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Cascade System with $Pr(X < Y < Z)$

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Abstract. The paper considers a n-cascade system with $P(X < Y < Z)$ where Y is the stress on the component subjected to two strengths X and Z. Reliability expressions of an n-cascade system is obtained when stress-strength both are either Exponential or Rayleigh or Lindley. It is also considered the cases when both strengths are one parameter exponential and stress follows Lindley distributions and when both strengths are one parameter exponential and stress follows two parameters gamma distributions. The numerical values $R(1)$, $R(2)$, $R(3)$ and R_3 have also been computed and provided in tabular forms for some specific values of the parameters.

1. Introduction

Cascade systems were first developed and studied by Pandit and Sriwastav [5]. Let us consider an n-cascade system and suppose that n components are numbered from 1 to n in their order of activation. Let X_i be the strengths of the ith component, and when activated faces the stress Y_i, $i = 1, 2, \dots, n$. For a cascade system with attenuation factor 'k' (a constant),

$$Y_i = k^{i-1} Y_1, \quad i = 1, 2, \dots, n.$$

In stress strength model the reliability, R of a component (or system) is defined as the probability that its strength X, is not less than the stress Y working on it, where X and Y are random variables. i.e.

$$R = Pr(X \geq Y).$$

But sometimes a component (or system) can work only when the stress Y on it is not only less than certain values, say Z, but also must be greater than some other value, say X, i.e. stress is within certain limits. For example, many electronic components cannot work at very high voltage or at a very low voltage. Here the limits are generally constant for a particular equipment. Similarly a person's blood pressure has two limits-systolic and diastolic pressure. For a healthy person his

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blood pressure must lie within these two limits. Both these pressure may vary within certain ranges beyond which a person cannot survive. The reliability of a component (or system) under such a situation may be defined as

$$R = P(X < Y < Z), \tag{1.1}$$

where Y is the stress on the component and X and Z may termed as strengths. We shall call them lower and upper strengths. These X, Y and Z are random variables. Singh [6] has considered the estimation in stress-strength model under the assumption that strength of a component lies in an interval and estimates the probability

$$R = P(X_1 < Y < X_2),$$

where X_1 and X_2 are independent random stress variables and Y independent of X_1 and X_2 is random strength variable. Chandra and Owen [1] obtained the estimation of reliability of a component subjected to several different stresses. They obtained the estimate $R = P[\max(Y_1, Y_2, \ldots, Y_k) < X]$ when $(Y_1, Y_2, \ldots, Y_k)$ are i.i.d normal distributions and X as an another independent normal distribution. Hanagal [3] has estimated the reliability of a component subjected to two different stresses which are independent of the strength of a component. In this paper we have considered an n-cascade system with this model. We have not come across any study where cascade model is considered for such a model. The main aim of this paper is to obtain the system reliability R_n for this model where stress on the component is subjected to two strengths. The paper is organized as follows. In section 2 the general model is developed for an n-cascade system. In section 3 the reliability expressions of an n-cascade system is obtained when the stress-strength of the components follow particular distributions. In section 3.1 to 3.5 the expressions of R_n, is obtained when stress-strength are either exponential or Rayleigh or Lindley and when both strengths are one parameter exponential and stress follows Lindley and when both strengths are one parameter exponential and stress follows two parameters gamma distributions. Some numerical values of reliabilities $R(1)$, $R(2)$, $R(3)$ and R_3 are tabulated for each cases in section 4. Results and Discussions are given at the end.

2. **Mathematical Formulation**

Let us consider a system with n components working under the impact of stresses. Let X_i and Z_i be the lower and upper strengths, respectively, on the ith component and Y_i be the stress on it, $i = 1, 2, \ldots, n$. In cascade system after every failure the stress is modified by a factor k [2] such that

$$Y_2 = kY_1, Y_3 = kY_2 = k^2 Y_1, \ldots, Y_i = k^{i-1} Y_1 \quad \text{etc.}$$

where Y_1 is the stress on the first component. It is obvious that once the distribution of Y_1 is specified the distribution of $Y_2, Y_3, \ldots, Y_n$ are automatically specified.

The ith component works if the stress $k^{i-1}Y_1$ lie in the interval (X_i, Z_i). Whenever a stress falls outside these two limits, the component fails and another from standby takes the place of the failed component and the system continues to work. The system fails only if all the n components in cascade fail. It is further assumed that all the components work independently. Then the reliability, R_n, of the system is given by

$$R_n = R(1) + R(2) + \ldots + R(n), \tag{2.1}$$

where $R(r)$ is the marginal reliability due to the rth component.
Here we consider $X_1, X_2, \ldots, X_n$ and $Z_1, Z_2, \ldots, Z_n$ are i.i.d random variables and let $f(x)$, $h(z)$ be the probability density function of X, Z and $g(y_1)$ be the pdf of Y_1.
Now we have,

$$\begin{aligned} R(1) &= P(X < Y_1 < Z) \\ &= P(Y_1 > X) - P(Y_1 > X, Y_1 > Z) \\ &= \int_{-\infty}^{\infty} F(y_1)g(y_1)dy_1 - \int_{-\infty}^{\infty} F(y_1)H(y_1)g(y_1)dy_1 . \end{aligned} \tag{2.2}$$

where $F(x)$ and $H(z)$ are c.d.f's of X and Z respectively.

$$\begin{aligned} R(2) &= P(X < Y_1 < Z)^c P(X < kY_1 < Z) \\ &= [1 - R(1)][P(kY_1 > X) - P(kY_1 > X, kY_1 > Z)] \\ &= [1 - R(1)]\left[\int_{-\infty}^{\infty} F(ky_1)g(y_1)dy_1 - \int_{-\infty}^{\infty} F(ky_1)H(ky_1)g(y_1)dy_1\right]. \end{aligned} \tag{2.3}$$

Similarly

$$\begin{aligned} R(3) = {} & [1 - R(1)][1 - R(2)] \\ & \times \left[\int_{-\infty}^{\infty} F(k^2y_1)g(y_1)dy_1 - \int_{-\infty}^{\infty} F(k^2y_1)H(k^2y_1)g(y_1)dy_1\right]. \end{aligned} \tag{2.4}$$

In general, we get

$$\begin{aligned} R(r) = {} & [1 - R(1)][1 - R(2)]\ldots[1 - R(r-1)] \\ & \times \left[\int_{-\infty}^{\infty} F(k^{r-1}y_1)g(y_1)dy_1 - \int_{-\infty}^{\infty} F(k^{r-1}y_1)H(k^{r-1}y_1)g(y_1)dy_1\right]. \end{aligned} \tag{2.5}$$

3. **Stress-Strength follows Specific Distributions**

When Stress-Strength follow particular distributions we can evaluate the expression (2.5) to get $R(r)$ and thereby obtain the system reliability. In the following five sub-sections we assume different particular distributions of all the Stress-Strength involved and obtain expressions of system reliability.

3.1. *Stress-Strength follows Exponential Distributions*

Let the strengths of the n components be i.i.d with p.d.f $f(x)$ and $h(z)$ which follows exponential distributions with means $\frac{1}{\lambda}$ and $\frac{1}{\gamma}$ and the p.d.f of Y_1 be exponential density with parameter μ i.e.

$$f(x,\lambda) = \lambda e^{-\lambda x}, \quad x > 0,\ \lambda > 0;$$
$$h(z,\gamma) = \gamma e^{-\gamma z}, \quad z > 0,\ \gamma > 0;$$
$$g(y_1,\mu) = \mu e^{-\mu y_1}, \quad y_1 > 0,\ \mu > 0$$

then from (2.2) to (2.5) we have

$$R(1) = \frac{\mu}{\mu+\gamma} - \frac{\mu}{\mu+\gamma+\lambda}, \tag{3.1}$$

$$R(2) = [1-R(1)]\left[\frac{\mu}{\mu+\gamma k} - \frac{\mu}{\mu+\gamma k+\lambda k}\right], \tag{3.2}$$

$$R(3) = [1-R(1)][1-R(2)]\left[\frac{\mu}{\mu+\gamma k^2} - \frac{\mu}{\mu+\gamma k^2+\lambda k^2}\right]. \tag{3.3}$$

In general,

$$R(r) = [1-R(1)][1-R(2)]\dots[1-R(r-1)] \times \left[\frac{\mu}{\mu+\gamma k^{r-1}} - \frac{\mu}{\mu+\gamma k^{r-1}+\lambda k^{r-1}}\right]. \tag{3.4}$$

Substituting the values of $R(r)$, $r = 1,2,\dots,n$ in (2.1) we can obtain R_n, the reliability of the system.

3.2. *Stress-Strength follows Rayleigh Distributions*

Let the strengths of the n components be i.i.d with p.d.f $f(x)$ and $h(z)$ which follows rayleigh distributions with parameters σ_1 and σ_3, and the p.d.f of Y_1 be rayleigh density with parameter σ_2 i.e.

$$f(x,\sigma_1) = \tfrac{x}{\sigma_1^2} e^{-\frac{x^2}{2\sigma_1^2}}, \quad x > 0,\ \sigma_1 > 0;$$
$$h(z,\sigma_3) = \tfrac{z}{\sigma_3^2} e^{-\frac{z^2}{2\sigma_3^2}}, \quad z > 0,\ \sigma_3 > 0;$$
$$g(y_1,\sigma_2) = \tfrac{y_1}{\sigma_2^2} e^{-\frac{{y_1}^2}{2\sigma_2^2}}, \quad y_1 > 0,\ \sigma_2 > 0$$

then from (2.2) to (2.5) we have

$$R(1) = \frac{\sigma_3^2}{\sigma_2^2+\sigma_3^2} - \frac{\sigma_1^2\sigma_3^2}{\sigma_1^2\sigma_3^2+\sigma_2^2\sigma_3^2+\sigma_1^2\sigma_2^2}, \tag{3.5}$$

$$R(2) = [1 - R(1)]\left[\frac{\sigma_3^2}{k^2\sigma_2^2 + \sigma_3^2} - \frac{\sigma_1^2\sigma_3^2}{\sigma_1^2\sigma_3^2 + k^2\sigma_2^2\sigma_3^2 + k^2\sigma_1^2\sigma_2^2}\right], \tag{3.6}$$

$$R(3) = [1 - R(1)][1 - R(2)]\left[\frac{\sigma_3^2}{k^4\sigma_2^2 + \sigma_3^2} - \frac{\sigma_1^2\sigma_3^2}{\sigma_1^2\sigma_3^2 + k^4\sigma_2^2\sigma_3^2 + k^4\sigma_1^2\sigma_2^2}\right]. \tag{3.7}$$

In general,

$$\begin{aligned} R(r) &= [1 - R(1)][1 - R(2)]\dots[1 - R(r-1)] \\ &\times \left[\frac{\sigma_3^2}{k^{2r-2}\sigma_2^2 + \sigma_3^2} - \frac{\sigma_1^2\sigma_3^2}{\sigma_1^2\sigma_3^2 + k^{2r-2}\sigma_2^2\sigma_3^2 + k^{2r-2}\sigma_1^2\sigma_2^2}\right]. \end{aligned} \tag{3.8}$$

Substituting the values of $R(r)$, $r = 1, 2, \dots, n$ in (2.1) we can obtain R_n, the reliability of the system.

3.3. *Stress-Strength follows Lindley Distributions*

Let the strengths of the n components be i.i.d with p.d.f $f(x)$ and $h(z)$ which follows Lindley distributions [4] with parameters θ and γ the p.d.f of Y_1 be Lindley density with parameter μ i.e.

$$f(x,\theta) = \frac{\theta^2}{(1+\theta)}(1+x)e^{-\theta x}, \qquad x > 0,\ \theta > 0;$$

$$h(z,\gamma) = \frac{\gamma^2}{(1+\gamma)}(1+z)e^{-\gamma z}, \qquad z > 0,\ \gamma > 0;$$

$$g(y_1,\mu) = \frac{\mu^2}{(1+\mu)}(1+y_1)e^{-\mu y_1}, \quad y_1 > 0,\ \mu > 0$$

then from (2.2) to (2.5) we have

$$\begin{aligned} R(1) = \frac{\mu^2}{1+\mu}\Bigg[& \frac{1}{\mu+\gamma} + \frac{1}{(\mu+\gamma)^2} + \frac{\gamma}{(1+\gamma)(\mu+\gamma)^2} + \frac{2\gamma}{(1+\gamma)(\mu+\gamma)^3} \\ & - \frac{1}{\theta+\gamma+\mu} - \frac{1}{(\theta+\gamma+\mu)^2} - \frac{\theta}{(1+\theta)(\theta+\gamma+\mu)^2} \\ & - \frac{\gamma}{(1+\gamma)(\theta+\gamma+\mu)^2} - \frac{2\theta}{(1+\theta)(\theta+\gamma+\mu)^3} \\ & - \frac{2\gamma}{(1+\gamma)(\theta+\gamma+\mu)^3} - \frac{2\theta\gamma}{(1+\theta)(1+\gamma)(\theta+\gamma+\mu)^3} \\ & - \frac{6\theta\gamma}{(1+\theta)(1+\gamma)(\theta+\gamma+\mu)^4}\Bigg], \end{aligned} \tag{3.9}$$

$$\begin{aligned} R(2) = [1 - R(1)]\Bigg[& \frac{\mu^2}{1+\mu}\Bigg\{\frac{1}{\mu+\gamma k} + \frac{1}{(\mu+\gamma k)^2} + \frac{\gamma k}{(1+\gamma)(\mu+\gamma k)^2} \\ & + \frac{2\gamma k}{(1+\gamma)(\mu+\gamma k)^3} - \frac{1}{\theta k+\gamma k+\mu} - \frac{1}{(\theta k+\gamma k+\mu)^2} \end{aligned}$$

$$- \frac{\theta k}{(1+\theta)(\theta k+\gamma k+\mu)^2} - \frac{\gamma k}{(1+\gamma)(\theta k+\gamma k+\mu)^2}$$
$$- \frac{2\theta k}{(1+\theta)(\theta k+\gamma k+\mu)^3} - \frac{2\gamma k}{(1+\gamma)(\theta k+\gamma k+\mu)^3}$$
$$- \frac{2\theta\gamma k^2}{(1+\theta)(1+\gamma)(\theta k+\gamma k+\mu)^3}$$
$$- \frac{6\theta\gamma k^2}{(1+\theta)(1+\gamma)(\theta k+\gamma k+\mu)^4}\Bigg\}\Bigg], \tag{3.10}$$

$$R(3) = [1-R(1)][1-R(2)]\Bigg[\frac{\mu^2}{1+\mu}\Bigg\{\frac{1}{\mu+\gamma k^2} + \frac{1}{(\mu+\gamma k^2)^2}$$
$$+ \frac{\gamma k^2}{(1+\gamma)(\mu+\gamma k^2)^2} + \frac{2\gamma k^2}{(1+\gamma)(\mu+\gamma k^2)^3} - \frac{1}{\theta k^2+\gamma k^2+\mu}$$
$$- \frac{1}{(\theta k^2+\gamma k^2+\mu)^2} - \frac{\theta k^2}{(1+\theta)(\theta k^2+\gamma k^2+\mu)^2}$$
$$- \frac{\gamma k^2}{(1+\gamma)(\theta k^2+\gamma k^2+\mu)^2} - \frac{2\theta k^2}{(1+\theta)(\theta k^2+\gamma k^2+\mu)^3}$$
$$- \frac{2\gamma k^2}{(1+\gamma)(\theta k^2+\gamma k^2+\mu)^3} - \frac{2\theta\gamma k^4}{(1+\theta)(1+\gamma)(\theta k^2+\gamma k^2+\mu)^3}$$
$$- \frac{6\theta\gamma k^4}{(1+\theta)(1+\gamma)(\theta k^2+\gamma k^2+\mu)^4}\Bigg\}\Bigg]. \tag{3.11}$$

In general,

$$R(r) = [1-R(1)][1-R(2)]\dots[1-R(r-1)]\Bigg[\frac{\mu^2}{1+\mu}\Bigg\{\frac{1}{\mu+\gamma k^{r-1}}$$
$$+ \frac{1}{(\mu+\gamma k^{r-1})^2} + \frac{\gamma k^{r-1}}{(1+\gamma)(\mu+\gamma k^{r-1})^2} + \frac{2\gamma k^{r-1}}{(1+\gamma)(\mu+\gamma k^{r-1})^3}$$
$$- \frac{1}{\theta k^{r-1}+\gamma k^{r-1}+\mu} - \frac{1}{(\theta k^{r-1}+\gamma k^{r-1}+\mu)^2}$$
$$- \frac{\theta k^{r-1}}{(1+\theta)(\theta k^{r-1}+\gamma k^{r-1}+\mu)^2} - \frac{\gamma k^{r-1}}{(1+\gamma)(\theta k^{r-1}+\gamma k^{r-1}+\mu)^2}$$
$$- \frac{2\theta k^{r-1}}{(1+\theta)(\theta k^{r-1}+\gamma k^{r-1}+\mu)^3} - \frac{2\gamma k^{r-1}}{(1+\gamma)(\theta k^{r-1}+\gamma k^{r-1}+\mu)^3}$$
$$- \frac{2\theta\gamma k^{2(r-1)}}{(1+\theta)(1+\gamma)(\theta k^{r-1}+\gamma k^{r-1}+\mu)^3}$$
$$- \frac{6\theta\gamma k^{2(r-1)}}{(1+\theta)(1+\gamma)(\theta k^{r-1}+\gamma k^{r-1}+\mu)^4}\Bigg\}\Bigg]. \tag{3.12}$$

Substituting the values of $R(r)$, $r = 1, 2, \dots, n$ in (2.1) we can obtain R_n, the reliability of the system.

3.4. *Both Strength follows One Parameter Exponential and Stress follows Lindley Distributions*

Let the strengths of the n components be i.i.d with p.d.f $f(x)$ and $h(z)$ which follows one parameter exponential with means $\frac{1}{\lambda}$ and $\frac{1}{\theta}$ and the p.d.f of Y_1 be Lindley density with parameter μ i.e.

$$f(x,\lambda) = \lambda e^{-\lambda x}, \qquad x > 0,\ \lambda > 0;$$
$$h(z,\theta) = \theta e^{-\theta z}, \qquad z > 0,\ \theta > 0;$$
$$g(y_1,\mu) = \frac{\mu^2}{(1+\mu)}(1+y_1)e^{-\mu y_1}, \quad y_1 > 0,\ \mu > 0$$

then from (2.2) to (2.5) we have

$$R(1) = \frac{\mu^2}{1+\mu}\left[\frac{1}{\theta+\mu} + \frac{1}{(\theta+\mu)^2} - \frac{1}{\theta+\mu+\lambda} - \frac{1}{(\theta+\mu+\lambda)^2}\right], \tag{3.13}$$

$$R(2) = [1-R(1)]\left[\frac{\mu^2}{1+\mu}\left\{\frac{1}{\theta k+\mu} + \frac{1}{(\theta k+\mu)^2} - \frac{1}{\theta k+\lambda k+\mu} - \frac{1}{(\theta k+\lambda k+\mu)^2}\right\}\right], \tag{3.14}$$

$$R(3) = [1-R(1)][1-R(2)]\left[\frac{\mu^2}{1+\mu}\left\{\frac{1}{\theta k^2+\mu} + \frac{1}{(\theta k^2+\mu)^2} - \frac{1}{\theta k^2+\lambda k^2+\mu} - \frac{1}{(\theta k^2+\lambda k^2+\mu)^2}\right\}\right]. \tag{3.15}$$

In general,

$$R(r) = [1-R(1)][1-R(2)]\dots[1-R(r-1)]\left[\frac{\mu^2}{1+\mu}\left\{\frac{1}{\theta k^{r-1}+\mu} + \frac{1}{(\theta k^{r-1}+\mu)^2} - \frac{1}{\theta k^{r-1}+\lambda k^{r-1}+\mu} - \frac{1}{(\theta k^{r-1}+\lambda k^{r-1}+\mu)^2}\right\}\right]. \tag{3.16}$$

Substituting the values of $R(r)$, $r = 1,2,\dots,n$ in (2.1) we can obtain R_n, the reliability of the system.

3.5. *Both Strength follows One Parameter Exponential and Stress follows Two Parameter Gamma*

Let the strengths of the n components be i.i.d with p.d.f $f(x)$ and $h(z)$ which follows one parameter exponential with means $\frac{1}{\lambda}$ and $\frac{1}{\theta}$ and $g(y_1)$ be two parameters gamma densities with degrees of freedom γ and μ respectively and unit scale parameters i.e.

$$f(x,\lambda) = \lambda e^{-\lambda x}, \quad x > 0,\ \lambda > 0;$$
$$h(z,\theta) = \theta e^{-\theta z}, \quad z > 0,\ \theta > 0;$$

$$g(y_1, \gamma, \mu) = \frac{1}{\gamma^\mu \Gamma\mu} {y_1}^{\mu-1} e^{-\frac{y_1}{\gamma}}; \qquad y_1 > 0,\ \gamma, \mu > 0$$

then from (2.2) to (2.5) we have

$$R(1) = \frac{1}{(1+\theta\gamma)^\mu} - \frac{1}{(1+\theta\gamma+\lambda\gamma)^\mu}, \tag{3.17}$$

$$R(2) = [1-R(1)]\left[\frac{1}{(1+\theta\gamma k)^\mu} - \frac{1}{(1+\theta\gamma k+\lambda\gamma k)^\mu}\right], \tag{3.18}$$

$$R(3) = [1-R(1)][1-R(2)]\left[\frac{1}{(1+\theta\gamma k^2)^\mu} - \frac{1}{(1+\theta\gamma k^2+\lambda\gamma k^2)^\mu}\right]. \tag{3.19}$$

In general,

$$R(r) = [1-R(1)][1-R(2)]\dots[1-R(r-1)] \\ \times \left[\frac{1}{(1+\theta\gamma k^{r-1})^\mu} - \frac{1}{(1+\theta\gamma k^{r-1}+\lambda\gamma k^{r-1})^\mu}\right]. \tag{3.20}$$

Substituting the values of $R(r)$, $r = 1, 2, \dots, n$ in (2.1) we can obtain R_n, the reliability of the system.

4. Numerical Evaluation

For some specific values of the parameters involved in the expressions of $R(r)$, $r = 1, 2, 3$ we evaluate the marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 for the above five cases from their expressions obtained in the last section.

Table 1. Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when stress-strength are exponential variates

μ	γ	λ	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	0.3	0.3	0.1	0.1442	0.0235	0.0025	0.1702
1	0.5	0.5	0.1	0.1667	0.0361	0.0040	0.2067
1	0.7	0.7	0.1	0.1716	0.0475	0.0054	0.2245
2	0.3	0.3	0.2	0.1003	0.0247	0.0052	0.1302
2	0.5	0.5	0.2	0.1333	0.0375	0.0081	0.1789
2	0.7	0.7	0.2	0.1525	0.0486	0.0108	0.2120
3	0.3	0.3	0.3	0.0758	0.0254	0.0079	0.1090
3	0.5	0.5	0.3	0.1071	0.0387	0.0123	0.1581
3	0.7	0.7	0.3	0.1290	0.0500	0.0163	0.1953

Table 2. Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when stress-strength are Rayleigh variates

σ_1	σ_2	σ_3	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	3	3	0.1	0.4091	0.0479	0.0005	0.4575
1	5	5	0.1	0.4630	0.1055	0.0012	0.5697
1	7	7	0.1	0.4804	0.1681	0.0021	0.6506
2	3	3	0.2	0.2647	0.0563	0.0025	0.3235
2	5	5	0.2	0.3788	0.1158	0.0054	0.5000
2	7	7	0.2	0.4258	0.1756	0.0090	0.6144
3	3	3	0.3	0.1667	0.0583	0.0062	0.2312
3	5	5	0.3	0.2907	0.1241	0.0135	0.4256
3	7	7	0.3	0.3657	0.1805	0.0216	0.5678

Table 3. Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when stress-strength are Lindley variates

μ	θ	γ	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	3	4	2	0.0553	0.0279	0.0139	0.0972
1	5	6	2	0.0404	0.0200	0.0099	0.0703
1	7	8	2	0.0311	0.0154	0.0076	0.0541
2	3	4	3	0.1000	0.0415	0.0148	0.1564
2	5	6	3	0.0829	0.0312	0.0108	0.1248
2	7	8	3	0.0683	0.0246	0.0084	0.1013
3	3	4	4	0.1217	0.0481	0.0135	0.1833
3	5	6	4	0.1111	0.0366	0.0098	0.1575
3	7	8	4	0.0964	0.0293	0.0077	0.1334

Table 4. Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when stress-strength follows one parameter exponential and stress follows Lindley distributions

μ	θ	γ	k	$R(1)$	$R(2)$	$R(3)$	R_3
1	2	3	2	0.1250	0.0616	0.0302	0.2168
1	3	4	2	0.0859	0.0421	0.0206	0.1487
1	4	5	2	0.0650	0.0318	0.0156	0.1124
2	2	3	4	0.1990	0.0667	0.0179	0.2836
2	3	4	4	0.1554	0.0474	0.0124	0.2152
2	4	5	4	0.1270	0.0368	0.0095	0.1734
3	2	3	6	0.2236	0.0697	0.0130	0.3063
3	3	4	6	0.1900	0.0495	0.0089	0.2484
3	4	5	6	0.1642	0.0387	0.0068	0.2097

Table 5. Values of $R(1)$, $R(2)$, $R(3)$ and R_3 when stress-strength follows one parameter exponential and stress follows Lindley distributions

μ	γ	λ	θ	k	$R(1)$	$R(2)$	$R(3)$	R_3
0.1	0.2	0.3	0.4	2	0.0054	0.0096	0.0159	0.0308
0.2	0.3	0.4	0.5	2	0.0191	0.0310	0.0042	0.0944
0.3	0.4	0.5	0.6	2	0.0411	0.0590	0.0720	0.1721
0.1	0.2	0.3	0.4	3	0.0054	0.0131	0.0250	0.0435
0.2	0.3	0.4	0.5	3	0.0191	0.0395	0.0578	0.1165
0.3	0.4	0.5	0.6	3	0.0411	0.0700	0.0799	0.1910
0.1	0.2	0.3	0.4	4	0.0054	0.0160	0.0311	0.0525
0.2	0.3	0.4	0.5	4	0.0191	0.0456	0.0628	0.1275
0.3	0.4	0.5	0.6	4	0.0411	0.0765	0.0799	0.1955

Results and Discussions

From the Table 1, we notice that if the strength parameter λ and γ increases then the system reliability R_3 increase. When the stress parameter μ increases $R(1)$ decreases but $R(2)$ and $R(3)$ increases. For instance, if $\mu = 1$, $R(1) = 0.1442$ and if μ =2, $R(1) = 0.1003$. In general we see that when γ, λ increases then $R(2)$ and $R(3)$ will also increases. when the attenuation factor k increases then the marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and the system reliability R_3 decreases.

From the Table 2, it is clear that with some set of values of the parameters if σ_1 increases then the system reliability decrease. i.e., if $\sigma_1 = 1$, $R_3 = 0.4575$ and if $\sigma_1 = 2$, $R_3 = 0.3235$. But if the stress parameter σ_2 and strength parameter σ_3 increases then $R(1)$, $R(2)$ and $R(3)$ also increases. Here also see that when the attenuation factor k increases then the marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and the system reliability R_3 decreases.

From the Table 3, it is clear that when the strength parameters θ and γ increases then the system reliability R_3 and marginal reliabilities $R(1)$, $R(2)$, $R(3)$ decreases. But if the stress parameter μ and attenuation factor k increases reliabilities also increases.

From the Table 4, it is clear that with some set of values of the parameters if θ and λ increases then the system reliability decreases and $R(1)$, $R(2)$ and $R(3)$ also decreases. Here also see that when the attenuation factor k and stress parameter μ increases then the marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and the system reliability R_3 increases.

From the tabulated value of Table 5, we observe that when the strength parameters λ and θ and stress parameters μ and γ increases marginal reliabilities $R(1)$, $R(2)$, $R(3)$ and system reliability R_3 increases with constant value of k. When the attenuation factor k increases there are significant increase in the values of $R(2)$, $R(3)$ and R_3 but no significant difference in the values of $R(1)$.

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