$)

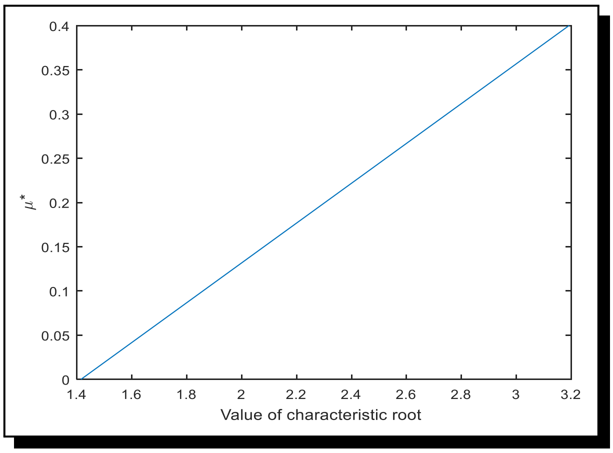

Figure 12. Correlation of characteristic root λ_1 and μ for L_2 ($A_4 = 0.0006$, $\sigma_1 = 0.0005$, $\sigma_2 = 0.0002$, $\sigma'_1 = 0.00008$, $\sigma'_2 = 0.00002$, $e = 0.0167$)

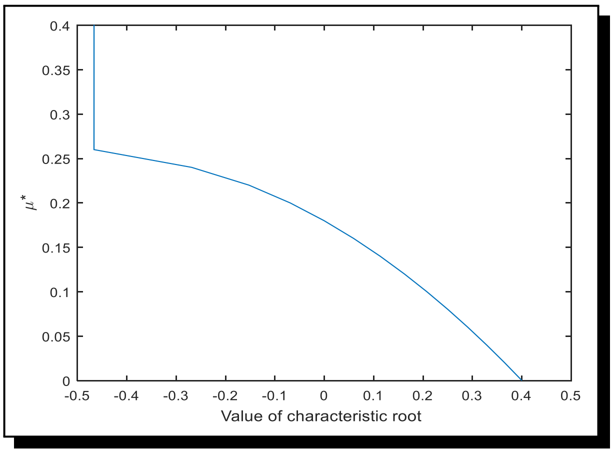

Figure 13. Correlation of characteristic root λ_1 and μ for L_3 ($A_4 = 0.0002$, $\sigma_1 = 0.00005$, $\sigma_2 = 0.00001$, $\sigma'_1 = 0.0005$, $\sigma'_2 = 0.0001$, $e = 0.0167$)

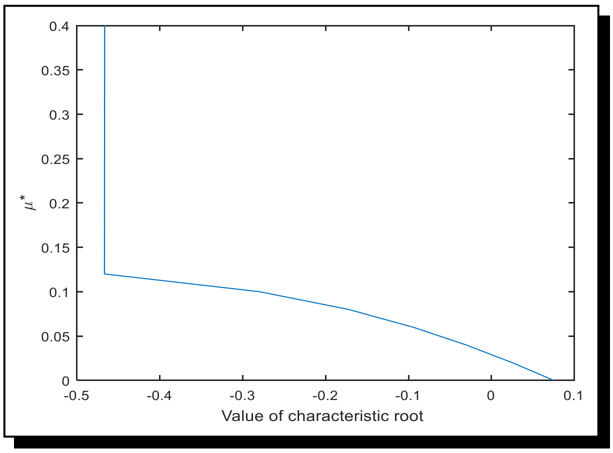

Figure 14. Correlation of characteristic root λ_1 and μ for L_3 ($A_4 = 0.00002$, $\sigma_1 = 0.0$, $\sigma_2 = 0.0$, $\sigma'_1 = 0.0005$, $\sigma'_2 = 0.0001$, $e = 0.0167$)

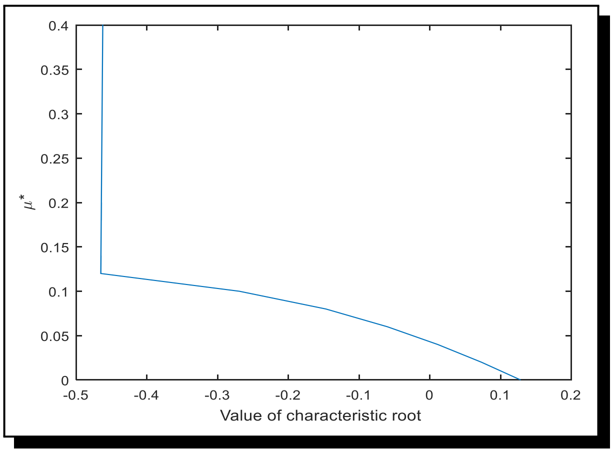

Figure 15. Correlation of characteristic root λ_1 and μ for L_3 ($A_4 = 0.00009$, $\sigma_1 = 0.00007$, $\sigma_2 = 0.00002$, $\sigma'_1 = 0.005$, $\sigma'_2 = 0.001$, $e = 0.0167$)

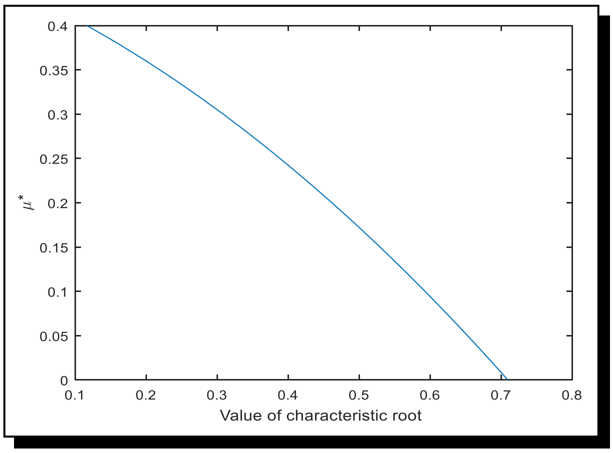

Figure 16. Correlation of characteristic root λ_1 and μ for L_3 ($A_4 = 0.0001$, $\sigma_1 = 0.0004$, $\sigma_2 = 0.0002$, $\sigma'_1 = 0.008$, $\sigma'_2 = 0.002$, $e = 0.0167$)

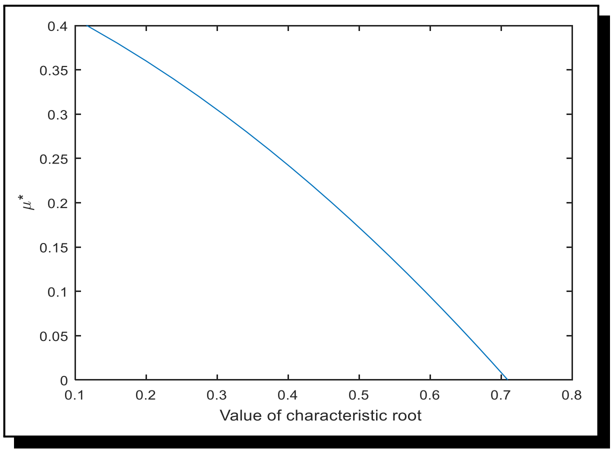

Figure 17. Correlation of characteristic root λ_1 and μ for L_3 ($A_4 = 0.00009$, $\sigma_1 = 0.00007$, $\sigma_2 = 0.00002$, $\sigma'_1 = 0.005$, $\sigma'_2 = 0.001$, $e = 0.0167$)

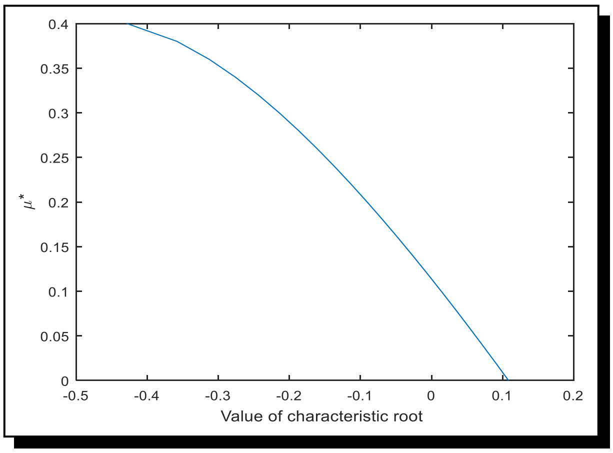

Figure 18. Correlation of characteristic root λ_1 and μ for L_3 ($A_4 = 0.0001$, $\sigma_1 = 0.0004$, $\sigma_2 = 0.0002$, $\sigma'_1 = 0.008$, $\sigma'_2 = 0.002$, $e = 0.0167$)

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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