



# Electron Acceleration by a Radially Polarized Laser Pulse in an Azimuthal Magnetic Field

Research Article

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**Abstract.** Laser acceleration by radially polarized laser beams takes advantage of the strong longitudinal electric field component at the beam centre. When the laser field intensity is sufficiently high, it can push electrons initially at rest at the beam waist outside the Rayleigh zone and accelerate them to relativistic velocities along the laser axis. To obtain the best results in terms of electron dynamics and energy estimation, we suggest that the electrons could be accelerated to a very high energy level by the radially polarized laser pulse. The additionally used azimuthal magnetic field helps to retain the electron energy during acceleration. In this paper, we describe the electron energy scales with laser power and we explain how the laser beam parameter and the magnetic field both can be optimized for maximal acceleration.

**Keywords.** Laser acceleration; Radially polarized laser beam

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## 1. Introduction

During the last decades very interesting research results have been found in particle acceleration by laser in vacuum. Direct electron acceleration in a short pulse and acceleration in the laser-excited wake-field can compete with each other. For sufficiently short and intense laser pulses, the direct electron acceleration in the pulse dominates [1]. Many theoretical and experimental

models were proposed and developed which target the sequential improvements in techniques for energy gains by electrons during interactions with laser fields. If an electron has large initial energy, then interaction time between the laser and electron increases and peak electron energy increases. To achieve high energy gain it is desirable that the laser intensity is high and pre-accelerated electrons are used [2]. Resonant interaction was suggested as a mechanism for accelerating electrons to highly relativistic energies [3, 4]. The magnetic field is generated by several mechanisms [5]. The electron rotates around the propagation direction of the laser pulse during the interaction with circularly polarized intense laser pulse. Betatron resonance occurs between the electrons and electric field of the laser pulse for two optimum values of the magnetic fields, and the electrons gain much higher energies [6]. The laser can trap electrons in transverse and accelerate them with the longitudinal ponderomotive force at the same time. Resonance occurs between the electric field of the laser pulse and the electron if a wiggler magnetic field of a suitable magnitude and period is externally applied [7]. The value of the optimum magnetic field decreases with initial electron energy. Most of the energy gain is in the longitudinal direction. The acceleration gradient increases with laser intensity [8]. The acceleration depends on the initial laser phase for a long laser pulse [9] and is independent for a short laser pulse [10]. The energy gained by electron increases with laser intensity and decreases with initial electron energy. The Rayleigh length for a wide enough laser beam waist is much larger than the drift distance of electron in the electromagnetic field of the laser; therefore, the change of laser beam waist's width can be neglected. However, if the width of laser beam waist is small, the corresponding Rayleigh length is comparable to or even shorter than the laser electron interaction drift distance. The intensity of laser interacting with the electron decreases due to diffraction during the falling part of the pulse, hence the electron gains net energy [11]. A resonance occurs between the electrons and the electric field of the laser if we apply an intense magnetic field in the direction of laser pulse propagation [12]. A small angle of injection for a sideways injection of electron about the axis of propagation of laser pulse is suggested for better trapping of electron in laser field and stronger betatron resonance under the influence of axial magnetic field [13–17].

In this paper we have employed an RP laser pulse for electron acceleration in the presence of magnetic field in vacuum. The electron experiences a force due to longitudinal component of electric field when interacting with an RP laser pulse. Also all the equations are solved by Runge-Kutta method using Matlab software. This paper is organized as follows. In section 2, we present the physical model used to study electron acceleration. In section 3, we show how electron energy scales with laser power in the ideal case where the particle remains perfectly synchronized with the laser pulse throughout the interaction. Finally, in section 4 we make closing remarks.

## 2. Electron Dynamics

The equation for an RP laser paraxial approximation which propagating along z-axis with electric field  $E = \hat{r}E_r + \hat{z}E_z$ . The electric field component can be written as

$$E_r(r, t, z) = \frac{E_0}{f(z)} \cos(\varphi) \exp\left(-\left(\frac{1}{\tau^2} \left(t - \frac{z - z_L}{c}\right)^2\right) - \frac{r^2}{r_0^2 f^2}\right), \tag{2.1}$$

$$E_z(r, t, z) = E_0 \frac{2}{k_0 r_0 f^2} \left[ \left(1 - \frac{r^2}{r_0^2 f^2}\right) \sin(\varphi) - \frac{z r^2}{z_R r_0 f^2} \cos(\varphi) \right] \cdot \exp\left(-\left(\frac{1}{\tau^2} \left(t - \frac{z - z_L}{c}\right)^2\right) - \frac{r^2}{r_0^2 f^2}\right), \tag{2.2}$$

where  $E_0$  is the amplitude of electric field,  $\varphi$  is the Gaussian beam phase,  $\tau$  is the pulse duration,  $r$  is the radial coordinate,  $Z_L$  is the initial position of the pulse peak,  $r^2 = x^2 + y^2$ ,  $r_0$  is the minimum laser spot size, and  $c$  is the velocity of light. The Gaussian beam parameters is defined as  $f(z) = \sqrt{1 + \xi^2}$ , where  $f(z)$  is the laser beam width parameter, and  $\xi = z/Z_R$  is the normalized propagation distance,  $Z_R = kr_0^2/2$  is the Rayleigh length,  $k$  is the laser wave number. And  $\phi = \omega t - kz + \tan^{-1}(z/Z_R) - \frac{zr^2}{(Z_R r_0^2 f^2)} + \phi_0$ ,  $\omega_0$  is the laser frequency and  $\phi_0$  is the initial phase.

Using Maxwell's equations the magnetic field component related to laser pulse can be given by

$$B_L = -\left(\frac{c}{\omega_0}\right) (\vec{k} \times \vec{E}). \tag{2.3}$$

The azimuth magnetic field used in this calculation can be written by

$$B_\theta = (-x\hat{y}) \frac{B_0}{r_0} \exp\left(-\frac{x^2}{2r_0^2}\right). \tag{2.4}$$

To analyse the electron dynamics, we write the equations governing electron momentum and energy as follows:

$$\frac{dp_r}{dt} = -eE_r + e\beta_z B_\theta, \tag{2.5}$$

$$\frac{dp_\theta}{dt} = -e\beta_z, \tag{2.6}$$

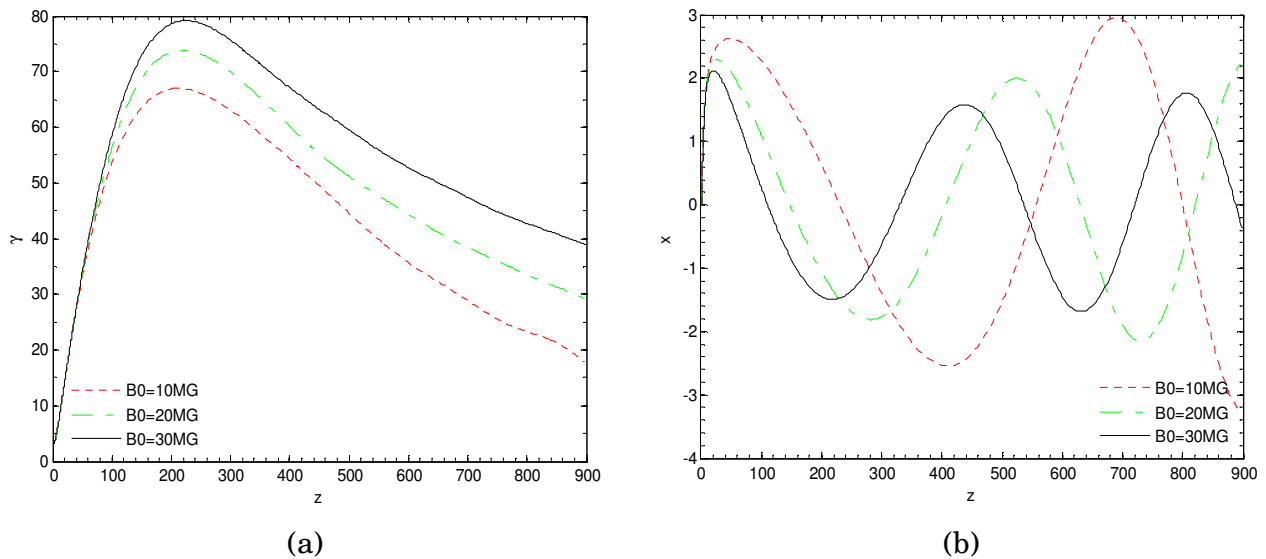
$$\frac{dp_z}{dt} = -eE_z - e\beta_r B_\theta, \tag{2.7}$$

$$\frac{d\gamma}{dt} = -e(\beta_r E_r + \beta_z E_z), \tag{2.8}$$

where  $\gamma^2 = 1 + \frac{p_r^2 + p_\theta^2 + p_z^2}{m_0 c^2}$  is the Lorentz factor and  $p_r$  is the radial component of the electron momentum,  $p_\theta$  is the azimuth component of the electron momentum,  $p_z$  is the longitudinal component of the electron momentum and  $\beta_r$  and  $\beta_z$  are the radial and longitudinal components of normalized velocity,  $\beta = \frac{v}{c}$ , respectively. Electronic charge  $e = 1.6 \times 10^{-19}$  J and rest mass of the electron  $m_0 = 9.1 \times 10^{-31}$  Kg. Equations (2.5) to (2.8) form a set of coupled ordinary differential equations. We solved these equations numerically with Runge-Kutta method

using Matlab for electron trajectory and energy. Throughout this paper time, length, velocity, momentum, energy and frequency are normalized by  $1/\omega$ ,  $1/k$ ,  $c$ ,  $m_0c$ ,  $m_0c^2$  and  $k$ . Following dimensionless variables  $a_0 = eE_0/m_0\omega c$  and  $b_0 = eB_0/m_0\omega$  have been used.

### 3. Results and Discussion

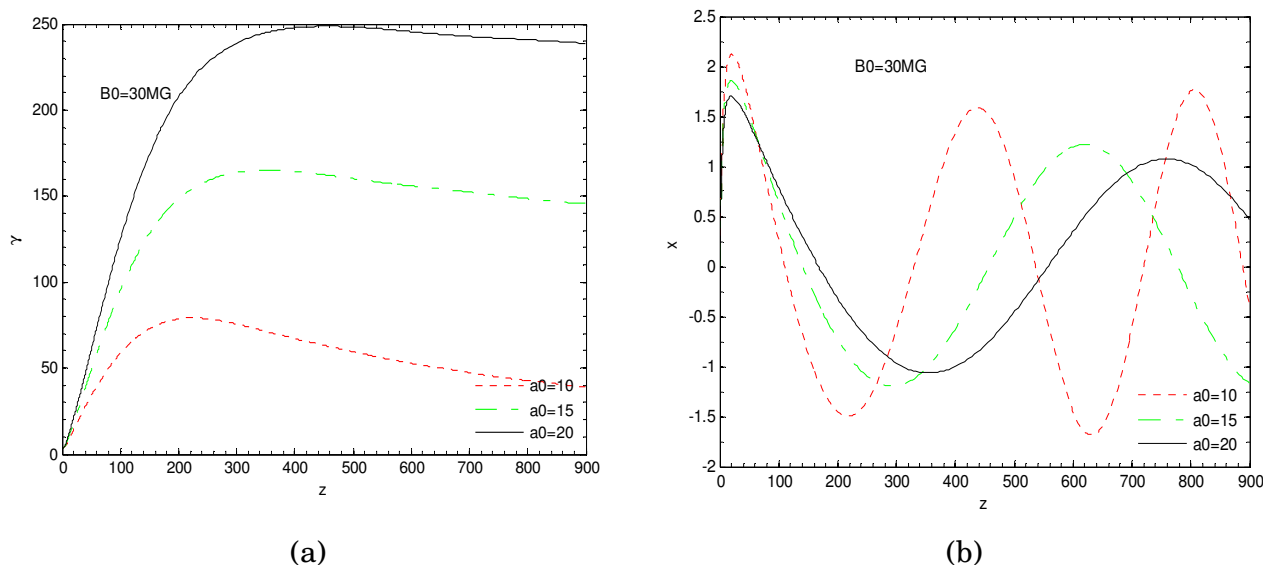


**Figure 1.** Electron energy gain with the propagation distance for different magnetic field strength of 10, 20, 30 MG. The laser intensity parameter is  $a_0 = 10$  corresponding to the laser wavelength of 1 micrometre with intensity of  $1.38 \times 10^{20}$  W/cm<sup>2</sup> and  $r_0 = 20$  (b) the corresponding electron trajectory for the same parameters and  $V_{x_0} = 0.3c$ ,  $V_{z_0} = 0.9c$

The equations (2.5)-(2.8) have been solved using Runge-Kutta method in MATLAB software. Figure 1(a) shows the electron energy with the propagation distance  $Z$  for the strength of magnetic field of  $b_0$  is equal to 0.1, 0.2 and 0.3, respectively. The energy of electrons increases with the initial electron momentum. The above variation of electron energy gains with normalized distance is taken for  $a_0 = 10$  corresponding to the laser wavelength of 1 micrometre with intensity of  $1.38 \times 10^{20}$  W/cm<sup>2</sup>. The black curve is for the magnetic field of  $B_0 = 30$  MG, the green is for  $B_0 = 20$  MG, and the red is for  $B_0 = 10$  MG. The electron energy gain increases and then decreases for all the values of magnetic field that is  $B_0 = 30$  MG, 20 MG and 10 MG, respectively corresponding to  $a_0 = 10$  and  $r_0 = 20$ . Figure 1(b) shows the electron trajectory in  $X-Z$ . If we analysed this trajectory for larger propagation distance then the behaviour of trajectory is look likes as a sinusoidal wave. This shows that the electron is gaining energy and stay in the interaction region to retain it. The Other parameters are taken as  $V_{x_0} = 0.3c$ ,  $V_{z_0} = 0.9c$  for these results.

Figure 2 shows the electron energy gain variation for normalized distance. For this figure we took three different values of the laser intensity parameter  $a_0 = 10, 15$  and 20 at constant

Magnetic field  $B_0 = 30$  MG for which  $r_0 = 20$  and  $b_0 = 0.3$  is adopted. The energy gained by the electron and acceleration gradient increases with the laser intensity the laser intensities for red, green and black lines are  $1.38 \times 10^{20}$  W/cm<sup>2</sup>,  $3.1 \times 10^{20}$  W/cm<sup>2</sup> and  $5.52 \times 10^{20}$  W/cm<sup>2</sup> at wavelength =  $\mu$  m. The initial parameters for above fig are  $V_{x_0} = 0.3c$ ,  $V_{z_0} = 0.9c$  and  $r_0 = 2.914$ . Also Figure 2(b) is the trajectory in X-Z plane the trajectory is the graph between Z and X axis, the trajectory is plotted for a higher value of the magnetic field with laser intensity parameters  $a_0 = 10, 15$  and  $20$  respectively. The electron traverses more distance in the transverse direction for higher laser intensity. The electron trajectories in the other cases are similar the behaviour looks like Figure 1(b) we also take the parameters  $V_{x_0} = 0.3c$ ,  $V_{z_0} = 0.9c$  and  $r_0 = 2.914$ .

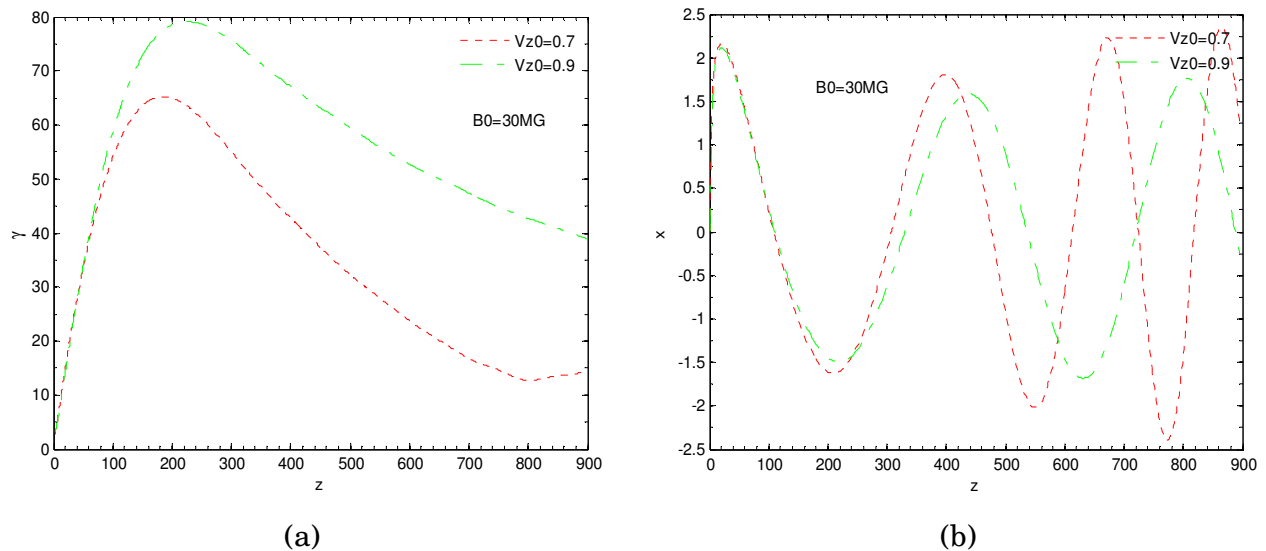


**Figure 2.** Electron energy gain with the propagation distance for different laser intensity in the presence of a magnetic field of 30 MG,  $r_0 = 20$  and  $b_0 = 0.3$ . (b) the corresponding electron trajectory for the laser intensity parameters  $a_0 = 10, 15$  and  $20$ , respectively

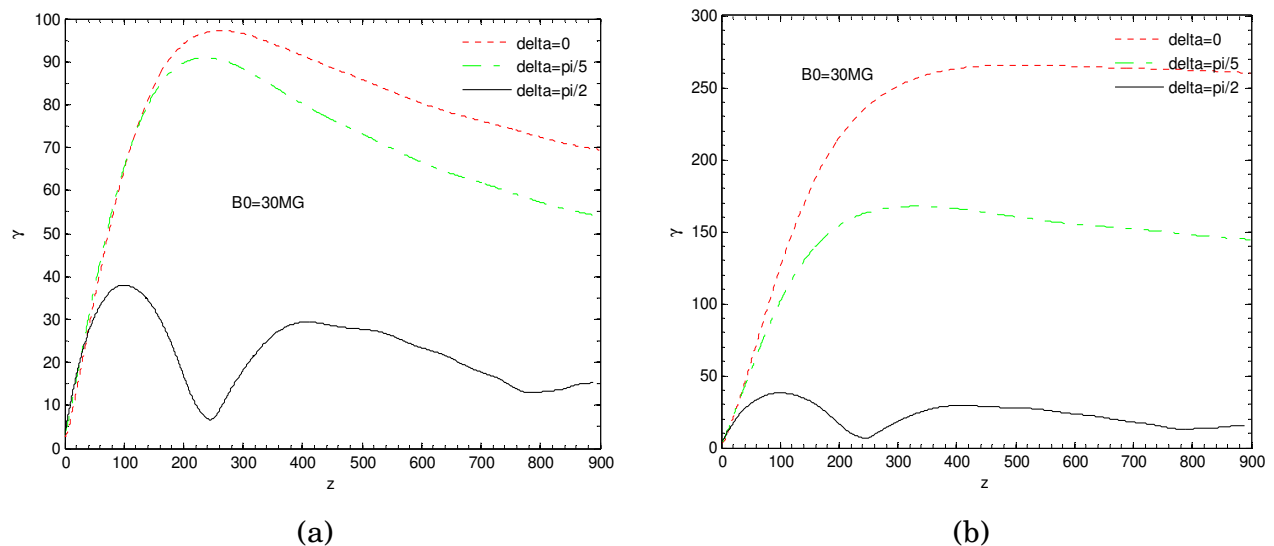
Figure 3 is a plotting of electron energy gain with normalised distance for different values of initial velocity of the electron  $V_{z_0} = 0.9c$  and  $0.7c$ . Here the laser intensity parameter is  $a_0 = 10$ , the electron energy gain is higher for the values of  $V_{z_0} = 0.9c$  as compared to  $V_{z_0} = 0.7c$ . This energy taken at high magnetic field of 30 MG for which  $b_0 = 0.3$  and the laser port size is 20. Figure 3(b) is a trajectory that is the plotting between Z and X for the trajectory the parameters are same in Figure 3(a) but the behaviour of the trajectory is not same as in Figure 2(b) the electron energy gain is different for the different values of velocity components.

Figure 4 shows the relativistic factor  $\gamma$  (electron energy) as a function of the propagation distance at  $a_0 = 10$ ,  $b_0 = 0.3$  and  $r_0 = 20$  and phase ( $\delta$ ) is taken for different values i.e.  $\delta = 0, \pi/5$  and  $\pi/2, r$  respectively. As the propagation distance increases the electron energy gain enhances for all values of the initial phase of the wave ( $\delta$ ). The magnetic field is taken 30 MG for these estimation. The energy gained by the electron increases with the laser intensity and decreases

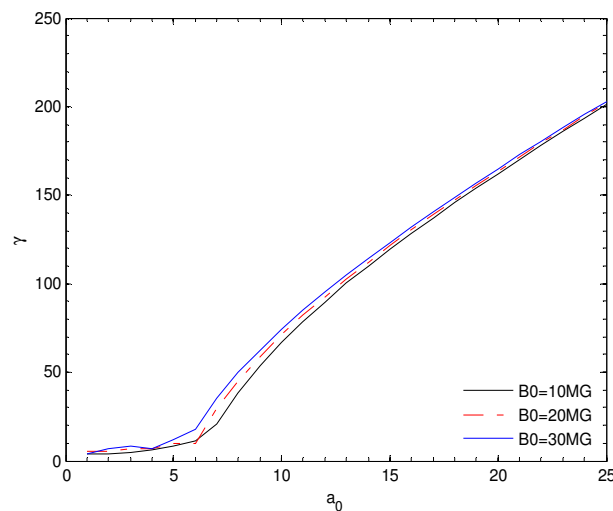
with the initial electron energy. Furthermore, there is a reduction in acceleration gradient with initial momentum with the laser intensity. The resonance between the electric field and the electrons of the laser pulse is stronger at higher and an optimum magnetic field. Consequently, due to this reason the acceleration gradient increases with decrease in initial electron energy.



**Figure 3.** Electron energy gain with the propagation distance for different initial electron energy in the presence of a magnetic field of 30 MG, The laser intensity parameter is  $a_0 = 10$  corresponding to the laser wavelength of 1 micrometre with intensity of  $1.38 \times 10^{20}$  W/cm<sup>2</sup> and  $V_{z_0} = 0.9c$  and  $0.7c$ , (b)  $V_{z_0} = 0.9c$  and  $0.7c$  corresponding electron trajectory for the same parameters.

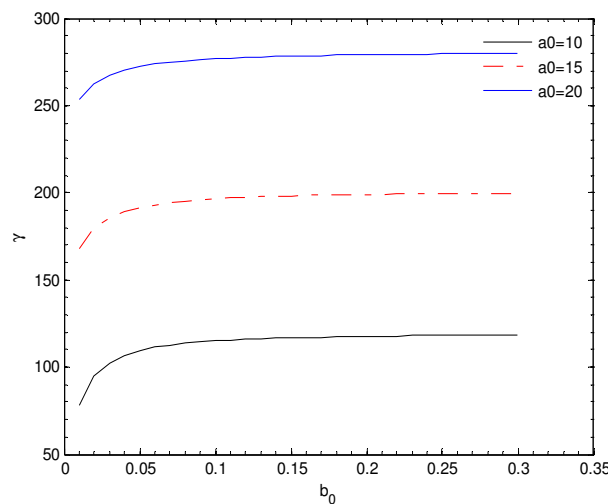


**Figure 4.** Electron energy gain with the propagation distance for different initial phase of the laser in the presence of a magnetic field of 30 MG and the laser intensity parameter is  $a_0 = 10$  corresponding to the laser wavelength of 1 micrometre with intensity of  $1.38 \times 10^{20}$  W/cm<sup>2</sup> and  $r_0 = 20$ , (b) the corresponding electron trajectory for the same parameters and  $V_{x_0} = 0.3c$ ,  $V_{z_0} = 0.9c$ . Also  $\delta = 0, \pi/5$  and  $\pi/2$ , respectively for both the figures.



**Figure 5.** Electron energy gain with the propagation distance for different magnetic field at zero initial phase and  $r_0 = 20$  and  $V_{x_0} = 0.3c$ ,  $V_{z_0} = 0.9c$ .

Figure 5 shows the variation of electron energy gain with  $a_0$  at different values of magnetic field of 10 MG, 20 MG and 30 MG for this variation the values of phase is single that is zero degree and  $r_0 = 20$  as  $a_0$  increases the electron energy gain increases slowly but when  $a_0$  is greater 6 the electron energy games increases quickly and gains more energy this energy for  $B_0 = 30$  MG is higher than other values of the magnetic field which are lower than this value. So above graph also shows that a relation between electron energy gain and  $a_0$  which is theoretically proved.

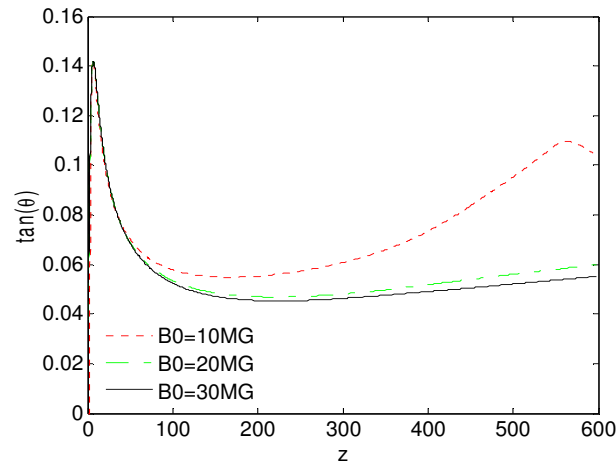


**Figure 6.** The variation of energy verses magnetic field for different values of the laser intensity parameter is  $a_0 = 10, 15$  and  $20$ .

Figure 6 because the related variation of electron energy gain  $\gamma$  with normalized magnetic field  $b_0$ . The electron energy gain in Figure 7 is analysed at different values of normalized



electric fields  $a_0 = 10, 15$  and  $20$ . For a smaller value of  $b_0$  the electron energy gain is much higher when  $a_0 = 20$  as compared to other values of  $a_0$ . After sometime increasing the values of magnetic field the curve saturates.



**Figure 7.** Electron energy scattering for different magnetic field at  $a_0 = 10, 15$  and  $20$

## 4. Conclusions

Laser acceleration by radially polarized laser beams takes advantage of the strong longitudinal electric field component at the beam centre has been investigated in this paper also we conclude that the electron energy scales with laser power. The laser beam parameter and the magnetic field both can be optimized for maximal acceleration. The energy gained by the electron and acceleration gradient increases with the laser intensity the laser intensities and the electron traverses more distance in the transverse direction for higher laser intensity. The propagation distance increases the electron energy gain enhances for all values of the initial phase of the wave ( $\delta$ ). The electron energy increases quickly and gains more energy at different values of magnetic field of 10 MG, 20 MG and 30 MG. At 30 MG the max value of energy for a single values of phase is observed. The electron energy scattering for different magnetic field it is observed that the electron energy gain is much higher for a smaller value of  $b_0$ . The azimuthal magnetic field helps to retain the electron energy during acceleration.

When the laser field intensity is sufficiently high, it can push electrons initially at rest at the beam waist outside the Rayleigh zone and accelerate them to relativistic velocities along the laser axis. To obtain the best results in terms of electron dynamics and energy estimation, we suggest that the electrons could be accelerated to a very high energy level by the radially polarized laser pulse. The additionally used aximuthal magnetic field helps to retain the electron energy during acceleration. In this paper, we describe the electron energy scales with laser power and we explain how the laser beam parameter and the magnetic field both can be optimized for maximal acceleration.



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## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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