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Review Article

A Review on Dynamical Nature of Systems of Nonlinear Difference Equations

İnci Okumuş and Yüksel Soykan*

Department of Mathematics, Art and Science Faculty, Zonguldak Bülent Ecevit University, 67100, Zonguldak, Turkey
***Corresponding author:** yuksel_soykan@hotmail.com

Abstract. The goal of this paper is to review about the dynamical behavior of the positive solutions of the systems of difference equations. The present study gives review of recent studies in systems of difference equations. We focus on papers dealing with two-dimensional, third-dimensional and multi-dimensional systems of nonlinear difference equations.

Keywords. Difference equations; Equilibrium point; Boundedness, Stability; Periodicity

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1. Introduction

Difference equation or discrete dynamical system is a diverse field which impact almost every branch of pure and applied mathematics. Every dynamical system $x_{n+1} = f(x_n, x_{n-1}, \dots, x_{n-k})$ determines a difference equation and vice versa.

Recently, there has been great interest in studying the systems of difference equations. One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real life situations in population biology, economics, probability theory, genetics, psychology and so forth.

The theory of difference equations occupies a central position in applicable analysis. There is no doubt that the theory of difference equations will continue to play an important role in mathematics as a whole. Nonlinear difference equations of order greater than one are of paramount importance in applications. Such equations also appear naturally as discrete analogues and as numerical solutions of differential and delay differential equations which model various diverse phenomena in biology, ecology, psychology, physics, engineering, and economics. It is very interesting to investigate the behavior of solutions of a system of nonlinear difference equations and to discuss the local asymptotic stability of their equilibrium points. Though difference equations are very simple in their form, it is quite hard to understand throughly the global behavior of their solutions.

The study of properties of nonlinear difference equations and systems of rational difference equations, systems of max-type difference equations and systems of exponential type difference equations has been an area of interest in recent years. There are many papers in which systems of difference equations have been studied (see [1–47]).

In this paper, we consider the systems of difference equations as two-dimensional, three dimensional and high-dimensional. First section is about two-dimensional systems of difference equations such as rational-type, exponential-type and max-type. Second section consists of three-dimensional systems of rational-type difference equations and third section is about high-dimensional systems of max-type difference equations.

2. Review on Two-Dimensional Systems of Nonlinear Difference Equations

This section is concerned with review dynamical behavior of solutions of the systems of two-dimensional nonlinear difference equations.

2.1 Systems of Rational Type Difference Equations

In [26], Papaschinopoulos and Schinas considered the system of difference equations

$$x_{n+1} = A + \frac{y_n}{x_{n-p}}, \quad y_{n+1} = A + \frac{x_n}{y_{n-q}}, \quad n = 0,1,\ldots, \tag{2.1}$$

where $A \in (0,\infty)$, p, q are positive integers and $x_{-p},\ldots,x_0$, $y_{-q},\ldots,y_0$ are positive numbers.

They investigated the oscillatory behavior, the boundedness of the solutions, and the global asymptotic stability of the positive equilibrium of the system (2.1). As a result, they prove that:

- Every positive nontrivial solution $\{(x_n, y_n)\}$ of system (2.1) oscillates about the positive equilibrium of system (2.1).
- If $A > 0$ and one at least of p, q is an odd number (resp. $A > 1$ and p, q are both even numbers), then any positive solution of system (2.1) is bounded away from zero and infinity.
- If $A > 1$, then the positive equilibrium (c, c) of system (2.1) is globally asymptotically stable.

In [25], Papaschinopoulos and Schinas studied the oscillatory behavior, the periodicity and the asymptotic behavior of the positive solutions of the system of difference equations

$$x_{n+1} = A + \frac{x_{n-1}}{y_n}, \quad y_{n+1} = A + \frac{y_{n-1}}{x_n}, \quad n = 0,1,\ldots, \tag{2.2}$$

where A is a positive constants and initial conditions are positive numbers.

They established conditions so that a positive solution (x_n, y_n) of system (2.2) oscillates about positive equilibrium of the system (2.2). Moreover, they found

- For the case $0 \le A < 1$,
  - The unique positive equilibrium (c, c) of (2.2) is not stable.
  - The system (2.2) has unbounded solutions.
- For the case $A = 1$,
  - For every $\mu \in (1, \infty)$, there exist positive solutions (x_n, y_n) of system (2.2) which tend to the positive equilibrium $\left(\mu, \frac{\mu}{\mu-1}\right)$.
  - Every positive solution of system (2.2) tends to a period 2 solution as $n \to \infty$.
- For the case $A > 1$,
  - The unique positive equilibrium (c, c) of (2.2) is locally asymptotically stable.
  - The positive equilibrium (c, c) of system (2.2) is globally asymptotically stable.

In [2], Camouzis and Papaschinopoulos studied the boundedness, persistence, and the global asymptotic behavior of the positive solutions of the system of difference equations

$$x_{n+1} = 1 + \frac{x_n}{y_{n-m}}, \quad y_{n+1} = 1 + \frac{y_n}{x_{n-m}}, \quad n = 0, 1, \ldots, \tag{2.3}$$

where x_i, y_i, $i = -m, -m+1, \ldots, 0$ are positive numbers and m is a positive integer.

Then the following results were exhibited in their paper:

- Every positive solution of system (2.3) is bounded and persists,
- System (2.3) has an infinite number of positive equilibrium solutions (x, y) with x, $y \in (1, \infty)$ that satisfy equation $xy = x + y$,
- Every positive solutions of system (2.3) converges to a positive equilibrium solution of system (2.3) as $n \to \infty$.

In [40], Yang studied the behavior of positive solutions of the system of difference equations

$$x_{n+1} = A + \frac{y_{n-1}}{x_{n-p}y_{n-q}}, \quad y_{n+1} = A + \frac{x_{n-1}}{x_{n-r}y_{n-s}}, \quad n = 1, 2, \ldots, \tag{2.4}$$

where $p \ge 2$, $q \ge 2$, $r \ge 2$, $s \ge 2$, A is a positive constant, and $x_{1-\max\{p,r\}}$, $x_{2-\max\{p,r\}}$, $\ldots$, x_0, $y_{1-\max\{q,s\}}$, $y_{2-\max\{q,s\}}$, $\ldots$, y_0 are positive real numbers.

He demonstrated that:

- The system (2.4) has the unique positive equilibrium
$$(c, c) = \left(\frac{A + \sqrt{A^2 + 4}}{2}, \frac{A + \sqrt{A^2 + 4}}{2}\right),$$
- When $A > 1$, every positive solution of system (2.4) is bounded,
- When $A > 2/\sqrt{3}$, (c, c) is locally asymptotically stable,
- When $A > \sqrt{2}$, every positive solution of system (2.4) approaches (c, c),
- When $A > \sqrt{2}$, the positive equilibrium (c, c) of (2.4) is globally asymptotically stable for all positive solutions.

In [47], Zhang et al. considered the behavior of positive solutions of the system of difference equations

$$x_{n+1} = A + \frac{1}{y_{n-p}}, \quad y_{n+1} = A + \frac{1}{x_{n-r}y_{n-s}}, \quad n = 1,2,\ldots, \tag{2.5}$$

where $p \geq 1$, $r \geq 1$, $s \geq 1$, $A \geq 0$, and $x_{1-r}, x_{2-r}, \ldots, x_0, y_{1-\max\{p,s\}}, y_{2-\max\{p,s\}}, \ldots, y_0$ are positive real numbers.

They obtained the following results:

- If $A > 0$, every positive solution of system (2.5)) is bounded,
- If $A = 0$, all positive solutions of system (2.5) are periodic,
- If $A > 2/\sqrt{3}$ and $\max\{p,r,s\} \geq 2$, the positive equilibrium (c,c) of (2.5) is locally asymptotically stable where $(c,c) = \left(\frac{A+\sqrt{A^2+4}}{2}, \frac{A+\sqrt{A^2+4}}{2}\right)$,
- If $A > \sqrt{2}$, every positive solution of system (2.5) approaches (c,c),
- If $A > \sqrt{2}$ and $\max\{p,r,s\} \geq 2$, the positive equilibrium (c,c) of (2.5) is globally asymptotically stable for all positive solutions.

In [46], Zhang et al. studied the system of rational difference equations

$$x_{n+1} = A + \frac{y_{n-m}}{x_n}, \quad y_{n+1} = A + \frac{x_{n-m}}{y_n}, \quad n = 0,1,\ldots. \tag{2.6}$$

They investigated the dynamic behavior of positive solutions of system (2.6) for the cases of $A < 1$, $A = 1$, and $A > 1$.

For the case $A < 1$, they obtained that the system (2.6) has unbounded solutions.

For the case $A = 1$, they proved that every positive solution of the system (2.6) is bounded and persists with interval $\left[L, \frac{L}{L-1}\right]$ and has prime two periodic solutions.

For the case $A > 1$, the global asymptotic stability of the unique equilibrium point of the system (2.6) is established. For this case, they proved that:

- Every positive solution of the system (2.6) is bounded and persists by interval $\left[L, \frac{L}{L-A}\right]$,
- The positive equilibrium point (c,c) of system (2.6) is locally asymptotically stable where $c = A + 1$,
- Every positive solution of system (2.6) converges to (c,c).

In [44], Zhang et al. considered the behavior of the symmetrical system of rational difference equations

$$x_{n+1} = A + \frac{y_{n-k}}{y_n}, \quad y_{n+1} = A + \frac{x_{n-k}}{x_n}, \quad n = 0,1,\ldots \tag{2.7}$$

where $A > 0$ and $x_i, y_i \in (0,\infty)$, for $i = -k, -k+1, \ldots, 0$.

They investigated the dynamic behavior of positive solutions of system (2.7) for the cases of $0 < A < 1$, $A = 1$, and $A > 1$.

In the case $0 < A < 1$, they obtained similar results as in [46] for k is odd. However, they said that they *can't* get some useful results for k is even.

In the case $A = 1$, the results which are obtained are similar to results in [46].

In the case $A > 1$, the following results were established:

- Every positive solution of the system (2.7) is bounded and persists by interval $\left[L, \frac{L}{L-A}\right]$,

- Every positive solution of the system (2.7) converges to the equilibrium as $n \to \infty$.

In [19], Kurbanli, Çinar and Yalcinkaya studied the behavior of the positive solutions of the system of difference equations

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1}+1}, \quad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1}+1}, \tag{2.8}$$

where the initial conditions are arbitrary non-negative real numbers.

They found the equilibrium point and all solutions of the system (2.8). Also, they obtained the followings where $y_0 = a$, $y_{-1} = b$, $x_0 = c$ and $x_{-1} = d$ are arbitrary non-negative real numbers:

- If $b \neq 0$ and $c = 0$, $x_{2n} = 0$ and $y_{2n-1} = b$,
- If $b = 0$ and $c \neq 0$, $x_{2n} = c$ and $y_{2n-1} = 0$,
- If $d = 0$ and $a \neq 0$, $y_{2n} = a$ and $x_{2n-1} = 0$,
- If $a = 0$ and $d \neq 0$, $y_{2n} = 0$ and $x_{2n-1} = d$.

In [20], Kurbanli et al. investigated the periodicity of the solutions of the system of difference equations

$$x_{n+1} = \frac{x_{n-1}+y_n}{y_n x_{n-1}-1}, \quad y_{n+1} = \frac{y_{n-1}+x_n}{x_n y_{n-1}-1}, \tag{2.9}$$

where x_0, x_{-1}, y_0, $y_{-1} \in \mathbb{R}$.

They proved that the solutions of x_n and y_n are six periodic under the special conditions.

In [39], Wang, Zhang and Fu considered the system of difference equations

$$x_{n+1} = \frac{x_{n-2k+1}}{A y_{n-k+1} x_{n-2k+1}+\alpha}, \quad y_{n+1} = \frac{y_{n-2k+1}}{B x_{n-k+1} y_{n-2k+1}+\beta}, \quad n \geq 0, \tag{2.10}$$

where k is a positive integer, A, B, α, β and the initial conditions are positive real numbers.

Under the specific conditions, they established the convergence of the positive solutions of the system (2.10) and showed that the system (2.10) has unbounded solutions.

In [45], Zhang et al. concerned with the dynamical behavior of positive solutions of the system of two rational difference equations

$$x_{n+1} = A + \frac{x_n}{\sum_{i=1}^{k} y_{n-i}}, \quad y_{n+1} = B + \frac{y_n}{\sum_{i=1}^{k} x_{n-i}}, \quad n = 0, 1, \ldots, \tag{2.11}$$

where A, B are positive constants and the initial conditions x_{-i}, $y_{-i} \in (0, \infty)$, $i = 0, 1, \ldots, k$.

They proved that under the case $A > 1/k$, $B > 1/k$ and assuming that $\frac{k^2AB-1}{kA-1} + \frac{k^2AB-1}{kB-1} < 1$:

- Every positive solution of system (2.11) is persistent and bounded,
- The system (2.11) has a unique positive equilibrium given by $x = \frac{k^2AB-1}{k(kB-1)}$, $y = \frac{k^2AB-1}{k(kA-1)}$,
- Every positive solution of the system (2.11) tends to the positive equilibrium of system (2.11) as $n \to \infty$,
- The unique positive equilibrium of the system (2.11) is locally asymptotically stable,
- The unique positive equilibrium of the system (2.11) is globally asymptotically stable.

In [41], Zhang and Zhang investigated the solutions, stability character and asymptotic behavior of the system of high-order nonlinear difference equations

$$x_{n+1} = \frac{x_{n-k}}{q + \prod_{i=0}^{k} y_{n-i}}, \quad y_{n+1} = \frac{y_{n-k}}{p + \prod_{i=0}^{k} x_{n-i}}, \quad k \in \mathbb{N}^+, \quad n = 0,1,\ldots, \tag{2.12}$$

where p, $q \in (0,\infty)$, $x_{-i} \in (0,\infty)$, $y_{-i} \in (0,\infty)$ and $i = 0,1,\ldots,k$.

First, they obtained the equilibrium points of system (2.12) as follows:

- $(0,0)$ and $\left(\sqrt[k+1]{1-p}, \sqrt[k+1]{1-q}\right)$ are equilibrium points if $p < 1$ and $q < 1$,
- Every point on the x-axis is an equilibrium point if $q = 1$,
- Every point on the y-axis is an equilibrium point if $p = 1$,
- $(0,0)$ is the unique equilibrium point if $p > 1$ and $q > 1$.

Then, they proved the following results:

- If $p > 1$ and $q > 1$, then the unique equilibrium point $(0,0)$ of system (2.12) is locally asymptotically stable,
- If $p < 1$ and $q < 1$, then the unique equilibrium point $(0,0)$ of system (2.12) is unstable,
- If $p < 1$ and $q < 1$, then the positive equilibrium point $\left(\sqrt[k+1]{1-p}, \sqrt[k+1]{1-q}\right)$ of system (2.12) is unstable,
- Every solutions of system (2.12) is bounded,
- If $p > 1$ and $q > 1$, then the unique equilibrium point $(0,0)$ of system (2.12) is globally asymptotically stable.

In [42], Zhang et al. studied the behavior of solutions of the following system

$$x_{n+1} = A + \frac{x_n}{y_{n-1}y_{n-2}}, \quad y_{n+1} = A + \frac{y_n}{x_{n-1}x_{n-2}}, \quad n = 0,1,\ldots, \tag{2.13}$$

where A is positive constant and x_{-i}, $y_{-i} \in (0,\infty)$, $i = 0,1,2$.

They obtained the results which are listed below:

- If $A > 1$, every positive solution of system (2.13) is bounded,
- If $A > 2/\sqrt{3}$, (c,c) is locally asymptotically stable,
- If $A > \sqrt{3}$, every positive solution of system (2.13) approaches (c,c),
- If $1 < A < 2/\sqrt{3}$, (a_1,b_1) and (a_2,b_2) are locally asymptotically stable.

In [34], Stevic et al. considered the following system of difference equations

$$x_{n+1} = A + \frac{y_n^p}{x_{n-1}^q}, \quad y_{n+1} = A + \frac{x_n^p}{y_{n-1}^q}, \quad n \in \mathbb{N}_0 \tag{2.14}$$

where parameters A, p and q are positive and investigated the boundedness character of positive solutions of system (2.14).

They proved the following results:

- If $p^2 \geq 4q > 4$, or $p \geq 1+q$, $q \leq 1$, then system (2.14) has positive unbounded solutions where $A > 0$,
- If $p^2 < 4q$, or $2\sqrt{q} \leq p < 1+q$, $q \in (0,1)$, then all positive solutions of system (2.14) are bounded.

In [1], Bao investigated the local stability, oscillation and boundedness character of positive solutions of the system of difference equations

$$x_{n+1} = A + \frac{x_{n-1}^p}{y_n^p}, \quad y_{n+1} = A + \frac{y_{n-1}^p}{x_n^p}, \quad n = 0,1,\dots, \tag{2.15}$$

where $A \in (0,\infty)$, $p \in [1,\infty)$ and initial conditions x_i, $y_i \in (0,\infty)$, $i = -1,0$.

He proved that the system (2.15) has a positive equilibrium point $(\overline{x},\overline{y}) = (A+1, A+1)$ and the equilibrium point of system (2.15) is locally asymptotically stable if $A > 2p-1$, is unstable if $0 < A < 2p-1$ and is a sink or an attracting equilibrium if $p/(A+1) < \sqrt{2}-1$. Also, he indicated that the positive solution of system (2.15) which consists of at least two semicycles is oscillatory and the system (2.15) has unbounded solutions.

In [12], Gümüş and Soykan considered the dynamical behavior of positive solutions for a system of rational difference equations of the following form

$$u_{n+1} = \frac{\alpha u_{n-1}}{\beta + \gamma v_{n-2}^p}, \quad v_{n+1} = \frac{\alpha_1 v_{n-1}}{\beta_1 + \gamma_1 u_{n-2}^p}, \quad n = 0,1,\dots, \tag{2.16}$$

where the parameters α, β, γ, α_1, β_1, γ_1, p and the initial values u_{-i}, v_{-i} for $i = 0,1,2$ are positive real numbers.

First, they reduced the system (2.16) to the following system of difference equations

$$x_{n+1} = \frac{r x_{n-1}}{1 + y_{n-2}^p}, \quad y_{n+1} = \frac{s y_{n-1}}{1 + x_{n-2}^p}, \quad n = 0,1,\dots, \tag{2.17}$$

by the change of variables $u_n = (\beta_1/\gamma_1)^{1/p} x_n$ and $v_n = (\beta/\gamma)^{1/p} y_n$ with $r = \alpha/\beta$ and $s = \alpha_1/\beta_1$.

Then, they found the equilibrium points of the system (2.17) under the certain conditions and investigated their local asymptotical behavior. Also, they proved that

- If $r < 1$ and $s < 1$, the zero equilibrium point of system (2.17) is globally asymptotically stable,
- For r, $s \in (1,\infty)$, the system (2.17) has unbounded solutions,
- If $r = s = 1$, the system (2.17) possesses the prime period two solution.

In [6], Din studied the qualitative behavior of positive solutions of following second-order system of rational difference equations

$$x_{n+1} = \frac{\alpha_1 + \beta_1 y_{n-1}}{a_1 + b_1 x_n}, \quad y_{n+1} = \frac{\alpha_2 + \beta_2 x_{n-1}}{a_2 + b_2 y_n}, \tag{2.18}$$

where the parameters α_i, β_i, a_i, b_i for $i \in \{1,2\}$ and initial conditions are positive real numbers.

He determined the following results:

- Every positive solution of system (2.18) is bounded and persists when $\beta_1\beta_2 < a_1 a_2$,
- The unique positive equilibrium point of system (2.18) is global attractor when $a_1 a_2 \neq \beta_1\beta_2$,
- Under the some specific conditions the unique positive equilibrium point of system (2.18) is globally asymptotically stable.
- The system (2.18) has no prime period-two solutions when $a_1 a_2 \neq \beta_1\beta_2$.

In [23], Mansour et al. got the exact form of the solutions and the periodic nature of the following systems of difference equations

$$x_{n+1} = \frac{x_{n-5}}{-1 + x_{n-5}y_{n-2}}, \quad y_{n+1} = \frac{y_{n-5}}{\pm 1 \pm y_{n-5}x_{n-2}}, \tag{2.19}$$

where the initial conditions are real numbers.

In [8], Elsayed and El-Metwally had the periodic nature and the form of the solutions of some systems of difference equations

$$x_{n+1} = \frac{x_n y_{n-2}}{y_{n-1}(\pm 1 \pm x_n y_{n-2})}, \quad y_{n+1} = \frac{y_n x_{n-2}}{x_{n-1}(\pm 1 \pm y_n x_{n-2})}, \tag{2.20}$$

where the initial conditions are nonzero real numbers.

In [10], Elsayed obtained the form of the solutions and the periodicity of the following systems of second-order rational difference equations

$$x_{n+1} = \frac{x_n y_{n-1}}{y_n(\pm 1 \pm x_n y_{n-1})}, \quad y_{n+1} = \frac{y_n x_{n-1}}{x_n(\pm 1 \pm y_n x_{n-1})}, \tag{2.21}$$

with the initial conditions are nonzero real numbers.

In [3], Clark and Kulenovic investigated the asymptotic and global stability behavior of solutions of the following systems of difference equations

$$x_{n+1} = \frac{x_n}{a + c y_n}, \quad y_{n+1} = \frac{y_n}{b + d x_n}, \quad n = 0, 1, \ldots, \tag{2.22}$$

where the parameters are positive numbers and the initial conditions are arbitrary nonnegative numbers.

Then, in [4], Clark et al. completed the investigation studied in [3] of the global behavior of system (2.22).

In [16], Kulenovic and Nurkanovic studied the system of difference equations

$$x_{n+1} = A x_n \frac{y_n}{1 + y_n}, \quad y_{n+1} = B y_n \frac{x_n}{1 + x_n}, \quad n = 0, 1, \ldots, \tag{2.23}$$

where the parameters A and B are in $(0, \infty)$ and the initial conditions are arbitrary nonnegative numbers. Under the special circumstances of parameters, they established the global asymptotic stability of the equilibrium points of the system (2.23) .

Also, there are many similar woks, see [5], [9], [13], [14], [43].

2.2 Systems of Exponential-Type Difference Equations

In this section, we review on some papers studied related to system of difference equations of exponential form.

In [28], Papaschinopoulos, Radin and Schinas studied the boundedness, the persistence and the asymptotic behavior of the positive solutions of the system of two difference equations of exponential form

$$x_{n+1} = a + b x_{n-1} e^{-y_n}, \quad y_{n+1} = c + d y_{n-1} e^{-x_n} \tag{2.24}$$

where a, b, c, d are positive constants, and the initial values x_{-1}, x_0, y_{-1}, y_0 are positive real values.

They investigated the boundedness character and the existence of invariant intervals of system (2.24). Then, they found the following results. Under the conditions that $be^{-c} < 1$ and $de^{-a} < 1$, every positive solution of system (2.24) is bounded and persists. Also, they proved that the unique positive equilibrium $(\overline{x},\overline{y})$ of system (2.24) is globally asymptotically stable under appropriate conditions.

In [27], Papaschinopoulos and Schinas considered the following systems of difference equations

$$x_{n+1} = a + by_{n-1}e^{-x_n}, \quad y_{n+1} = c + dx_{n-1}e^{-y_n} \tag{2.25}$$

$$x_{n+1} = a + by_{n-1}e^{-y_n}, \quad y_{n+1} = c + dx_{n-1}e^{-x_n} \tag{2.26}$$

where the constants are positive real numbers and the initial values x_{-1}, x_0, y_{-1}, y_0 are positive real numbers.

They investigated the boundedness and the persistence of the positive solutions, the existence of a unique positive equilibrium and the global asymptotic stability of the above mentioned systems. As a result, they established that every solution of the systems (2.25) and (2.26) is positive, bounded and persists if $p = bde^{-a-c} < 1$. Also, under the specific conditions, they indicated that the systems (2.25) and (2.26) have a unique positive equilibrium and every solution of these systems tends to the unique positive equilibrium of their as $n \to \infty$, each one positive equilibrium of these systems is globally asymptotically stable and finally, these systems have unbounded solutions.

In [29], Papaschinopoulos et al. investigated the boundedness, the persistence and the asymptotic behavior of the positive solutions of the following systems of difference equations

$$x_{n+1} = \frac{\alpha + \beta e^{-y_n}}{\gamma + y_{n-1}}, \quad y_{n+1} = \frac{\delta + \epsilon e^{-x_n}}{\zeta + x_{n-1}}, \tag{2.27}$$

$$x_{n+1} = \frac{\alpha + \beta e^{-y_n}}{\gamma + x_{n-1}}, \quad y_{n+1} = \frac{\delta + \epsilon e^{-x_n}}{\zeta + y_{n-1}}, \tag{2.28}$$

$$x_{n+1} = \frac{\alpha + \beta e^{-x_n}}{\gamma + y_{n-1}}, \quad y_{n+1} = \frac{\delta + \epsilon e^{-y_n}}{\zeta + x_{n-1}}, \tag{2.29}$$

where α, β, γ, δ, ϵ, ζ are positive constant and the initial values x_{-1}, x_0, y_{-1}, y_0 are positive constant.

They got the results are given below:

- For the system (2.27)
  - Every positive solution of the system (2.27) is bounded and persists,
  - If $\epsilon < \gamma$ and $\beta < \zeta$, the system (2.27) has a unique positive equilibrium and every solution of the system (2.27) tends to the unique positive equilibrium of the system (2.27) as $n \to \infty$,
  - If $\frac{\beta\epsilon + (\beta+\epsilon)e - 1}{\gamma\zeta} + \frac{(\alpha+\beta)(\delta+\epsilon)}{\gamma^2\zeta^2} < 1$, the unique positive equilibrium of the system (2.27) is globally asymptotically stable.
- For the system (2.28)
  - Every positive solution of the system (2.28) is bounded and persists,
  - If $\beta\epsilon < \gamma\zeta$, the system (2.28) has a unique positive equilibrium and every solution of the system (2.28) tends to the unique positive equilibrium of the system (2.28) as $n \to \infty$,

- If $\frac{\alpha+\beta}{\gamma^2}+\frac{\delta+\epsilon}{\zeta^2}+\frac{\beta\epsilon}{\gamma\zeta}+\frac{(\alpha+\beta)(\delta+\epsilon)}{\gamma^2\zeta^2}<1$, the unique positive equilibrium of the system (2.28) is globally asymptotically stable.

- For the system (2.29)
  - Every positive solution of the system (2.29) is bounded and persists,
  - If $\beta<\gamma$ and $\epsilon<\zeta$, the system (2.29) has a unique positive equilibrium and every solution of the system (2.29) tends to the unique positive equilibrium of the system (2.29) as $n\to\infty$,
  - If $\frac{\beta}{\gamma}+\frac{\epsilon}{\zeta}+\frac{\beta\epsilon}{\gamma\zeta}+\frac{(\alpha+\beta)(\delta+\epsilon)}{\gamma^2\zeta^2}<1$, the unique positive equilibrium of the system (2.29) is globally asymptotically stable.

In [7], Elettreby and El-Metwally considered the system of difference equations, which describes an economic model,

$$x_{n+1}=(1-\alpha)x_n+\beta x_n(1-x_n)e^{-(x_n+y_n)}, \tag{2.30}$$

$$y_{n+1}=(1-\alpha)y_n+\beta y_n(1-y_n)e^{-(x_n+y_n)}, \quad n=0,1,\dots, \tag{2.31}$$

where α and $\beta\in(0,\infty)$ with the initial conditions x_0 and $y_0\in(0,\infty)$.

They studied the boundedness and the invariant of the solutions of system (2.30) and also investigated global convergence for the solutions of system (2.30). Then, they obtained the following main results:

- Every positive solution $\{(x_n,y_n)\}_{n=0}^{\infty}$ of system (2.30) is bounded. Moreover,
$$\lim_{n\to\infty}\sup x_n\le\frac{\beta}{\alpha e},\ \lim_{n\to\infty}\sup y_n\le\frac{\beta}{\alpha e}.$$
- When $\alpha\ge\beta$, the zero equilibrium $(0,0)$ is a global attractor of all positive solutions of system (2.30) .
- When $\alpha+\beta e^{-2}<1$, the unique positive equilibrium point $(\overline{x},\overline{x})$ of system (2.30) is a global attractor of all positive solutions of system (2.30) .
- When $\beta\left(\alpha e-\beta\right)\ge\alpha^2e^3$, the unique positive equilibrium point $(\overline{x},\overline{x})$ of system (2.30) is a global attractor of all positive solutions of system (2.30) .
- When one of the following conditions hold
  (i) $5\beta\le 4e^2(1-\alpha)$
  (ii) $\alpha+\beta<1$

the unique positive equilibrium point $(\overline{x},\overline{x})$ of system (2.30) is a global attractor of all positive solutions of system (2.30).

In [30], Papaschinopoulos et al. studied the asymptotic behavior of the positive solutions of the system of difference equations

$$x_{n+1}=ay_n+bx_{n-1}e^{-y_n},\ y_{n+1}=cy_n+dy_{n-1}e^{-x_n},\quad n=0,1,\dots, \tag{2.32}$$

where a, b, c, d are positive constants and the initial values x_{-1}, x_0, y_{-1}, y_0 are positive numbers.

Then, they prove that under the condition that $a,\ b,\ c,\ d\in(0,1)$, $a+b>1$, $c+d>1$;

- Every positive solution of system (2.32) is bounded and persists.
- Every positive solution of system (2.32) tends to the unique positive equilibrium $(\overline{x},\overline{y})$ of system (2.32) as $n\to\infty$, when suppose that either relations

  $$c\le a,\ b\le c,\ d\le c$$

  or

  $$a\le c,\ b\le a,\ d\le a.$$

  Under the condition that $a+b\le 1$, $c+d\le 1$;
- Every positive solution of system (2.32) tends to the zero equilibrium $(0,0)$ of system (2.32) as $n\to\infty$.

Finally, they established that where a, b, c, d are positive constants such that either

$$a+b<1,\ c+d<1$$

or

$$a+b=1,\ c+d=1,$$

the zero equilibrium $(0,0)$ of system (2.32) is globally asymptotically stable.

In [15], Khan investigated the qualitative behavior of positive solutions of the following two systems of exponential rational difference equations

$$x_{n+1}=\frac{\alpha e^{-y_n}+\beta e^{-y_{n-1}}}{\gamma+\alpha y_n+\beta y_{n-1}},\ y_{n+1}=\frac{\alpha_1 e^{-x_n}+\beta_1 e^{-x_{n-1}}}{\gamma_1+\alpha_1 x_n+\beta_1 x_{n-1}},\quad n=0,1,\dots, \tag{2.33}$$

$$x_{n+1}=\frac{\alpha e^{-x_n}+\beta e^{-x_{n-1}}}{\gamma+\alpha y_n+\beta y_{n-1}},\ y_{n+1}=\frac{\alpha_1 e^{-y_n}+\beta_1 e^{-y_{n-1}}}{\gamma_1+\alpha_1 x_n+\beta_1 x_{n-1}},\quad n=0,1,\dots, \tag{2.34}$$

where α, β, γ, α_1, β_1, γ_1 and the initial conditions are positive real numbers.

They obtained the results are given below:

- For the system (2.33)
  - Every positive solution of the system (2.33) is bounded and persists,
  - If $(\alpha+\beta)e^{-L_2}<\overline{x}(\gamma+(\alpha+\beta)L_2)$ and $(\alpha_1+\beta_1)e^{-L_1}<\overline{y}(\gamma_1+(\alpha_1+\beta_1)L_1)$, the unique positive equilibrium point of the system (2.33) is globally asymptotically stable.
- For the system (2.34)
  - Every positive solution of the system (2.34) is bounded and persists,
  - If $(\alpha+\beta)e^{-L_1}<\overline{x}(\gamma+(\alpha+\beta)L_2)$ and $(\alpha_1+\beta_1)e^{-L_2}<\overline{y}(\gamma_1+(\alpha_1+\beta_1)L_1)$, the unique positive equilibrium point of the system (2.34) is globally asymptotically stable.

2.3 Systems of Max-Type Difference Equations

In [31], Simsek, Demir and Cinar considered the behavior of the solutions of the following system of difference equations

$$x_{n+1}=\max\left\{\frac{A}{x_n},\frac{y_n}{x_n}\right\},\quad y_{n+1}=\max\left\{\frac{A}{y_n},\frac{x_n}{y_n}\right\}, \tag{2.35}$$

where the constant A and the initial conditions are positive real numbers. They proved that the system (2.35) has unbounded solutions for special cases.

But then, in [38], Stevic corrected the results given in [31] and showed that the general solution to the max-type system of difference equations (2.35) for the case y_0, $x_0 \geq A > 0$, $y_0/x_0 \geq \max\{A, 1/A\}$, is given by:

$$x_n = \left(\frac{A^{f_{k(n)-1-\alpha(n)}} x_0^{f_{k(n)}}}{y_0^{f_{k(n)}}} \right)^{(-1)^n}, \quad n \in \mathbb{N},$$

and

$$y_n = \left(\frac{y_0^{f_{k(n-1)+1}}}{A^{f_{k(n-1)+\alpha(n)-1}} x_0^{f_{k(n-1)+1}}} \right)^{(-1)^n}, \quad n \geq 2.$$

In [11], Fotiades and Papaschinopoulos studied the periodic character of the solutions of the system of the difference equations

$$x_{n+1} = \max\left\{A, \frac{y_n}{x_{n-1}}\right\}, \quad y_{n+1} = \max\left\{B, \frac{x_n}{y_{n-1}}\right\}, \tag{2.36}$$

where A, B are positive constants and the initial values x_{-1}, x_0, y_{-1}, y_0 are positive numbers.

The authors established that every solution of system (2.36) is eventually periodic for the cases:

$1 \leq A \leq B$, $A < 1 \leq B$, $A \leq B < 1$, $1 \leq B \leq A$, $B < 1 \leq A$, $B \leq A < 1$.

In [35], Stevic studied behavior of positive solutions of the max-type system of difference equations

$$x_{n+1} = \max\left\{c, \frac{y_n^p}{x_{n-1}^p}\right\}, \quad y_{n+1} = \max\left\{c, \frac{x_n^p}{y_{n-1}^p}\right\}, \quad n \in \mathbb{N}_0 \tag{2.37}$$

where $p, c \in (0, \infty)$. In his work, boundedness character and global attractivity are investigated for some special cases.

For the case $p \in (0,4)$ and $c > 0$, boundedness of all positive solutions of system (2.37) is determined. Also, for $p \in (0,4)$ and $c \geq 1$, it is given that every positive solution $(x_n, y_n)_{n\geq 1}$ of system (2.37) is eventually equal to (c, c). Besides, the system (2.37) has positive unbounded solutions when $p \geq 4$ and $c > 0$. Finally, every positive solution of system (2.37) converges to $(1,1)$ when $p \in (0,1)$ and $c \in (0,1)$.

In [33], Stevic et al. studied the boundedness character of positive solutions of system of difference equations

$$x_{n+1} = \max\left\{A, \frac{y_n^p}{x_{n-1}^q}\right\}, \quad y_{n+1} = \max\left\{A, \frac{x_n^p}{y_{n-1}^q}\right\}, \quad n \in \mathbb{N}_0 \tag{2.38}$$

with $\min\{A, p, q\} > 0$.

Consequently, the following statements are obtained:

- All positive solutions of system (2.38) are bounded when $A > 0$, $2\sqrt{q} \leq p < 1 + q$ and $q \in (0,1)$.
- All positive solutions of system (2.38) are bounded when $A > 0$, $p > 0$ and $p^2 < 4q$.
- All positive solutions of system (2.38) are bounded when $A > 0$, $p = 1 + q$ and $q \in (0,1)$.
- The system (2.38) has positive unbounded solutions if $A > 0$, $p^2 \geq 4q \geq 4$, or $p > 1 + q$ and $q \in (0,1)$.

3. Review on Three-Dimensional Systems of Nonlinear Difference Equations

This section is concerned with review dynamical behavior of solutions of the systems of three-dimensional nonlinear difference equations.

3.1 Systems of Rational Type Difference Equations

In [17], Kulenovic and Nurkanovic studied the global behavior of solutions of the system of difference equations

$$x_{n+1} = \frac{a + x_n}{b + y_n}, \quad y_{n+1} = \frac{c + y_n}{d + z_n}, \quad z_{n+1} = \frac{e + z_n}{f + x_n}, \quad n = 0, 1, \ldots, \tag{3.1}$$

where the parameters a, b, c, d, e and f are in $(0, \infty)$ and the initial conditions are arbitrary non-negative numbers.

They indicated that the equilibrium of system (3.1) is locally asymptotically stable if $b \geq 1$, $d \geq 1$, $f \geq 1$, and obtained the global asymptotic stability of the unique positive equilibrium for several cases depending of some special values of the parameters.

In [22] Kurbanli, in [21] Kurbanli and in [18] Kurbanli et al. investigated the behavior of the solutions of the difference equations systems

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \; y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \; z_{n+1} = \frac{z_{n-1}}{y_n z_{n-1} - 1} \tag{3.2}$$

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \; y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \; z_{n+1} = \frac{1}{y_n z_n}, \tag{3.3}$$

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \; y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \; z_{n+1} = \frac{x_n}{y_n z_{n-1}}, \tag{3.4}$$

where the initial conditions are arbitrary real numbers, respectively.

They found all exact solutions of systems (3.2), (3.3), and (3.4) under special conditions and showed that the systems have unbounded solutions.

In [24], Özkan and Kurbanli studied the periodical solutions of the systems of difference equations

$$x_{n+1} = \frac{y_{n-2}}{-1 \pm y_{n-2} x_{n-1} y_n}, \; y_{n+1} = \frac{x_{n-2}}{-1 \pm x_{n-2} y_{n-1} x_n}, \; z_{n+1} = \frac{x_{n-2} + y_{n-2}}{-1 \pm x_{n-2} y_{n-1} x_n}, \quad n \in \mathbb{N}_0,$$

where the initial conditions are arbitrary real numbers. They obtained all six-period solutions of given systems under special conditions.

In [36], Stevic showed that the following system of difference equations

$$x_{n+1} = \frac{a_1 x_{n-2}}{b_1 y_n z_{n-1} x_{n-2} + c_1}, \; y_{n+1} = \frac{a_2 y_{n-2}}{b_2 z_n x_{n-1} y_{n-2} + c_2}, \; z_{n+1} = \frac{a_3 z_{n-2}}{b_3 x_n y_{n-1} z_{n-2} + c_3}, \quad n \in \mathbb{N}_0,$$

where the parameters and the initial conditions are real numbers, can be solved.

4. Review on High-Dimensional Systems of Nonlinear Difference Equations

This section is concerned with review dynamical behavior of solutions of the systems of high-dimensional nonlinear difference equations.

4.1 Systems of Max-Type Difference Equations

In [37], Stevic studied the system of max-type difference equations

$$\begin{aligned} x_n^{(1)} &= \max_{1\le i\le m_1}\left\{f_{1i}\left(x^{(1)}_{n-k^{(1)}_{i,1}}, x^{(2)}_{n-k^{(1)}_{i,2}}, \dots, x^{(l)}_{n-k^{(1)}_{i,l}}, n\right), x^{(1)}_{n-s}\right\}, \\ x_n^{(2)} &= \max_{1\le i\le m_2}\left\{f_{2i}\left(x^{(1)}_{n-k^{(2)}_{i,1}}, x^{(2)}_{n-k^{(2)}_{i,2}}, \dots, x^{(l)}_{n-k^{(2)}_{i,l}}, n\right), x^{(2)}_{n-s}\right\}, \\ &\vdots \\ x_n^{(l)} &= \max_{1\le i\le m_l}\left\{f_{li}\left(x^{(1)}_{n-k^{(l)}_{i,1}}, x^{(2)}_{n-k^{(l)}_{i,2}}, \dots, x^{(l)}_{n-k^{(l)}_{i,l}}, n\right), x^{(l)}_{n-s}\right\}, \end{aligned} \tag{4.1}$$

$n \in \mathbb{N}_0$, where s, l, m_j, $k^{(j)}_{i,t} \in \mathbb{N}$, j, $t \in \{1,\dots,l\}$ and for a fixed j, $i \in \{1,\dots,m_j\}$, and where the functions $f_{ij} : (0,\infty)^l \times \mathbb{N}_0 \to (0,\infty)$, $j \in \{1,\dots,l\}$, $i \in \{1,\dots,m_j\}$.

He proved that every positive solution to system (4.1) is eventually periodic with period s under some conditions. Also, he proved some related results for the corresponding system of min-type difference equations.

In [32], Stevic and Iricanin investigated the long-term behavior of positive solutions of the cyclic system of difference equations

$$x^{(i)}_{n+1} = \max\left\{\alpha, \frac{\left(x_n^{(i+1)}\right)^p}{\left(x_{n-1}^{(i+2)}\right)^q}\right\}, \quad i = 1,\dots,k,\ n \in \mathbb{N}_0, \tag{4.2}$$

where $k \in \mathbb{N}$, $\min\{\alpha, p, q\} > 0$.

They showed that the system (4.2) has bounded and unbounded solutions depending on the status of the parameters and gave some sufficient conditions which guaranty the global attractivity of all positive solutions of system (4.2) .

5. Future Work

Next, we will concentrate on the dynamical behavior of solutions of systems of difference equations and difference equations which their solutions associated with integer sequences such as Fibonacci and Tribonacci sequences.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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