



Coherent Population Trapping in An Inhomogeneously Broadened Λ -System with Multiple Excited States

Research Article

Paramjit Kaur^{1,2,*} and Ajay Wasan¹

¹ Indian Institute of Technology, Roorkee, India

² Guru Nanak Dev University, Amritsar, India

*Corresponding author: pjit12@gmail.com

Abstract. We present a theoretical model using density matrix approach to study the phenomenon of *coherent population trapping* (CPT) in the Λ -type Doppler broadened system with multiple excited states. A multi-level Λ -system is formed by considering the closely spaced hyperfine levels in the D2 line of ^{87}Rb . The presence of closely spaced hyperfine levels affects the transparency window and cause asymmetry in absorption profiles. We observe the sharp CPT dip when the frequency difference of applied fields is equal to the frequency separation between two ground levels. This dip demonstrates that the system is trapped in the dark state. By performing a thermal averaging in the Doppler-broadened Λ -system, we have shown that the transparency window becomes narrower and its linewidth decreases at higher temperature. This study has potential applications in precision measurements due to the small linewidth of the transparency window.

Keywords. Coherent population trapping; Doppler-broadened system; Transparency window; Absorption and multiple excited states

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1. Introduction

In the last two decades, light-atom interaction is a topic of great interest in the field of atom optics and laser spectroscopy. This interaction induces the coherence between the atomic

states that modifies the optical properties of a medium. Consequently, this has led to many important phenomena such as *electromagnetically induced transparency* (EIT) [1, 2], *coherent population trapping* (CPT) [3, 4] and *lasing without inversion* (LWI) [5]. But in recent years, the phenomenon of coherent population trapping (CPT) has attracted substantial attention in several theoretical and experimental studies in the three-level atomic systems. Now a days, CPT is extensively used for precision measurement applications such as chip-scale atomic clocks [6], atomic magnetometry [7], laser cooling [8] and trapping of atoms in optical lattices [9] etc.

CPT is said to have occurred when two ground hyperfine levels of alkali atoms are optically coupled to a common excited level by two coherent electromagnetic (e.m.) fields give rise to trapping of population in a coherent superposition of the ground levels, provided that the frequency difference of two applied e.m. fields exactly matches ground-state splitting frequency. By this technique, the atoms are prepared in a non-absorbing state called dark state. Once the population will all be optically pumped into the dark state through spontaneous emission from the intermediate state, then there is no process to remove the population. Thus the population is trapped in the dark state [4].

In the present paper, we propose an extensive study to see the influence of a multiple excited states on the Λ -type CPT system and it has been compared with a similar three-level system. We consider the case of a stationary as well as a moving atoms and perform thermal averaging of velocities at room temperature atomic vapor using Maxwell-Boltzmann distribution function.

This paper is organized as follows: in Section 2, a theoretical model is presented by using a density matrix approach. The numerical results of the five-level system are discussed and compared with the similar three-level system in Section 3. Finally, Section 4 gives a conclusion of this theoretical work.

2. Theoretical Model

We consider a five-level system in ^{87}Rb atom to show the influence of hyperfine levels of atomic excited states on the Λ -type CPT system. The energy level diagram of ^{87}Rb D_2 line is shown in Figure 1. The five-level system involves two hyperfine levels of a ground state $5^2\text{S}_{\frac{1}{2}}$ and three hyperfine levels of the excited state $5^2\text{P}_{\frac{3}{2}}$. The ground state $|1\rangle \equiv |F=1, m_F=1\rangle$ coupled with the excited states $|3\rangle \equiv |F=0, m_F=0\rangle$, $|4\rangle \equiv |F=1, m_F=0\rangle$ and $|5\rangle \equiv |F=2, m_F=0\rangle$ by a weak σ^- polarized probe field. The second set of atomic transitions is excited by the strong σ^+ polarized control field coupling the ground state $|2\rangle \equiv |F=1, m_F=-1\rangle$ with the same excited states $|3\rangle$, $|4\rangle$ and $|5\rangle$. The detuning of the probe with frequency ω_p and control field with frequency ω_c from the atomic transition $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |3\rangle$ will be denoted, $\Delta_p = \omega_p - \omega_{31}$ and $\Delta_c = \omega_c - \omega_{32}$, respectively. Here $\omega_{jk} = (E_j - E_k)/\hbar$ is defined as the atomic transition frequency between levels j and k ($j > k$), and E_j is the energy of the unperturbed atomic state $|j\rangle$. The decay rate from state $5^2\text{P}_{\frac{3}{2}}$ ($\Gamma_3, \Gamma_4, \Gamma_5$) is 6.1 MHz. Our five-level system can be reduced to simple three-level system if only $|1\rangle$, $|2\rangle$ and $|3\rangle$ states are present.

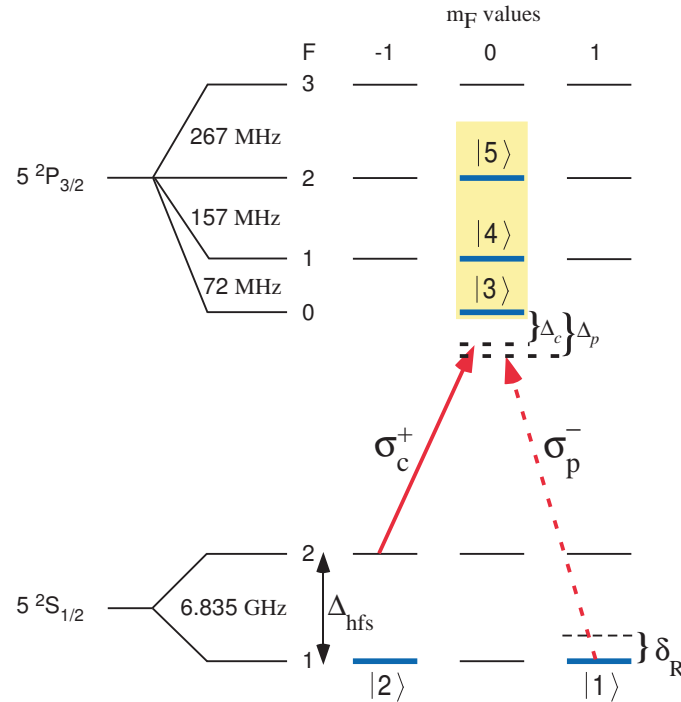


Figure 1. A multilevel Λ -system of ^{87}Rb -D2 line with only the considered m_F values. In the five-level system, $|1\rangle$ and $|2\rangle$ are the ground states and $|3\rangle$, $|4\rangle$ and $|5\rangle$ are the excited states. The three-level approximation involves only $|1\rangle$, $|2\rangle$ and $|3\rangle$ states.

The expression for total Hamiltonian after carrying out rotating wave approximation is written as:

$$\hat{H} = \hbar \left((\delta_R + \Delta_{hfs}) |2\rangle \langle 2| + \sum_{j=3}^5 (\delta_R + \Delta_{hfs} - \Delta_c + \omega_{j3}) |j\rangle \langle j| \right) + \frac{\hbar}{2} \left(\sum_{j=1}^3 (\Omega_{p1j} |1\rangle \langle j| + \Omega_{c2j} |2\rangle \langle j|) + h.c. \right) \quad (1)$$

where $h.c.$ is the hermitian conjugate of the preceding off diagonal terms, Ω_{xjk} (with $x = p, c$ for the σ^- -probe and σ^+ -control fields, respectively) are Rabi frequencies of the applied fields between $|j\rangle$ and $|k\rangle$ states. The conjugates of the Rabi frequencies are given as $\Omega_{xkj}^* = \Omega_{xjk}$. The terms Δ_{hfs} and δ_R denote the ground state splitting and Raman detuning, respectively. The Raman detuning is defined as $\delta_R = (\omega_p - \omega_c) - \Delta_{hfs}$. In the framework of semiclassical theory, the time evolution of system is governed by *Liouville equation* for a density matrix (ρ) and incorporating the decay rates by a relaxation matrix (Γ) [10]:

$$i\hbar\dot{\rho} = [H, \rho] - i\hbar\frac{1}{2}(\Gamma\rho + \rho\Gamma). \quad (2)$$

Here Γ is a diagonal matrix of the decay rate of each state. We have considered that our five-level system is closed since all of atom from $5^2P_{3/2}$ decays to the ground state $5^2S_{1/2}$ with unit branching ratio. So, closure of the system requires that

$$\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} + \rho_{55} = 1. \quad (3)$$

By substituting expression (1) into (2), we obtain the various coupled density matrix equations by using the procedure given in [11]. We have solved these coupled density-matrix equations for a steady-state solution ($\dot{\rho} = 0$). The steady state solution for ρ_{13} is given by:

$$\begin{aligned} \rho_{13} = & -i \frac{\Omega_{p13}}{2\gamma_{13}x_{13}} + i \frac{\Omega_{c23}}{8\gamma_{12}\gamma_{13}x_{12}x_{13}} \left[\frac{\Omega_{p14}\Omega_{c24}^*}{\gamma_{14}x_{14}} + \frac{\Omega_{p15}\Omega_{c25}^*}{\gamma_{15}x_{15}} \right] \\ & - i \frac{\Omega_{c23}(|\Omega_{c25}|^2\Omega_{c24}^*\Omega_{p14} + |\Omega_{c24}|^2\Omega_{c25}^*\Omega_{p15})}{32\gamma_{12}^2\gamma_{13}\gamma_{14}\gamma_{15}x_{12}^2x_{13}x_{14}x_{15}} \\ & + i \frac{\Omega_{c23}|\Omega_{c24}|^2|\Omega_{c25}|^2\Omega_{c25}^*\Omega_{p15}}{128\gamma_{12}^3\gamma_{13}\gamma_{14}\gamma_{15}^2x_{12}^3x_{13}x_{14}x_{15}^2}, \end{aligned} \quad (4)$$

where

$$\begin{aligned} x_{15} &= 1 + \frac{|\Omega_{c25}|^2}{4\gamma_{12}\gamma_{15}x_{12}}, \\ x_{14} &= 1 + \frac{|\Omega_{c24}|^2}{4\gamma_{12}\gamma_{14}x_{12}} - \frac{|\Omega_{c24}|^2|\Omega_{c25}|^2}{16\gamma_{12}^2\gamma_{14}\gamma_{15}x_{12}^2x_{15}}, \\ x_{13} &= 1 + \frac{|\Omega_{c23}|^2}{4\gamma_{12}\gamma_{13}x_{12}} - \frac{|\Omega_{c23}|^2}{16\gamma_{12}^2\gamma_{13}x_{12}^2} \left[\frac{|\Omega_{c24}|^2}{\gamma_{14}x_{14}} + \frac{|\Omega_{c25}|^2}{\gamma_{15}x_{15}} \right] + \frac{|\Omega_{c23}|^2|\Omega_{c24}|^2|\Omega_{c25}|^2}{32\gamma_{12}^3\gamma_{13}\gamma_{14}\gamma_{15}x_{12}^3x_{14}x_{15}} \\ & - \frac{|\Omega_{c23}|^2|\Omega_{c24}|^2|\Omega_{c25}|^2}{256\gamma_{12}^4\gamma_{13}\gamma_{14}\gamma_{15}^2x_{12}^4x_{14}x_{15}^2}, \\ x_{12} &= 1 + \frac{|\Omega_{p13}|^2}{4\gamma_{12}\gamma_{32}} + \frac{|\Omega_{p14}|^2}{4\gamma_{12}\gamma_{42}} + \frac{|\Omega_{p15}|^2}{4\gamma_{12}\gamma_{52}} \end{aligned}$$

and

$$\begin{aligned} \gamma_{12} &= i(\delta_R + \Delta_{hfs}), \\ \gamma_{13} &= \left(-\frac{\Gamma_3}{2} + i(\delta_R + \Delta_{hfs} - \Delta_c) \right), \\ \gamma_{14} &= \left(-\frac{\Gamma_4}{2} + i(\delta_R + \Delta_{hfs} - \Delta_c + \omega_{43}) \right), \\ \gamma_{15} &= \left(-\frac{\Gamma_5}{2} + i(\delta_R + \Delta_{hfs} - \Delta_c + \omega_{53}) \right), \\ \gamma_{32} &= \left(-\frac{\Gamma_3}{2} + i\Delta_c \right), \\ \gamma_{42} &= \left(-\frac{\Gamma_4}{2} + i(\Delta_c - \omega_{43}) \right), \\ \gamma_{52} &= \left(-\frac{\Gamma_5}{2} + i(\Delta_c - \omega_{53}) \right). \end{aligned}$$

If we apply above solution to three-level system ($|1\rangle$, $|2\rangle$ and $|3\rangle$ states are present), then it takes following form:

$$\rho_{13} = \frac{-i\Omega_{p13}}{\gamma_{13} + \frac{2}{\frac{|\Omega_{c23}|^2}{4}}}$$

This is a well-known solution for the three-level Λ system. So we can say that the solution form of the five-level system is in a right direction. We observe that in equation (4), the second, third and fourth terms arise due to the presence of hyperfine levels in the excited states of the atom, which give additional contributions that can interfere constructively or destructively with first term. The absorption and the dispersion of the probe field is determined by $\text{Im}(\rho_{13})$ and $\text{Re}(\rho_{13})$, respectively. Here, we focus only on the probe absorption.

3. Result and Discussion

In this section, we discuss results of the probe absorption for a stationary as well as for a moving atoms and compare these results with similar three-level system. For a moving atom, we perform thermal averaging at the room temperature to see the effect of all velocities groups on probe absorption profiles. The solid curves are for the five-level system and the dashed curves are for the respective three-level system.

3.1 Stationary Atom

We plot the imaginary part of solution given by eq. (4) versus Raman detuning δ_R in Figure 2 for different values of the control Rabi frequency (Ω_{c23}). In a three-level system, if the control field is considered in resonance with $|2\rangle \rightarrow |3\rangle$ transition, i.e. $\Delta_c = 0$, one clearly seen that the absorption peak splits into a doublet and a sharp CPT dip appears near 6.835 MHz modulation frequency as shown in Figure 2(a) [11]. These doublet peaks are two symmetric dressed states created by the control field. Now as the Rabi frequency of control field Ω_{c23} increases, the separation between these peaks, i.e. the linewidth of transparency window or CPT dip increases. This CPT dip corresponds to the formation of dark state. This means that our system is trapped in dark state. The dark state is the superposition of two ground states $|1\rangle$ and $|2\rangle$, i.e it has no contribution from state $|3\rangle$. Once the system is in dark state, then there is no possibility of excitation to $|3\rangle$ and subsequent spontaneous emission.

Now we discuss the results of the five-level system. Our theoretical calculations shows that the absorption profiles become asymmetric due to the presence of multiple excited states, as shown in Figure 2. Besides this, the transparency window also shifts towards a less value of Raman detuning. This asymmetric behavior and shift of profiles arise because our five-level system has three Λ -channels with different dipole moments for various transitions.

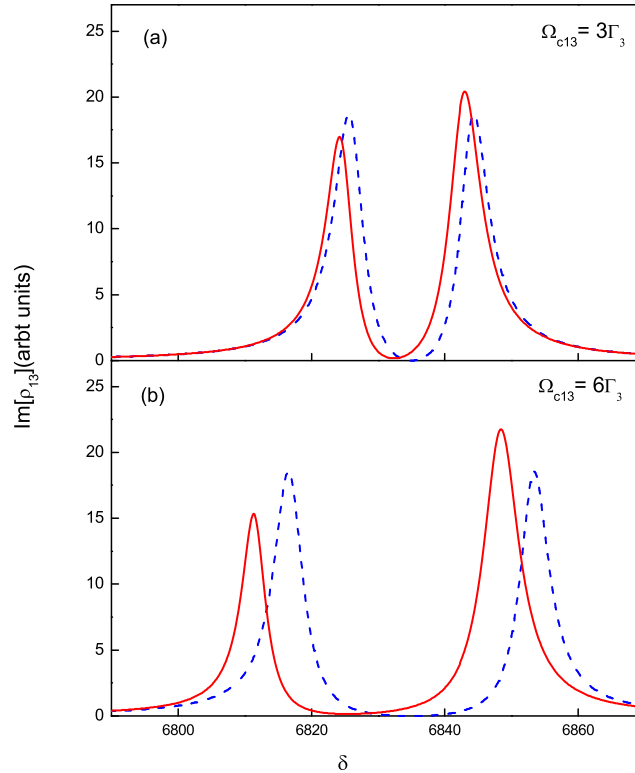


Figure 2. The probe absorption $\text{Im}(\rho_{13})$ for atoms as a function of Raman detuning (δ_R) in the three-level (blue dashed line) and five-level (red solid line) systems for different values of Ω_{c13} . In this calculation, the control and pump field detunings are $\Delta_c = \Delta_s = 0$.

3.2 Thermal Averaging for Moving Atoms

The results presented in previous section are for stationary atoms. In this section, we are interested to see the effects of a room temperature on the probe absorption of the Λ -system with multiple excited states [12, 13]. However, for a atomic vapour at room temperature we have to account for thermal velocity distribution.

To make our calculations Doppler-free, we consider an atom with velocity v interacting with the co-propagating probe and control fields. The Doppler shifts (Δ_D) for probe and control fields are given by $\pm v/\lambda_p$ and $\pm v/\lambda_c$, where λ_p , and λ_c are the wavelengths of the probe and control fields, respectively. The sign \pm depends on whether the atom is moving away from or toward the applied fields. To perform thermal averaging over all the velocities, we use a Maxwell-Boltzman velocity distribution at the room temperature (297K). The most probable velocity corresponding to the room temperature is 237.52 ms^{-1} .

Figure 3 shows the probe absorption curves versus Raman detuning δ_R at 297K. The solid curve is for the five-level system and the dashed curve is for the three-level system. First we consider the three-level system, when the control field is on resonance ($\Delta_c = 0$), our result shows a narrowing of CPT dip near 6.835 MHz of Raman detuning δ_R as shown in Figure 3. This narrowing of dip or decrease in linewidth of dip arises due to summing over all atoms velocities contributions. For a five-level system, we observe that the CPT dip disappears as

shown in Figure 3. This significant modifications in the absorption profiles are due to the presence of multiple excited states in our system. Our numerical calculations show that the probe absorption are reduced significantly after carrying out the thermal averaging.

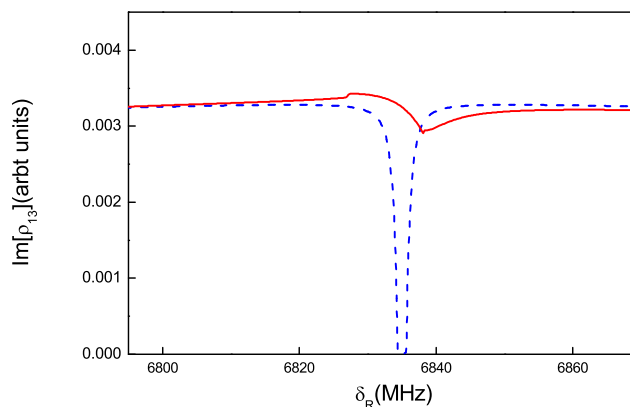


Figure 3. The Doppler-broadened probe absorption as a function of Raman detuning (δ_R) at room temperature 297 K. The red solid and blue dashed lines are for the five-level and three-level systems, respectively. All the graphs are normalized by the same factor. The reduction in the overall scale of absorption and dispersion at room temperature is due to the absorption spreads over all the velocity classes.

4. Conclusion

We have theoretically investigated the influence of multiple excited states on the Λ -type CPT system. Our numerical calculations demonstrate that these multiple excited states lead to a asymmetric absorption profiles as compare with a three-level system.

In the case of a stationary atom, we observe the CPT dip which corresponds to a formation of a dark state. This dip arises when the frequency difference of applied fields is equal to the frequency separation between two ground levels. In a Doppler-broadened system, the probe absorption becomes narrow after carrying thermal averaging at room temperature vapor. This variation in the linewidth of probe absorption may be used for applications in precision measurements.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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