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Research Article

Secure Triple Connected Domination Number of a Graph

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Abstract. Secure domination is a well-studied concept [2,4,6]. In this domination, a vertex outside has the chance of coming inside the dominating set by replacing an element of the set without affecting domination. This idea is combined with the concept of triple connected domination, by considering a path between any three vertices of a graph [11,13,16], to introduce a new parameter called secure triple connected domination. A secure dominating set S of V of a nontrivial graph G is said to be secure triple connected dominating set, if the induced sub graph $\langle S \rangle$ is triple connected. Among all the secure triple connected dominating sets of the graph G, a set having the minimum cardinality is called the secure triple connected domination number denoted by γ_{stc} of G. We have determined the exact values of secure triple connected domination number for some standard graphs and obtained bounds for this new parameter. NORDHAUS-GADDUM type results and the relationship of this parameter with other graph theoretical parameters are also discussed.

Keywords. Domination number; Secure domination number; Triple connected domination number; Secure connected domination number; Secure triple connected dominating set; Secure triple connected domination number

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1. Introduction

By a ***graph*** we mean a finite, simple, connected and undirected graph $G = (V,E)$, where V denotes its vertex set and E its edge set. Unless otherwise stated, the graph G has n vertices and m edges. ***Degree*** of a vertex v is denoted by $d(v)$, the ***maximum degree*** of a graph G is denoted by $\Delta(G)$. We denoted a cycle on n vertices by C_n, a ***path*** on n vertices by P_n, and a ***complete graph*** on n vertices by K_n. A graph G is ***connected*** if any two vertices of G are connected by a path. A maximal connected sub graph of a graph G is called a ***component*** of G. The number of component of G is denoted by $\omega(G)$. The *complement* $\bar{G}$ of G is the graph with vertex set V in which two vertices are adjacent if and only if they are not adjacent in G. A ***tree*** is a connected acyclic graph. A ***bipartite graph*** (or ***bigraph***) is a graph whose vertex set can be divided into two disjoint sets V_1 and V_2 such that every edge has one end in V_1 and another end in V_2. A ***complete bipartite graph*** is a bipartite graph where every vertex of V_1 is adjacent to every vertex in V_2. The complete bipartite graph with partitions of order $|V_1| = r$ and $|V_2| = s$ is denoted by $K_{r,s}$. A star, denoted by $K_{1,n-1}$ is a tree with one root vertex and $n-1$ pendant vertices. A bistar $B(r,s)$ is the graph obtained by joining the root vertices of the the stars $K_{1,r}$ and $K_{1,s}$. The ***friendship graph***, denoted by F_n can be constructed by identifying n copies of the cycle at a common vertex. A ***wheel graph***, denoted by W_n is a graph with p vertices, formed by connecting a single vertex to all vertices of C_{n-1}. A ***helm graph***, denoted by H_n is a graph obtained from the wheel W_n by attaching a pendant vertex in the outer cycle of W_n. ***Corona*** of two graphs G_1 and G_2 denoted by $G_1 \circ G_2$ is the graph obtained by taking one copy of G_1 and $|V_1|$ copies of G_2 ($|V_1|$ is the order G_1) in which ith vertex of G_1, is joined to every vertex in ith copy of G_2. If S is a subset of V, then $\langle S \rangle$ denotes the vertex induced sub graph of G induced by S. The ***open neighborhood*** of a set S of vertices of a graph G, denoted by $N(S)$ is the set of all vertices adjacent to some vertex in S and $N(S) \cup S$ is called the ***closed neighborhood*** of S denoted by $N[S]$. The ***diameter*** of a connected graph is maximum distance between two vertices in G and is denoted by $\operatorname{diam}(G)$. A cut-vertex (cut edge) of a graph G is a vertex (edge) whose removal increases the numbers of components. A ***vertex cut***, or ***separating*** set of a connected graph G is a set of vertices whose removal results in a disconnected graph.

The ***connectivity*** or ***vertex connectivity*** of a graph G, denoted by $\kappa(G)$ (where G is not complete) is the size of a smallest vertex cut. A connected sub graph H of a connected graph G is called a ***H-cut*** if $\omega(G-H) \geq 2$. The ***chromatic number*** of a graph G, denoted by $\chi(G)$ is the smallest number of colors needed to color all the vertices of a graph G in which adjacent vertices receive different colors. For any real number x, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x. A ***Nordhaus-Gaddum type*** results is a (tight) lower or upper bound on the sum or product of parameter of a graph and its complement. Terms not defined here are used in the sense of [1].

A subset S of V is called a ***dominating set*** of G if every vertex in $V-S$ is adjacent to at least one vertex in S. The ***domination number*** $\gamma(G)$ of G is the minimum cardinality taken over all dominating sets in G. A dominating set S of a connected graph G is said to be a ***connected dominating set*** of G if the induced sub graph $\langle S \rangle$ is connected. The minimum cardinality taken over all connected dominating sets is the ***connected domination*** number and is denoted by γ_c. A subset S of V is a ***secure dominating set*** [2, 4–6] if for each $u \in V-S$, there exist a vertex v such that $v \in N(u) \cap S$ and $(S-\{v\}) \cup \{u\}$ is a dominating set of G. The minimum cardinality of a secure dominating set is the ***secure domination number*** of G, denoted by $\gamma_s(G)$. A secure dominating set of G of cardinality $\gamma_s(G)$ is called $\gamma_s(G)$-set. Let S be a connected dominating set in G, a vertex $v \in E$ is said to ***S-defend*** u [3] where u$\in V-S$, if $uv \in EG$) and $(S-\{v\}) \cup \{u\}$ is a connected dominating set in G. S is a ***secure connected dominating set*** in G if for each $u \in V-S$, there exists $v \in S$ such that v S-defend u. The ***secure connected domination number*** γ_{SC} of G is the smallest cardinality of a secure connected dominating set in G.

Many authors have introduced different types of domination parameters by imposing conditions on the domination set. Recently, the concept of triple connected graphs has been introduced by Paulraj Joseph *et al.* [14], by considering the existence of a path containing any three vertices of G. They have studied the properties of triple connected graphs and established many results on them. A graph G is said to be ***triple connected*** if any three vertices lie on a path in G. All paths, cycles, complete graphs and wheels are some standard examples of triple connected graphs in G. Mahadevan *et al.* [11] was introduced the concept of triple connected domination number of a graph. A subset S of V of a non-trivial graph G is said to be a ***triple***

connected dominating set if S is a dominating set and the induced sub graph $\langle S \rangle$ is triple connected. The minimum cardinality taken over all triple connected dominating sets is called the ***triple connected domination number*** of $G_{tc}(G)$. Any triple connected dominating set with γ_{tc} vertices is called a γ_{tc}-set of G.

In this paper we use this idea to develop the concept of secure triple connected dominating set and secure triple connected domination number of a graph.

Theorem 1.1 ([14])**.** *A tree is triple connected if and only if* $T \cong P_n$, $n \geq 3$.

Theorem 1.2 ([14])**.** *A connected graph is not triple connected if and only if there exists a* H*-cut with* $\omega(G-H) \geq 3$ *such that* $|V(H) \cap N(C_i)| = 1$ *for atleast three components of* c_1, c_2 *and* c_3 *of* $G-H$.

Theorem 1.3 ([11])**.** *If the induced sub graph of each connected dominating set of G has more than two pendent vertices, then G does not contain a triple connected dominating set.*

Notation 1.4. Let G be a connected graph with n vertices $v_1, v_2, \ldots, v_n$. The graph obtained from G by attaching r_1 times a pendent vertex of P_{l_1} on the vertex v_1, r_2 times a pendent vertex of P_{l_2} on the vertex v_2 and so on, is denoted by $G(r_1P_{l_1}, r_2P_{l_2}, r_3P_{l_3}, \ldots, r_nP_{l_n})$ where r_i, $l_i \geq 0$ and $l \leq i \leq n$.

Example 1.5. Let v_1, v_2, v_3, v_4 be the vertices of K_4. The graph $K_4(2P_2, P_3, P_4, 2P_3)$ is obtained from K_4 by attaching 2 times a pendent vertex of P_2 on v_1, 1 time a pendent vertex of P_3 on v_2, 1 time a pendent vertex of P_4 on v_3 and 2 times a pendent vertex of P_3 on v_4 is shown in Figure 1.1.

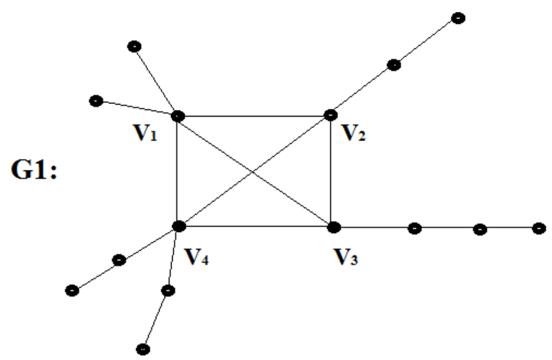

Figure 1.1

2. Secure Triple Connected Domination Number

Definition 2.1. A subset S of V of a nontrivial connected graph G is called a ***secure triple connected dominating set***, if S is a secure dominating set and the induced sub graph $\langle S \rangle$ is triple connected. The minimum cardinality taken over all ***secure triple connected domination number*** of G, is denoted by $\gamma_{stc}(G)$. Any ***secure triple connected dominating set*** with $\gamma_{stc}(G)$ vertices is called γ_{stc}-set of G.

Example 2.2. For the graph G_2, $\gamma_2 = 3$, $\gamma_{tc} = 3$, $\gamma_{stc} = 3$.

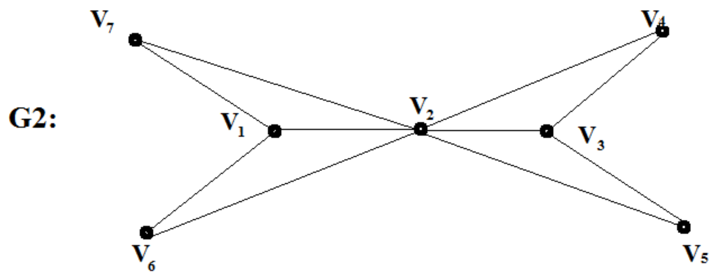

Figure 2.1. Graph with $\gamma_s = \gamma_{sc} = \gamma_{tc} = \gamma_{stc} = 3$

Example 2.3. For the graph G_3, γ_2 and $\gamma_{stc} = 3$.

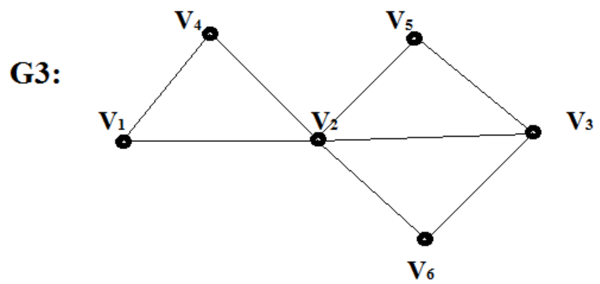

Figure 2.2. Graph with $\gamma_s < \gamma_{stc}$

3. Results

Proposition 3.1. *For any graph G : $\gamma_s(G) \leq \gamma_{sc}(G) \leq_{stc} (G)$.*

Proof. For any graph G, it is clear that secure triple connected dominating set is a secure connected dominating set which in turn, is a secure dominating set. □

Example 3.2. For the connected graph G_4, $\gamma_s = \gamma_{sc} = 1$ and $\gamma_{stc} = 3$.

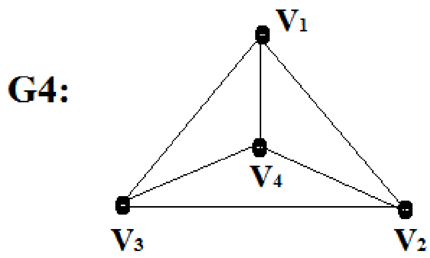

Figure 3.1. Graph with $\gamma_s = \gamma_{sc} < \gamma_{stc}$

Proposition 3.3. *For any connected graph G, $\gamma_{tc}(G) \leq_{stc} (G)$.*

Note 3.4 (Chain Rule). $\gamma_c \leq \gamma_s \leq \gamma_{sc} \leq \gamma_{tc} \leq \gamma_{stc}$.

Observation 3.5. Secure triple connected dominating set does not exist for all graph and if it exists then $\gamma_{stc}(G) \geq 3$.

Observation 3.6. The complement of a secure triple connected dominating set need not be a secure triple connected dominating set.

Example 3.7. For the graph G_5, $\gamma_{stc} = 3$.

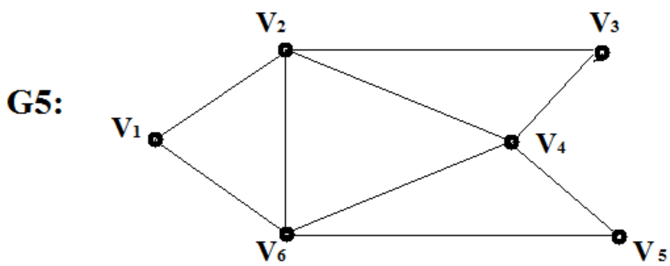

Figure 3.2. Graph without complement γ_{stc}-set

Theorem 3.8. *A tree is a secure triple connected dominating set if and only if $T \cong P_n$, $n \geq 3$* [11].

Proof. Let $T \cong P_n$, $n \geq 3$. It is clear that for every P_n, $n \geq 3$ there exists a secure triple connected domination set. Thus $T \cong P_4$, also has a secure triple connected domination set. Conversely, assume that, $T \ncong P_n$, $n \geq 3$ since T is a tree, every vertex is a cut vertex. Moreover, as $T \ncong P_n$, there is a vertex with $d(v_i) \geq 3$, whose removal will result in $\omega(T - v_i) \geq 3$. Then by Theorem 1.2, we cannot find a triple connected domination set T. Thus, when $T \ncong P_n$, $n \geq 3$, there is no secure triple connected dominating set. □

4. Exact Values of Some Standard Graphs

(i) $\gamma_{stc}(P_n) = \begin{cases} 3, & \text{if } n = 3,4 \\ n-2, & \text{if } n \geq 5. \end{cases}$

(ii) $\gamma_{stc}(K_n) = 3$, if $n \geq 3$.

(iii) $\gamma_{stc}(C_n) = \begin{cases} 3, & \text{for } n \leq 5 \\ n-2, & \text{for } n > 5. \end{cases}$

(iv) $\gamma_{stc}(K_r, s) = 3$, for $r, s \geq 2$ and $r + s = n \geq 4$.

(v) For any wheel graph, with $n \geq 4$

$$\gamma_{stc}(W_n) = \begin{cases} 3, & \text{if } n < 9 \\ n-6, & \text{if } n \geq 10. \end{cases}$$

(vi) For Peterson graph G, $\gamma_{stc}(G) = 5$.

(vii) For Fan graph with $n \geq 3$

$$\gamma_{stc}(f_n) = \begin{cases} 3, & \text{for } n < 10 \\ n-6, & \text{for } n \geq 10. \end{cases}$$

(viii) For any helm graph of order $n \geq 7$

$$\gamma_{stc}(H_n) = \left(\frac{n-1}{2}\right) + 1, \quad \text{where } 2r - 1 = n, \ r \text{ is the order of } W_r.$$

(ix) For any corona graph of order $2n$, $n \geq 2$

$$\gamma_{stc}(G \circ K_1) = n, \ n = \text{order of } G.$$

Observation 4.1. If a spanning sub graph H of a graph G has a secure triple connected dominating set, then G also has a secure triple connected dominating set [11].

Observation 4.2. Let G be a connected graph and H be a spanning sub graph of G. If H has a secure triple connected dominating set, then $\gamma_{stc}(G) \leq \gamma_{stc}(H)$ [11].

Example 4.3. For the connected graph G_6, $\gamma_{stc}(G) = 4$, $\gamma_{stc}(H_1) = 5$, $\gamma_{stc}(H_2) = 6$.

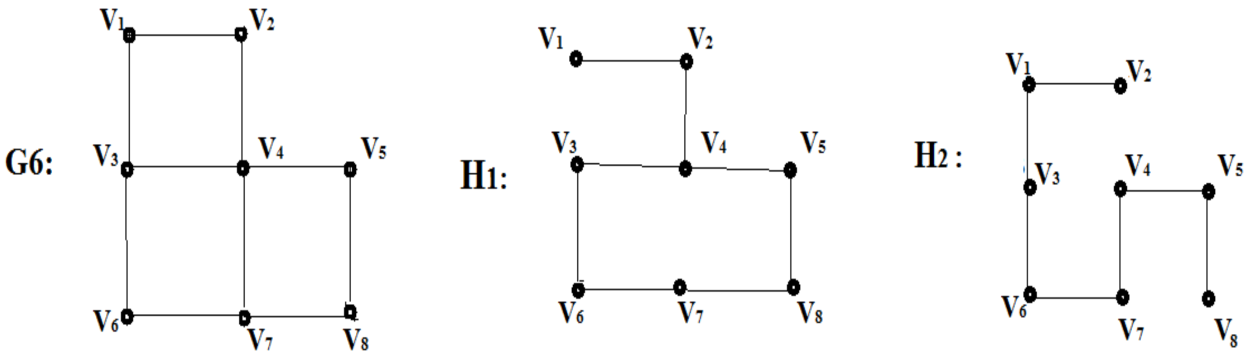

Figure 4.1. Graph with $\gamma_{stc}(H) \geq \gamma_{stc}(G)$

Theorem 4.4. *For any connected graph G with order $n \geq 4$, $3 \leq \gamma_{stc}(G) \leq 2m - n + 1$ and the bound is sharp.*

Proof. The lower bound follows clearly from the Definition 2.1, and for $n \geq 4$, $\gamma_{stc}(G) \leq n - 1 = 2(n-1) - n + 1 \leq 2m - n + 1$. Also, from the Observation 3.5, $\gamma_{stc}(G) \geq 3$. Hence $3 \leq \gamma_{stc}(G) \leq 2m - n + 1$. The above bound is sharp for the graph P_4. □

Observation 4.5. For any connected graph G of order n, $\gamma_{stc}(G) = n$, if and only if $G \cong P_3$ or C_3.

Theorem 4.6. *For any connected graph G of order n, $\gamma_{stc}(G) = n - 1$ if and only if $G \cong P_4, C_4, K_4, K_4 - (e), C_3(P_2), K_2, W_n$.*

Proof. Assume, $G \cong P_4$, C_4, K_4, $K_4 - (e)$, $C_3(P_2)$, K_{22}. Then it is clear that $\gamma_{stc}(G) = 3 = n - 1$. Conversely, let G be a connected graph of order n such that $\gamma_{stc}(G) = n - 1$. Let $S = \{v_1, v_2, \ldots, v_{n-1}\}$ be the γ_{stc}-set of G. Let $x \in V - S$. Since S is the minimum secure triple connected dominating set, then there must exists a vertex $v_i \in S$ such that $v_i x \in E(G)$. Now, if $n \geq 5$ then starting from the vertex v_i, we can construct a secure triple connected dominating set with fewer then $n - 1$ vertices, a contradiction.

Hence, the graph must be of order $n \leq 4$. Now, $\gamma_{stc} = n - 1$ implies that the minimum secure dominating set S of G will have, $S = \{v_1, v_2, v_3\}$ and the set $V - S = \{v_4\}$. Since S is the minimum secure triple connected dominating set, the induced sub graph $\langle S \rangle$ will either be a or P_3 or C_3.

Case (i): $\langle S \rangle = P_3$ $(= v_1 v_2 v_3)$

Since G is a connected graph, the vertex v_4 is adjacent to either or securely v_1 or v_3 to both v_1

and v_3. In either case $G \cong P_4$ or $K_{2,2}$.

Case (ii): $\langle S \rangle = C_3 = v_1v_2v_3v_1$

G being a connected graph, the vertex v_4 may be adjacent to v_1 (or v_2 or v_3) securely which implies $G \cong C_3$ (P_2). Now, adding edges to P_4 or C_3 (P_2) without affecting the $\gamma_{stc}(G)$, we get $G \cong C_4, K_4, K_4 - \{e\}, W_n$. □

Observation 4.7. For any connected graph G with order n, $3 \le \gamma_{stc}(G) \le n-2$.

Theorem 4.8. *For any connected graph G with order $n \ge 5$, $\gamma_{stc}(G) = n-2$ if and only if $G \cong P_n$ or C_n.*

Proof. If we assume G is isomorphic to P_n or C_n, then it is clear that $\gamma_{stc}(G) = n-2$. Conversely, let G be a connected graph with order $n \ge 5$ and S be the $\gamma_{stc}(G)$-set. Then $V - S = \{v_{n-1}, v_n\}$ and it is clear that the induced sub graph $\langle V - S \rangle$ will be either K_2 or $\bar{K}_2$.

Claim: $\langle S \rangle$ is a tree.

If $\langle S \rangle$ is not a tree, then $\langle S \rangle$ must contain a cycle. Without loss of generality, let $C = v_1, v_2, \dots, v_q$, v_1 ($q \le n-2$) be a cycle of shortest length in $\langle S \rangle$. Now, assume $\langle V - S \rangle = v_{n-1}v_n = K_2$. Since G is connected and S is the minimum secure triple connected dominating set of G either v_{n-1} (or v_n) is adjacent to a vertex v_k in $\langle S \rangle$. If v_k is in C then $S = \{v_{n-1}, v_i, \dots, v_{i-3}\} \cup \{x \in V(G), x \in C\}$ forms a γ_{stc}-set of G, so that $\gamma_{stc}(G) < n-2$, a contradiction. Suppose v_{n-1} (or v_n) is adjacent to a vertex v_i in $S - C$, then we can construct a $\gamma_{stc}(G)$-set with fewer than $n-2$ vertices containing v_{n-1}, v_i, again a contradiction.

Similarly, if $\langle V - S \rangle = \bar{K}_2$, we can prove that no graph exists. Hence T has to be a tree. Since S is the minimum secure triple connected dominating set of G, by the $\langle S \rangle \cong P_{n-2}$.

Case (i): $\langle V - S \rangle = K_2 = v_{n-1}, v_n$.

Since G is connected and S is a γ_{stc}-set of G, there exists a vertex, say v_i in P_{n-2} which is securely adjacent to a vertex say v_{n-1} in K_2. If $v_i = v_1$, then $G \cong P_n$. If $v_i = v_1$ is securely adjacent to $v_{n+1}v_n$ with v_{n-1}, then $G \cong C_n$. If $v_i = v_j$ for $j = 2, 3, \dots, n-3$ then, $S_1 = S - \{v_1v_{n-2}\} \cup \{v_{n-1}\}s$ is a secure triple connected dominating set of cardinality $n-3$. Hence $\gamma_{stc}(G) \le n-3$, contradiction.

Case (ii): $\langle V - S \rangle = \bar{K}_2$.

Since G is connected and S is a $\gamma_{stc}(G)$-set of G there exists a vertex say v_i in P_{n-2} which is securely adjacent to both vertices v_{n-1} and v_n if $v_i = v_1$ (or v_{n-2}), then by taking v_1 (or v_{n-2}) we can construct a secure triple connected dominating set with fewer than $n-2$ vertices, a contradiction. Thus no graph exists. If $v_i = v_j$, $j = 2, 3, \ldots, n-3$ then by considering v_j we can construct secure triple connected dominating set with fewer than $n-2$, again a contradiction. □

Corollary 4.9. *Let G be a connected graph with $n > 5$ vertices, with $\gamma_{stc}(G) = n-2$, then $\kappa(G) = 1$ or 2, $\Delta(G) = 2$, $\chi(G) = 2$ or 3 and* $\mathrm{diam}(G) = (n-1)$ *or* $\lceil \frac{n}{2} \rceil$.

Proof. Given G is a connected graph with $n > 5$ and has $\gamma_{stc} = (n-2)$.

Then by Theorem 4.8, $G \cong P_n$ or C_n.

When $G \cong P_n$, then $\kappa(G) = 1$, $\Delta(G) = 2$, $\chi(G) = 2$ and $\mathrm{diam}(G) = (n-1)$.

When $G \cong C_n$, we have $\kappa(G) = 2$, $\Delta(G) = 2$, $\chi(G) = \begin{cases} 2, & n \text{ is even} \\ 3, & n \text{ is odd} \end{cases}$ and $\mathrm{diam}(G) = \left\lceil \frac{n}{2} \right\rceil$. □

5. Nordhaus-Gaddum Type Results

Theorem 5.1. *Let G and $\bar{G}$ be connected graph of order $n \geq 4$, then $\gamma_{stc}(G) + \gamma_{stc}(\bar{G}) \leq 2n-2$ and $\gamma_{stc}(G)\gamma_{stc}(\bar{G}) \leq (n-1)^2$.*

Proof. The bound directly follows from Theorem 4.4. For P_4, both the bounds are attained. □

6. Relation between Secure Triple Connected Dominating Set and other parameters

Theorem 6.1. *For any connected graph G with $n \geq 5$, $\gamma_{stc}(G) + \kappa(G) \leq 2n-3$. The bound is sharp if and only if $G \cong K_5$* [11].

Proof. Let G be a connected graph with $n \geq 5$ vertices. We know, $\kappa(G) \leq n-1$. By Theorem 4.8 we have $\gamma_{stc} \leq n-2$. Thus $\gamma_{stc}(G) + \kappa(G) \leq 2n-3$. Suppose G is isomorphic to K_5 then clearly $\gamma_{stc}(G) + \kappa(G) = 2n-3$. Conversely, let $\gamma_{stc}(g) + \kappa(G) = 2n-3$, this is possible only if $\gamma_{stc} = n-2$

and $\kappa(G) = n-1$. But $\kappa(G) = n-1$ implies G is isomorphic to K_n for which $\gamma_{stc} = n-2$ is possible only when $n = 5$. Hence $G \cong K_5$. □

Theorem 6.2. *For any connected graph G with $n \geq 5$, $\gamma_{stc}(G) + \chi(G) \leq 2n-2$. The bound is sharp if and only if $G \cong K_5$* [11].

Proof. Let G be a connected graph with $n \geq 5$ vertices. We know, $\chi(G) \leq n$. By Theorem 4.8 we have $\gamma_{stc} \leq n-2$. Thus $\gamma_{stc}(G) + \chi(G) \leq 2n-2$. Suppose G is isomorphic to K_5 then clearly $\gamma_{stc}(G) + \chi(G) = 2n-2$. Conversely, let $\gamma_{stc}(G) + \chi(G) = 2n-2$, this is possible only if $\gamma_{stc} = n-2$ and $\chi(G) = n$. But $\chi(G) = n$ implies G is isomorphic to K_n for which $\gamma_{stc} = n-2$, is possible only when $n = 5$. Hence $G \cong K_5$. □

Theorem 6.3. *For any connected graph G with $n \geq 5$, $\gamma_{stc}(G) + \Delta(G) \leq 2n-3$. The bound is sharp if and only if $G \cong K_5, W_5$.*

Proof. Let G be a connected graph with $n \geq 5$ vertices. We know, $\Delta(G) \leq n-1$. By Theorem 4.8 we have $\gamma_{stc} \leq n-2$. Thus $\gamma_{stc}(G) + \Delta(G) \leq 2n-3$. Let G is isomorphic to K_5 then clearly $\gamma_{stc}(G) + \Delta(G) = 2n-3$. Conversely, let $\gamma_{stc}(G) + \Delta(G) = 2n-3$, this is possible only if $\gamma_{stc} = n-2$ and $\Delta(G) = n-1$. But $\Delta(G) = n-1$ implies G is isomorphic to K_n for which $\gamma_{stc} = n-2$, is possible only when $n = 5$. Hence $G \cong K_5, W_5$. □

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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