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Research Article

Metal-insulator Transition in $^{70}\text{Ge:Ga}$ Semiconductor by Applying the Scaling Laws

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Abstract. In this article, we focus on the scaling theory of Abraham *et al.* without and with a magnetic field on the metallic side of the *Metal-Insulator Transition* (MIT) for the three-dimensional system $^{70}\text{Ge:Ga}$, at very low temperatures. In particular, we have determined the zero temperature conductivity critical exponent when the MIT transition occurs with the variation of the impurity concentration ($\nu = 0.503$) and with the application of a magnetic field ($\nu = 1.06$). We have also estimated the critical magnetic field B_C that separates the metallic behavior ($B < B_C$) from the variable-range hopping regime ($B > B_C$). The data are for a $^{70}\text{Ge:Ga}$ sample prepared and reported by Itoh *et al.*, *Physical Review Letters* **77** (1996), 4058 and Watanabe *et al.*, *Physical Review B* **60** (1999), 15817.

Keywords. $^{70}\text{Ge:Ga}$ semiconductor; Scaling theory; Low temperature; Magnetic field; Metal-insulator transition; Metallic electrical conductivity; Transport properties; Localization

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1. Introduction

The *Metal-Insulator Transition* (MIT) is one of the most interesting problems in modern condensed matter physics [7, 9, 18, 22, 29–31, 37]. Remarkable progress has been made in the field of MIT, which improves our understanding of electrical transport mechanisms in semiconductors such as crystalline and amorphous semiconductors at low temperatures

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[1, 6, 8, 10], semiconductor alloys [24] and granular films [3, 14, 33].

The electrical conductivity σ of a sample strongly depends on the position of the mobility edge, E_C , (Mott [23]) with respect to the Fermi level E_F and the degree of disorder. Interestingly, by varying the magnetic field or the concentration of carriers or the composition of the samples, the Fermi energy can be lower than the mobility threshold, leading to the transition from a metallic state (delocalized state, $\sigma(T = 0 \text{ K}) \neq 0$) to an insulating state (localized state, where $(T = 0 \text{ K}) = 0$).

In this article, we have study the electrical transport properties of the $^{70}\text{Ge}:\text{Ga}$ system [15, 35], at low temperatures on the metallic side of the MIT. In fact, we checked the two scaling laws of electrical conductivity at $T = 0 \text{ K}$ in the absence and in the presence of the magnetic field. We also determined the critical magnetic field of the MIT.

2. Theoretical Background

In the metallic regime of MIT, the temperature dependence of the electrical conductivity of the 3D metallic samples at low temperatures can be described as follows [12, 25]:

$$\sigma = \sigma(T = 0 \text{ K}) + m T^{1/2}, \quad (2.1)$$

where $\sigma(T = 0)$ is the zero temperature conductivity, m the adjustable parameter, and T the temperature.

We recall that the MIT can be induced by the application of the magnetic field or by varying the impurity concentration. In addition, by analogy with the study of critical points in phase transitions, The scaling law of Abrahams *et al.* could be verified. Mainly following the work of Wegner [36], several critical behaviors have been predicted in the following:

When the MIT is induced by variation of the concentration of impurities n , Abrahams *et al.* [2, 4] showed that the zero temperature conductivity $\sigma(0)$ follows the scaling law:

$$\sigma(0) = \sigma_C \left(\frac{n}{n_C} - 1 \right). \quad (2.2)$$

Here n_C is the critical concentration of impurities that marks the boundary between the metallic and insulating sides and ν is the zero-temperature conductivity critical exponent (with $\nu = 0.5$ or $\nu = 1$).

It is interesting to note that when n approaches n_C , the zero-temperature conductivity approaches zero. This is in good agreement with the scaling theory of Abraham *et al.* [2, 4] who predicted that the minimum metallic conductivity does not exist in non-interacting electron systems.

In the case where the MIT is produced by the introduction of the magnetic field, the relation (2.2) can be written in the following way:

$$\sigma(0) = \sigma'_C \left(1 - \frac{B}{B_C} \right). \quad (2.3)$$

Here σ'_C is a constant that has the dimension of the conductivity, B_C is the critical magnetic field that separates the metallic and insulating phases, and ν is the critical exponent.

According to equation (2.3), the zero-temperature conductivity $\sigma(T = 0, B)$ is close to zero when B approaches B_C .

It is interesting to note that the scaling law have been verified in several systems.

Furthermore, the critical exponent ν depends on the material. We also note that the scale theory predicts that ν is of the order of 1 for three-dimensional materials. However, some authors show that ν takes the value 1/2 for certain materials. We would like to point out that. $\nu = 1$ for semiconductors: GaAs [19,20], Ge:Sb [32], InSb [21], and for the majority of amorphous alloys, as examples: $\text{Nb}_x\text{Si}_{1-x}$ [13], $\text{Bi}_x\text{Kr}_{1-x}$ [28] and amorphous silicon [29]. On the other hand,

$\nu = 1/2$ for uncompensated semiconductors such as Si:As [26], Ge:As [27]

3. Results and Discussion

3.1 Zero-Temperature Electrical Conductivity in the Absence Magnetic Field

We have reanalyzed experimental data for the $^{70}\text{Ge}:\text{Ga}$ system prepared and reported in refs. [15,35].

Figure 1 displays the temperature dependence of the electrical conductivity σ of ten metallic samples $^{70}\text{Ge}:\text{Ga}$ versus $T^{1/2}$, in the temperature interval 0.017-0.53 K and in the absence of a magnetic field. For the different values of the impurity concentration n varying between 1.861×10^{17} and $2.625 \times 10^{17} \text{ cm}^{-3}$.

We note that the critical impurity concentration that marks the boundary between the metallic and insulating samples is equal to $n_C = 1.856 \times 10^{17} \text{ cm}^{-3}$.

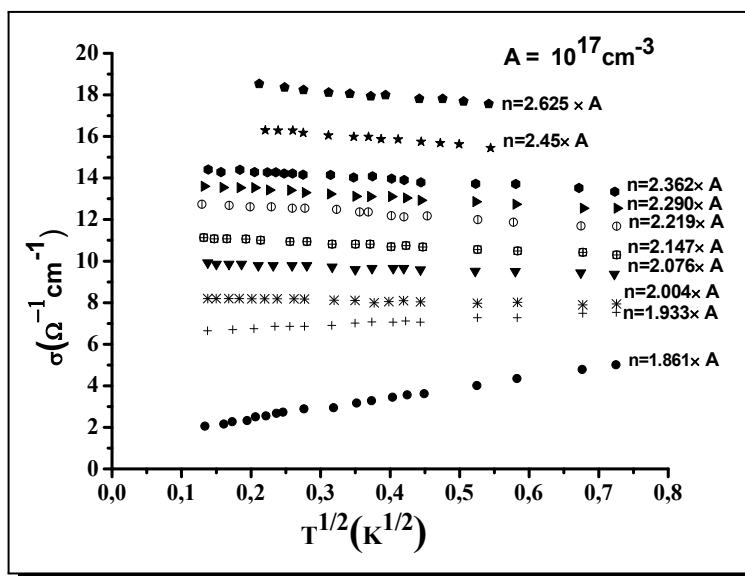


Figure 1. Electrical conductivity as a function of $T^{1/2}$ for different concentrations of impurities. As mentioned above, we have reanalyzed experimental data obtained in ref. [15]

We used this figure to extract the values of the zero-temperature electrical conductivity from ten metallic samples $^{70}\text{Ge}:\text{Ga}$ by the standard linear regression method. We then show the evolution of the logarithm of $\sigma(0)$ as a function of $\ln\left(\frac{n}{n_c} - 1\right)$ (see Figure 2). We have noted that the slope determined by the linear fit is very close to 0.5 ($\nu = 0.503$). We find that for the metallic $^{70}\text{Ge}:\text{Ga}$ system $\sigma(0)$ follows the scaling law given by equation (2.2). We also determined the constant σ_C is $\sigma_C = 29.16 (\Omega \text{ cm})^{-1}$.

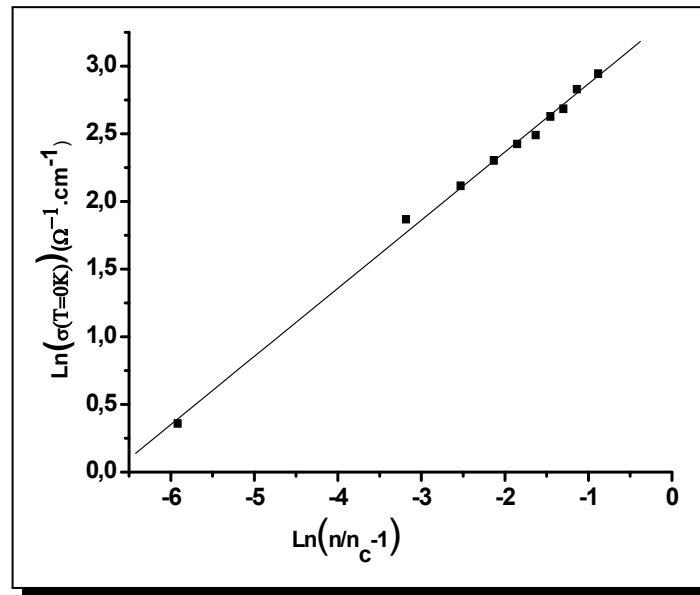


Figure 2. Variation of $\ln(\sigma(T = 0))$ against $\ln\left(\frac{n}{n_c} - 1\right)$

3.2 Zero-Temperature Conductivity as a Function of the Magnetic Field

Figure 3 shows the temperature variation of the electrical conductivity behavior versus $T^{1/2}$, for different values of the magnetic field B between 1 and 8 T, at very low temperatures in the range 0.65 to 0.048 K for sample $B2$ with impurity concentration $n = 2.004 \times 10^{17} \text{ cm}^{-3}$ of the metallic system $^{70}\text{Ge:Ga}$. According to the data shown in this figure, we find that the application of the magnetic field reduces the electrical conductivity and causes the sample to enter the insulating phase. We also obtain the values of $\sigma(T = 0, B)$ by extrapolating the curves $\sigma(T, B)$ as a function of $T^{1/2}$ (the linear regression method). The values of $\sigma(T = 0, B)$ are collected in Table 1.

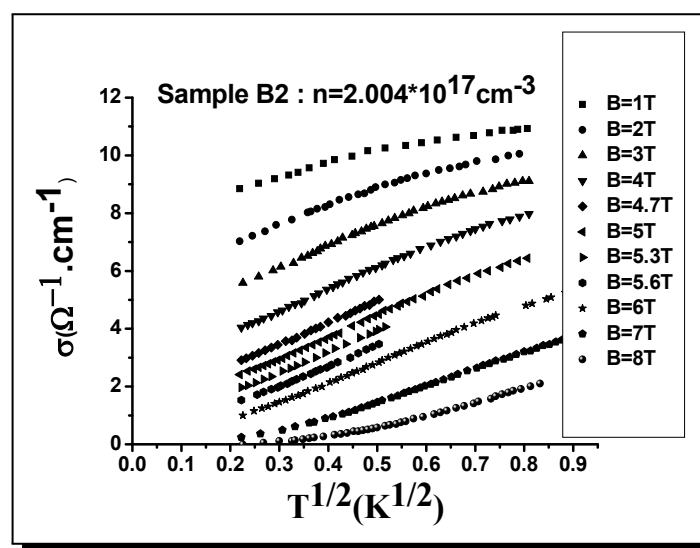
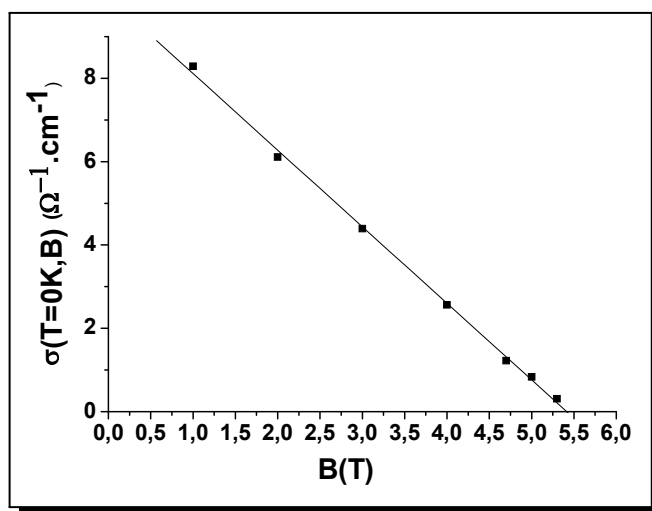


Figure 3. Temperature dependence of electrical conductivity with $T^{1/2}$ in the presence of magnetic fields between 1 and 8 T, as indicated. We re-analyzed the experimental data obtained in ref. [18].

Table 1. Table shows the values of magnetic fields (B) and Conductivity at $T = 0$ K ($\sigma(T = 0, B)$) deduced by the linear regression method (Figure 3)

B (Tesla)	$\sigma(T = 0, B)$ ($\Omega \text{ cm}^{-1}$)
1	8,2881
2	6,10946
3	4,38989
4	2,56061
4,7	1,22415
5	0,83393
5,3	0,30998
5,6	-0,08243
6	-0,53204
7	-1,24552
8	-1,16498

**Figure 4.** Zero-temperature conductivity versus magnetic field in the temperature range 0.048 to 0.65 K for Sample $B2$

In Figure 4, we show the variations of the zero-temperature electrical conductivity as a function of the magnetic field for the sample referenced $B2$. We note that $\sigma(T = 0, B)$ decreases linearly with the magnetic field (see Figure 4) and ends up canceling out for a value of the field B approximately equal to 5.45 Teslas ($B_C \approx 5.45$ T). This allows us to determine the critical magnetic field B_C in sample $B2$ to be 5.45 T. This result is in good agreement with the values collected in Table 1.

For $B > 5.3$ T, we found the negative values of the zero temperature conductivity that have no physical significance, but they allow us to note that this sample is on the insulating side of the MIT for $B > B_c$. In this case, the electronic states become localized and the electrical transport could be governed by variable range hopping.

In order to verify the scaling law of the conductivity (equation (2.3)), we show in Figure 5 the logarithmic variation of $\sigma(T = 0, B)$ as a function of $\ln(1 - B/B_C)$ for sample B2, in the temperature range between 0.048 and 0.65 K. We can see a straight line fit which allows us to determine the slope (see Figure 5).

We have found that the conductivity at $T = 0$ K follows the scaling law (2.3) with a critical exponent very close to the theoretical value $\nu = 1$ ($\nu = 1.06$). Indeed, there is a good agreement with the scaling theory of Abraham *et al.* (2.3) and we found the value of constant $\sigma'_C = 10.3056$ ($\Omega \text{ cm}$) $^{-1}$.

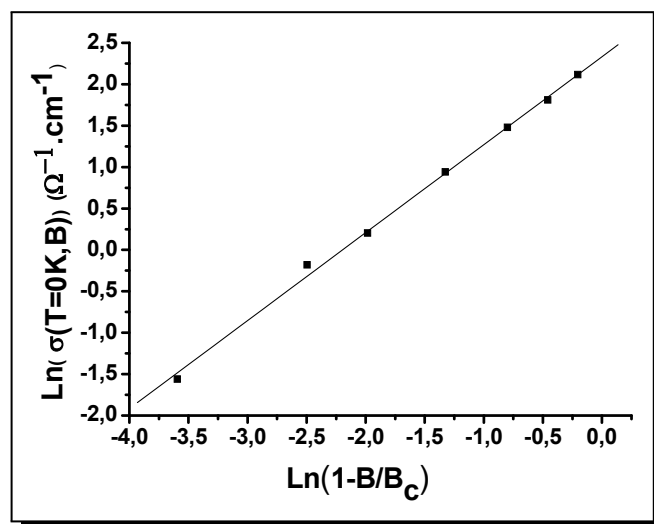


Figure 5. Variation of the logarithm of $\sigma(T = 0, B)$ as a function of $\ln(1 - B/B_C)$ for sample B2

4. Conclusion

In this paper, we have systematically investigated the electrical transport properties of the metallic side of MIT in the $^{70}\text{Ge}:\text{Ga}$ system, by verifying the two scaling laws by studying zero-temperature conductivity [5, 16, 34]. Indeed, when the MIT is induced by changing in the impurity concentration, we found that critical exponent close to 0.5. In addition, we obtained a critical exponent very close to unity, when the MIT is produced by varying the magnetic field. We have also estimated the critical magnetic field that marks the boundary between the metallic phase and the insulating phase. Moreover, we note that the application of the magnetic field causes the sample to return to the insulating phase [11, 17].

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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