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High Harmonic Generation in Quantum Dots in Short Pulse

Research Article

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Abstract. We consider high harmonic generation from a quantum dot (QD) in the presence of chirped laser pulse. The system dynamics is described by coupled equations which are solved numerically for various material parameters of QD and external laser field. Results show the effective controlling effect on the generation of High Harmonic Generation by changing the Chirping parameter of the laser pulse and the size of the QD which otherwise is not possible in other systems.

Keywords. Quantum dot; High harmonic generation; Laser; Runge-Kutta

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1. Introduction

One interesting phenomena studied continuously in the field of interaction of light with matter is the high harmonic generation (HHG) by laser irradiation. HHG is the process of frequency up-conversion induced by the nonlinear interaction processes which provides and important method to obtain coherent high frequency from a low frequency source. In the recent years HHG in atomic or molecular systems have been extensively explored due to its potential applications in attosecond laser pulse and X-ray coherent radiation [1–4]. More recently, the HHG study has been extended to semiconductor quantum dots [5–11] and bulk material [12–14]. Main motivation of the study of HHG in QD is to find an efficient way for frequency up-conversion by virtue of its controllable energy spectra and wavefunctions. In QDs, one can easily tune the energy levels and energy gaps by tailoring the size and shape of the QD or by external parameters like magnetic field and gate voltages. Further, QDs can be coupled to make it equivalent to a molecule. To facilitate the study of HHG in QDs, extremely short duration laser pulses are available for experiments [15] nowadays.

We have studied the interaction of laser pulses with QD leading to HHG. The non-perturbative time dependent Schrodinger equation is used for studying the HHG from QD. Although chirp effects on HHG and attosecond pulse generation has been investigated by Feng and Chu [16] but little has been reported in case of nanostructures. For this reason, in this paper we discuss the laser chirp effects on HHG from QD of different sizes. Results show that with laser chirping we can control the harmonic cutoff.

This paper is organized as follows: in Section 2, the theoretical framework is described; discussions of the results so obtained are presented in Section 3; and conclusions are drawn in Section 4.

2. Theoretical Framework

We consider a two dimensional harmonic potential QD with the charge carriers being free to move only on X-Y plane. We here take a Γ-type three level configuration in the 2-dimensional QD subjected to a laser pulse of the form

$$E(t) = \epsilon f(t) F_0 \cos\left(\omega_L t + \frac{1}{2}\chi t^2\right) \tag{2.1}$$

where 'ϵ' is the unit polarization vector, F_0 is the maximal electric field amplitude, $f(t)$ is the envelop function of the laser and χ is the chirping rate, and for the pulse width τ, the envelop function is defined as

$$f(t) = \begin{cases} 1, & 0 < t < \tau \\ 0, & \text{otherwise} \end{cases} \tag{2.2}$$

The Hamiltonian representing the QD subjected to the laser pulse is

$$H = H_0(\mathbf{r}) + H_I(\mathbf{r}, t) \tag{2.3}$$

where "$H_0(r)$" is the Hamiltonian of the isolated harmonic potential QD placed in external axial magnetic field [17–21] such that

$$(H_0)_i \, \phi_{nl}(r) = (E_{nl})_i \, \phi_{nl}(r) \tag{2.4}$$

where

$$\phi_{nl} = \frac{1}{\sqrt{2\pi a}} \sqrt{\frac{n!}{(n+|l|)!}} \left(\frac{r}{\sqrt{2}a_i}\right)^{|l|} e^{-\frac{r^2}{4a_i^2}} L_n^{|l|}\left(\frac{r^2}{2a_i^2}\right) e^{-il\theta} \tag{2.5}$$

and the effective radius of QD, $a_i^2 = \frac{\hbar^2}{2m_i^*}\frac{1}{(\omega_c)_i}$ where $(\omega_c)_i = \sqrt{(\omega_0)_i^2 + \frac{1}{4}(\omega_l)_i^2}$, $(\omega_0)_i$ and $(\omega_l)_i$ are the harmonic oscillator and cyclotron frequency, respectively. The subscript 'i' stands for conduction (c) and valence (v) bands.

The interaction Hamiltonian $H_I(r,t)$ of the QD with the laser field is

$$H_I(r,t) = -\mu(r)\, f(t)\, F_0 \cos\left(\omega_L t + \frac{1}{2}\chi t^2\right) \tag{2.6}$$

Here '$\mu(\mathbf{r})$' is the QD dipole moment operator.

The time dynamics of the system is dictated by the time-dependent Schrodinger equation, i.e.

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = H(\mathbf{r},t)\psi(\mathbf{r},t) \tag{2.7}$$

where

$$|\psi(t)\rangle = \sum_q c_q(t)\, e^{-i\omega_q t}\, \phi_q(\mathbf{r}) \tag{2.8}$$

Here for simplicity we have replaced 'nl' with 'q' such the eigen energy 'E_{nl} (E_q)' increase with increasing q.

Using the orthogonality condition of the eigenstates of H_0, equation (2.7) converts into the following set of coupled differential equations

$$i\frac{d}{dt}c_m(t) = \sum_n \mu_{mn}\, f(t) F_0 \cos\left(\omega_L t + \frac{1}{2}\chi t^2\right) e^{-i(\omega_m - \omega_n)t} c_n(t) \tag{2.9}$$

This set of coupled equations is solved to determine the expansion coefficients of the system wavefunction. Thus the expectation value of the time dependent dipole moment of QD is determined as

$$\langle \mu_{QD}(t)\rangle = \psi(t)\mu\psi(t) \tag{2.10}$$

The time dynamic dipole moment of the QD gives rise to the higher harmonic generation spectra. The coherent light spectrum is obtained by taking the Fourier transform as

$$S(\omega) = \left|\int \langle \mu_{QD}(t)\rangle\, e^{i\omega t}\, dt\right|^2 \tag{2.11}$$

The study of logarithm of $S(\omega)$ gives the HHG spectrum from the QD.

3. Results and Discussion

The GaAs material parameters are used for the numerical calculations. The energy band gap is 1.43 eV and carrier effective masses in conduction and valence band are 0.067 m_0 and 0.45 m_0 respectively, m_0 being the rest mass of an electron. A three level configuration is considered with lowest energy state being on the top of the valence band and the other two higher energy states are lying at the bottom of the conduction band. The configuration becomes a Γ type of transition scheme with an allowed transition between the lowest energy state and the first excited state and another between the two near lying excited states.

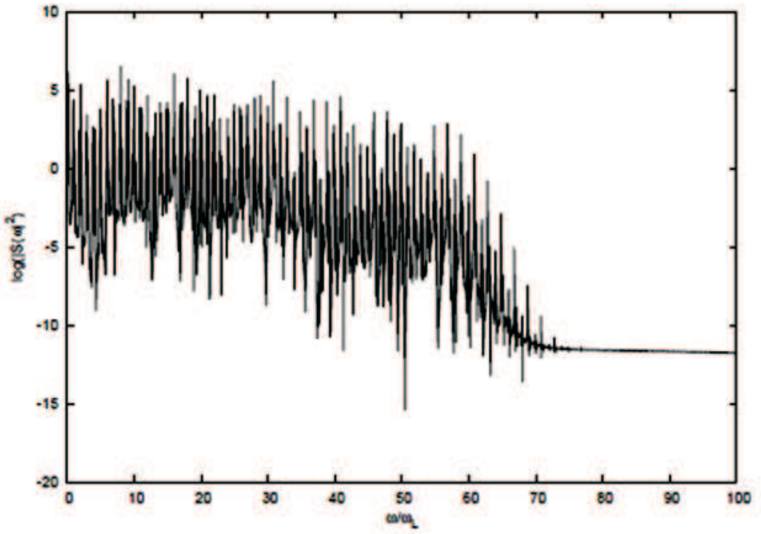


Figure 1. The HHG Spectrum of the QD when shined by a laser pulse of intensity 87.77T W/cm^2.

For that the QD is exposed to a laser pulse of 60 cycle duration with cycle period of 25 fs. The intensity of the pulse is varied from 87.78 TW/cm^2 to 224.65 TW/cm^2. Figure 1 shows the HHG spectrum from the QD system at laser intensity 87.78 TW/cm^2. The spectrum extends to 73rd order whereas in Figure 2, a harmonic generation of, as high as 153rd order is obtained on increasing the pulse intensity to 224.65 TW/cm^2, keeping other conditions identical as before. As the laser intensity increases, the interaction between the levels increases due to greater coupling between the levels resulting from power broadening. This effect results in the higher cutoff of the HHG spectrum in Figure 1 as compared to Figure 2.

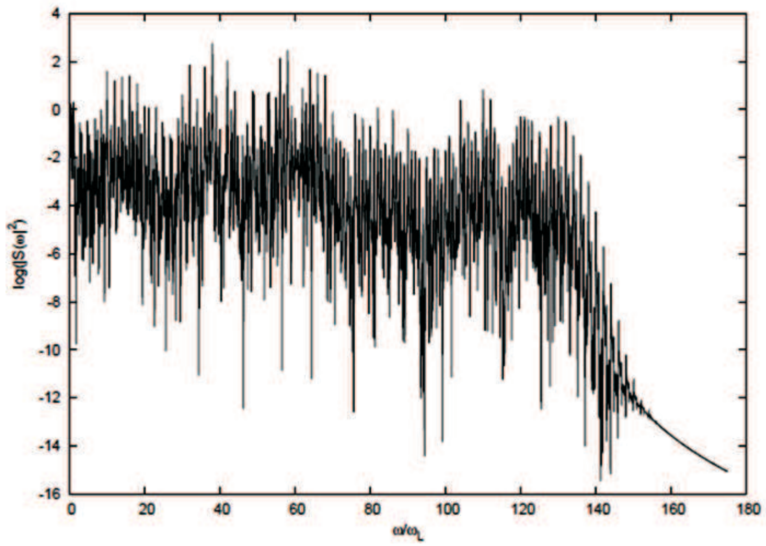
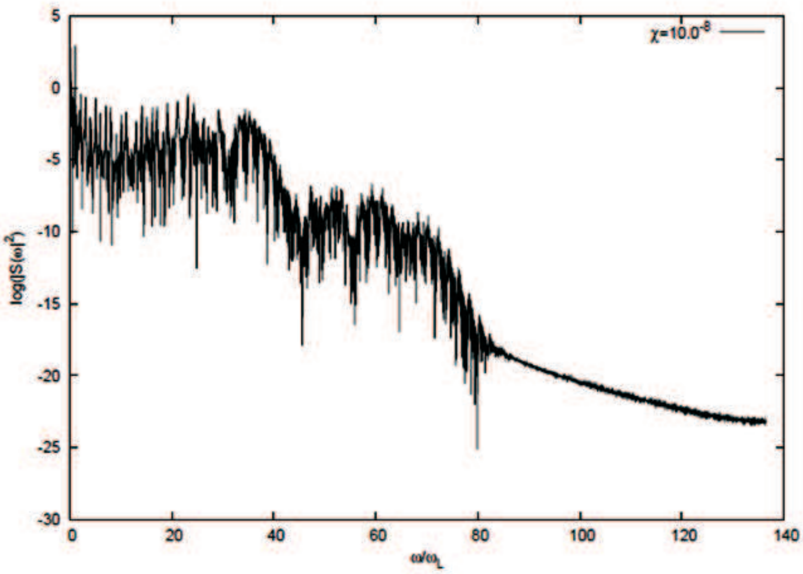
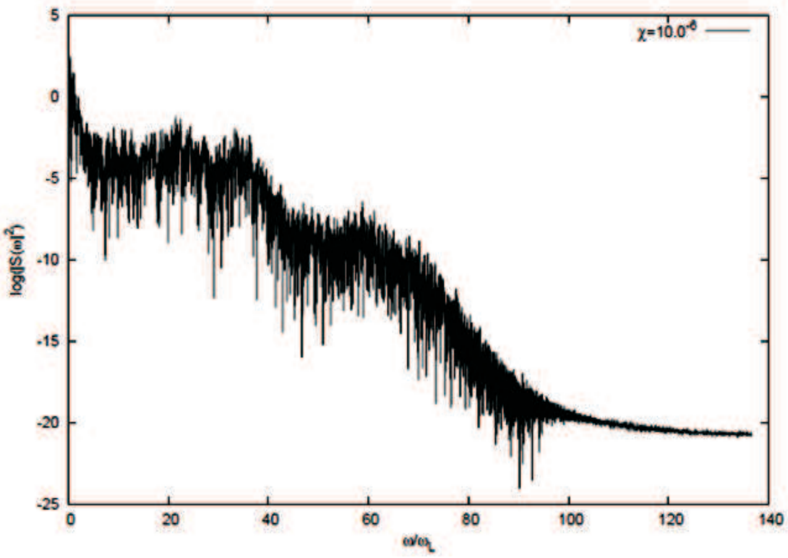


Figure 2. The HHG Spectrum of the QD when shined by a laser pulse of intensity 224.65 TW/cm^2.

It is observed that with the introduction of chirping into the driving laser, the HHG also changes in accordance with nonlinear effects. These changes are being presented as contrast between Figures 3a and 3b. The figures depict the HHG spectrum from the QD system when exposed to the sixty cycle 25fs period laser pulse of intensity 87.78 TW/cm^2 but with different chirping rates i.e. $1.2418 \times 10^{-21} s^{-2}$ and $1.2418 \times 10^{-19} s^{-2}$ respectively. As the chirping is introduced, the cutoff of the HHG increases from 73rd harmonic in Figure 1 to 83rd harmonic in Figure 3a, where the chirping rate is $1.2418 \times 10^{-21} s^{-2}$, the cutoff further increases to 105th order in Figure 3a on increasing the chirping rate to $1.2418 \times 10^{-19} s^{-2}$. The shape of the spectra also changes noticeably from Figure 1 to 3a-3b. These changes are resulted from the modification in the frequency of the driving laser pulse with time. With these modifications, the driving laser frequency and its integral multiple gets well tuned with the harmonic frequency.



(a)



(b)

Figure 3. The HHG Spectrum of the QD when shined by a laser pulse of intensity 87.77 TW/cm^2 with chirping rate (a) $1.2418 \times 10^{-21} s^{-2}$ (b) $1.2418 \times 10^{-19} s^{-2}$.

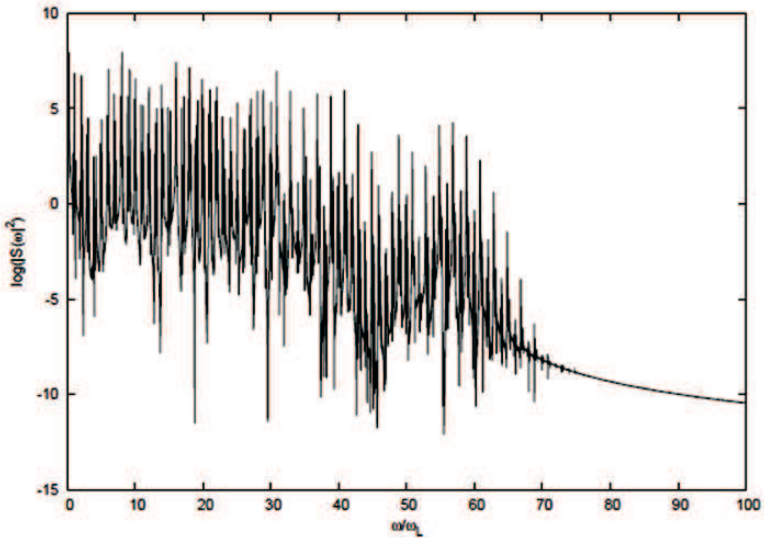


Figure 4. The HHG Spectrum of the QD when shined by a laser pulse of intensity 87.77 TW/cm^2 on QD of radius 22.96 nm.

In Figure 4, the HHG spectra of the QD of radius 22.96nm, which was obtained by placing the QD in a laser pulse of intensity 87.78 TW/cm^2 and sixty cycle duration with cycle period 25 fs. The HHG spectrum is observed to be extended to 73rd harmonic order. It is observed that, with increase in the size of the QD, the plateau region extends further. This effect is evident from Figure 5 where the highest harmonic of 117th order is obtained when the dot size is increased to the effective radius of 60.04 nm. This effect results from the enhancement in the polarizabilty of the QD due to the increase in the dipole moment associated with the intersubband and interband transitions, as the dipole moment for the transitions increases with the size of the QD.

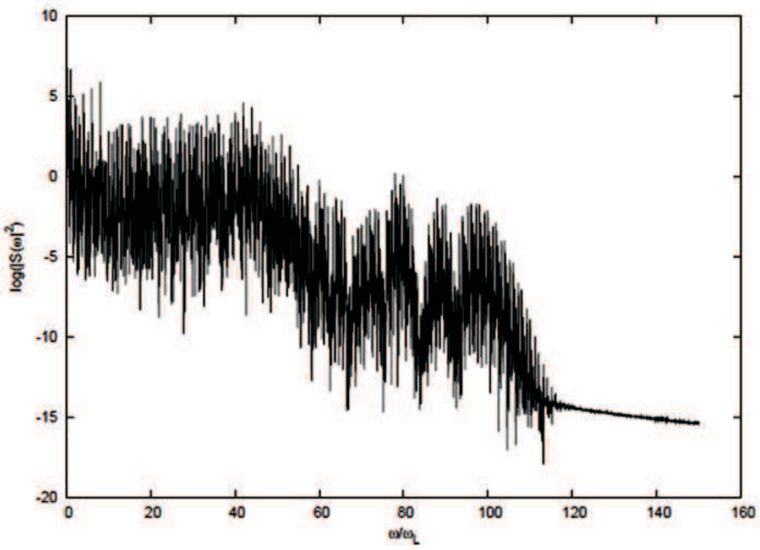


Figure 5. The HHG Spectrum of the QD when shined by a laser pulse of intensity 87.77 TW/cm^2 on QD of radius 60.04 nm.

4. Conclusion

We have demonstrated the high controllability of HHG cutoff as well as the spectra by the laser parameters, namely chirping rate and the intensity. The tailoring of the size of the QD is also shown to be an effective means to manipulate the HHG spectra from the QDs. The frequency up-conversion, thus, can be controlled ny using chirping rate of the laser as control parameter. The controlling can also be achieved by tailoring the size of the QD, as the HHG is shown to be controlled by QD size.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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