Communications in Mathematics and Applications
Vol. 9, No. 2, pp. 189–196, 2018
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications

http://www.rgnpublications.com

Research Article

On Generalization of ϕ-n-Absorbing Ideals in Commutative Rings

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Abstract. The aim of this paper is to extend the concept of n-absorbing, quasi-n-absorbing and ϕ-n-absorbing ideals given by Anderson and Badawi [2] to the context of ϕ-semi-n-absorbing ideals. Let $\phi : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ be a function where $\mathfrak{I}(R)$ is the set of all ideals of R. A proper ideal R of R is called a ϕ-semi-n-absorbing Ideal, if for each $a \in R$ with $a^{n+1} \in I - \phi(I)$, then $a^n \in I$. Some characterizations of semi-n-absorbing ideals are obtained. It is shown that if J is a ϕ-semi-n-absorbing ideal of R, then J/I is a ϕ_I-semi-n-absorbing ideal of R/I where $I \subseteq J$. A number of results concerning relationships between ϕ-semi-n-absorbing, ϕ_0-semi-n-absorbing, $\phi_\emptyset$-semi-2-absorbing and $\phi_{n\geq 2}$-semi-n-absorbing ideals of commutative rings. Finally, we obtain sufficient conditions of a semi-n-absorbing ideal in order to be a ϕ-semi-n-absorbing ideal.

Keywords. n-absorbing ideal; Quasi-n-absorbing; ϕ-n-absorbing ideal; Semi-n-absorbing ideal; ϕ-semi-n-absorbing ideal

MSC. 13A15; 13F05; 13G05

Received: December 6, 2017 **Accepted:** March 9, 2018

1. Introduction

Let R be a commutative ring with $1 \neq 0$ and n a positive integer. In 2007, Badawi [4] generalized the concept of prime ideals to 2-absorbing ideals in a different way. He defined a nonzero proper ideal I of R to be a 2-absorbing ideal of R if whenever $abc \in I$ for $a, b, c \in R$, then $ab \in I$ or $ac \in I$ or $bc \in I$. Anderson and Badawi [1] generalized the concept of 2-absorbing ideals to

n-absorbing ideals. According to their definition, a proper ideal I of R to be a n-absorbing ideal of R if whenever $a_1a_2\dots a_{n+1}\in I$ for $a_1,a_2,\dots,a_{n+1}\in R$, then there are n of the a_i's whose product is in I. In [5], Badawi and Darani generalized the concept of weakly prime ideals to weakly 2-absorbing ideals. According to their definition, a proper ideal I of R to be a weakly 2-absorbing ideal of R if whenever $0\neq abc\in I$ for $a,b,c\in R$, then $ab\in I$ or $ac\in I$ or $bc\in I$. In 2016, Mostafanasab *et al.* [8] introduced the concepts of a weakly n-absorbing ideal in commutative ring. According to their definition, a proper ideal I of R is called a weakly n-absorbing ideal if whenever $0\neq a_1a_2\dots a_{n+1}\in I$ for $a_1,a_2,\dots,a_{n+1}\in R$, then there are n of the a_i's whose product is in I. In 2017, Anderson and Badawi [2] generalized the concept of n-absorbing ideals to semi-n-absorbing ideals. According to their definition, a proper ideal I of R to be a semi-n-absorbing ideal of R if whenever $a^{n+1}\in I$ for $a\in R$, then $a^n\in I$. Mostafanasab and Darani [7] introduced the concepts of a quasi-n-absorbing ideal. According to their definition, a proper ideal I of R to be a quasi-n-absorbing ideal of R if whenever $a^nb\in I$ for $a,b\in R$, then $a^n\in I$ or $a^{n-1}b\in I$.

In 2008, Anderson and Batanieh [3] generalized the concept of prime and weakly prime ideals to φ-prime ideals. Let $\varphi:\mathfrak{I}(R)\to\mathfrak{I}(R)\cup\{\emptyset\}$ be a function where $\mathfrak{I}(R)$ is a set of ideals of R. Recall that a proper ideal I of R is called a φ-prime ideal of R as in [3] if whenever $ab\in I-\varphi(I)$ for $a,b\in R$, then $a\in I$ or $b\in I$. Ebrahimpour and Nekooei [6] generalized the concept of n-absorbing ideals to $(n-1,n)$-φ-prime ideals. According to their definition, a proper ideal I of R to be a $(n-1,n)$-φ-prime ideal of R if whenever $a_1a_2\dots a_n\in I$ for $a_1,a_2,\dots,a_n\in R$, then there are $n-1$ of the a_i's whose product is in I.

Motivated and inspired by the above works, the purposes of this paper are to introduce generalizations of n-absorbing, quasi-n-absorbing and ϕ-n-absorbing ideals to the context of ϕ-semi-n-absorbing ideals. A proper ideal I of R to be a ϕ-semi-n-absorbing ideal of R if whenever $a^{n+1}\in I-\phi(I)$ for $a\in R$, then $a^{n+1}\in I$. Let I be a ϕ_α-semi-n-absorbing ideal of R.

- If $\phi_\alpha(I)=\emptyset$ for every $I\in\mathfrak{I}(R)$, then we say that $\phi_\alpha=\phi_\emptyset$ and I is called a $\phi_\emptyset$-semi-n-absorbing ideal of R [2].
- If $\phi_\alpha(I)=\{0\}$ for every $I\in\mathfrak{I}(R)$, then we say that $\phi_\alpha=\phi_0$ and I is called a ϕ_0-semi-n-absorbing ideal of R.
- If $\phi_\alpha(I)=I^2$ for every $I\in\mathfrak{I}(R)$, then we say that $\phi_\alpha=\phi_2$ and I is called a ϕ_2-semi-n-absorbing ideal of R, and hence I is an almost semi-n-absorbing ideal of R.
- If $\phi_\alpha(I)=I^{n-1}$ for every $I\in\mathfrak{I}(R)$, then we say that $\phi_\alpha=\phi_{n\geq3}$ and I is called a ϕ_n-semi-n-absorbing ideal of R, and hence I is a n-semi-n-absorbing ideal of R.
- If $\phi_\alpha(I)=\bigcap_{i=1}^{\infty}I^i$ for every $I\in\mathfrak{I}(R)$, then we say that $\phi_\alpha=\phi_\omega$ and I is called a ϕ_ω-semi-n-absorbing ideal of R, and hence I is a ω-semi-n-absorbing ideal of R.

The aim of this paper is to investigate the concept of ϕ-semi-n-absorbing, ϕ_0-semi-n-absorbing, $\phi_\emptyset$-semi-n-absorbing, $\phi_{n\geq2}$-semi-n-absorbing and ϕ_ω-semi-n-absorbing ideals of commutative rings. A number of results concerning relationships between ϕ-semi-n-absorbing, ϕ_0-semi-n-absorbing, $\phi_\emptyset$-semi-n-absorbing and $\phi_{n\geq2}$-semi-n-absorbing ideals of commutative rings. Finally, we obtain sufficient conditions of a semi-n-absorbing ideal in order to be a ϕ-semi-n-absorbing ideal.

2. Properties of ϕ-Semi-n-Absorbing Ideals

The results of the following theorems seem to play an important role to study ϕ-2-absorbing quasi-primary ideals of commutative rings; these facts will be used frequently and normally we shall make no reference to this definition.

Definition 2.1. Let $\phi : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ be a function where $\mathfrak{I}(R)$ is a set of all ideals of R. A proper ideal I of R is called a ϕ-semi-n-absorbing ideal of R if for each $a \in R$ with $a^{n+1} \in I - \phi(I)$, then $a^n \in I$.

Remark 2.1. (1) It is easy to see that every ϕ-n-absorbing ideal is ϕ-semi-n-absorbing.

(2) It is easy to see that every semi-n-absorbing ideal is ϕ-semi-n-absorbing.

(3) It is easy to see that every quasi-n-absorbing ideal is ϕ-semi-n-absorbing.

The following example shows that the converse of Remark 2.1 is not true.

Example 2.1. (1) Let $R = \mathbf{Z}$ and $2 \le n \in \mathbf{Z}$. Consider the ideal $I = 3 \cdot 5^n \mathbf{Z}$ of R. Define $\phi : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ by $\phi(I) = \{0\}$, for every ideal I of R. It is easy to see that I is a ϕ-semi-n-absorbing ideal of R. Notice that $3 \cdot 5^n \in 3 \cdot 5^n \mathbf{Z} - \phi(3 \cdot 5^n \mathbf{Z})$, but $3 \cdot 5^{n-1} \notin 3 \cdot 5^n \mathbf{Z}$ and $5^n \notin 3 \cdot 5^n \mathbf{Z}$. Therefore I is not a ϕ-n-absorbing (quasi-n-absorbing) ideal of R.

(2) Let $R = \mathbf{Z} \times \mathbf{Z}$. Consider the ideal $I = \{0\} \times 8\mathbf{Z}$ of R. Define $\phi : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ by $\phi(I) = \{0\} \times 8\mathbf{Z}$, for every ideal I of R. It is easy to see that I is a ϕ-semi-2-absorbing ideal of R. Notice that $(0,2)^3 \in \{0\} \times 8\mathbf{Z}$, but $(0,2)^2 \notin \{0\} \times 8\mathbf{Z}$. Therefore I is not a semi-2-absorbing ideal of R.

Let I be n ideal of a ring R and let $\phi : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ be a function. Since $I - \phi(I) = I - (I \cap \phi(I))$, without loss of generality, we will assume that $\phi(I) \subseteq I$ [3]. Throughout this paper, as it is noted earlier, if ϕ is a function, then we always assume that $\phi(I) \subseteq I$. Define $\phi_I : \mathfrak{I}(R/I) \to \mathfrak{I}(R/I) \cup \{\emptyset\}$ by

$$\phi_I(J/I) = \begin{cases} (\phi(J) + I)/I \ ; & \phi(J) \neq \emptyset, \\ \emptyset \ ; & \phi(J) = \emptyset, \end{cases}$$

for every ideal J of R with $I \subseteq J$. In, 2008 Anderson and Bataineh in [3] gives relations between ϕ-prime ideals of R and ϕ_I-prime ideals of R/I. This leads us to give relations between ϕ-semi-2-absorbing ideals of R and ϕ_I-semi-2-absorbing ideals of R/I.

Theorem 2.1. *Let $\phi : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ be a function and let I, J be two ideals of R with $I \subseteq J$. If J is a ϕ-semi-n-absorbing ideal of R, then J/I is a ϕ_I-semi-n-absorbing ideal of R/I.*

Proof. Let $a \in R$ such that $a^{n+1} + I = (a+I)^{n+1} \in (J/I) - \phi_I(J/I) = (J/I) - (\phi(J)+I)/I = (J - \phi(J))/I$. Clearly, $a^{n+1} \in J - \phi(J)$. By Definition 2.1, $a^n \in J$. Therefore $(a+I)^n = a^n + I \in J/I$. Hence J/I is a ϕ_I-semi-n-absorbing ideal of R/I. □

Theorem 2.2. *Let $\phi : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ be a function and let I, J be two ideals of R with $I \subseteq J$. If J/I is a ϕ_I-semi-n-absorbing ideal of R/I such that $I \subseteq \phi(J)$, then J is a ϕ-semi-n-absorbing ideal of R.*

Proof. Let $a \in R$ such that $a^{n+1} \in J - \phi(J)$. Then $(a+I)^{n+1} = a^{n+1} + I \in (J - \phi(J))/I = J/I - (\phi(J)+I)/I = (J/I) - \phi_I(J/I)$. By Definition 2.1, $a^n + I = (a+I)^n \in J/I$. Thus $a^n \in J$. Hence J is a ϕ-semi-n-absorbing ideal of R. □

Now, by Theorems 2.1 and 2.2, we have the following corollary.

Corollary 2.1. *Let $\phi : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ be a function and let I, J be two ideals of R with $I \subseteq \phi(J)$. Then J is a ϕ-semi-n-absorbing ideal of R if and only if J/I is a ϕ_I-semi-n-absorbing ideal of R/I.*

Proof. The proof is similar to Theorem 2.1 and Theorem 2.2. □

We are finding additional condition to show that a semi-n-absorbing ideal is a ϕ-semi-n-absorbing ideal of R.

Theorem 2.3. *Let $\phi : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ be a function and let $\phi(I)$ be a semi-2-absorbing ideal of R. Then I is a ϕ-semi-n-absorbing ideal of R if and only if I is a semi-n-absorbing ideal of R.*

Proof. Suppose that I is a semi-2-absorbing ideal of R. Clearly, I is a ϕ-semi-n-absorbing ideal of R.

Conversely, assume that I is a ϕ-semi-2-absorbing ideal of R. Let $a \in R$ such that $a^{n+1} \in I$. If $a^{n+1} \notin \phi(I)$, then $a^{n+1} \in I - \phi(I)$. By Definition 2.1, $a^{n+1} \in I$. Now if $a^{n+1} \in \phi(I)$, then there is nothing to prove. Hence I is a semi-n-absorbing ideal of R. □

3. Properties of ϕ_α-Semi-n-Absorbing Ideals

We start with the following theorem that gives a relation between ϕ_α-semi-n-absorbing and ϕ-semi-n-absorbing ideal. Our starting points are the following definitions:

Definition 3.1. Let R be commutative ring and let $\mathfrak{I}(R)$ be the set of all ideals of R. Define the following functions $\phi_\alpha : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ and the corresponding ϕ_α-semi-n-absorbing ideals:

(1) If $\phi_\emptyset(I) = \emptyset$, then I is a semi-n-absorbing ideal of R [2].

(2) If $\phi_0(I) = \{0\}$, then I is a weakly semi-n-absorbing ideal of R.

(3) If $\phi_1(I) = I$, then I is a proper ideal of R.

(4) If $\phi_2(I) = I^2$, then I is an almost semi-n-absorbing ideal of R.

(5) If $\phi_m(I) = I^m$, then I is a m-semi-n-absorbing ideal of R.

(6) If $\phi_\omega(I) = \bigcap_{i=1}^{\infty} I^i$, then I is an ω-semi-n-absorbing ideal of R.

Remark 3.1. Let R be commutative ring and let $\mathfrak{I}(R)$ be the set of all ideals of R. For two functions $\phi_\alpha, \phi_\beta : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$. Define $\phi_\alpha \leq \phi_\beta$, if $\phi_\alpha(I) \subseteq \phi_\beta(I)$, for all $I \in \mathfrak{I}(R)$[2]. By Definition 3.1, observe that $\phi_\emptyset \leq \phi_0 \leq \phi_\omega \leq \ldots \leq \phi_{n+1} \leq \phi_n \leq \ldots \leq \phi_2 \leq \phi_1$.

Notice that for a commutative ring R, the zero ideal $\{0\}$ is always a ϕ_0-semi-n-absorbing ideal. In the following example, we give a commutative ring in which a ϕ-semi-n-absorbing ideal is not ϕ_0-semi-n-absorbing.

Example 3.1. Let $R = \mathbf{Z}$. Consider the ideal $I = 2^{n+1}\mathbf{Z}$ of R. Define $\phi : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ by $\phi(I) = I$, for every ideal I of R. It is easy to see that I is a ϕ-semi-n-absorbing ideal of R. Notice that $2^{n+1} \in 2^{n+1}\mathbf{Z} - \phi_0(2^{n+1}\mathbf{Z})$, but $2^n \notin 2^{n+1}\mathbf{Z}$. Therefore I is not a ϕ_0-semi-n-absorbing ideal of R.

Theorem 3.1. *Let $\phi_\alpha : \mathfrak{I}(R) \to \mathfrak{I}(R) \cup \{\emptyset\}$ be a function. Then the following hold.*

(1) *If I is a ϕ_β-semi-n-absorbing ideal of R such that $\phi_\beta \leq \phi_\gamma$, then I is a ϕ_γ-semi-n-absorbing ideal of R.*

(2) *If I is a semi-n-absorbing ideal of R, then I is a ϕ_0-semi-n-absorbing ideal of R.*

(3) *If I is a ϕ_0-semi-n-absorbing ideal of R, then I is an ω-semi-n-absorbing ideal of R.*

(4) *If I is a ϕ_ω-semi-n-absorbing ideal of R, then I is a ϕ_n-semi-n-absorbing ideal of R.*

(5) *If I is a ϕ-semi-n-absorbing ideal of R, then I is a ϕ-semi-$(n+m)$-absorbing ideal of R.*

Proof. (1) Let $a \in R$ such that $a^{n+1} \in I - \phi_\gamma(I)$. Since $\phi_\beta \leq \phi_\gamma$, we have $\phi_\beta(I) \subseteq \phi_\gamma(I)$. Then $a^{n+1} \in I - \phi_\gamma(I) \subseteq I - \phi_\beta(I)$. By Definition 3.1, $a^n \in I$. Hence I is a ϕ_γ-semi-n-absorbing ideal of R.

(2) It is obvious.

(3) It is obvious.

(4) It is obvious.

(5) It is obvious. □

From the above definitions we obtain immediately the following implication chart for the considered types of ideals:

semi-n-absorbing $\Rightarrow$ weakly semi-n-absorbing $\Rightarrow$ ω-semi-n-absorbing $\Rightarrow$
$\phi_{n\geq 2}$-semi-n-absorbing $\Rightarrow$ almost semi-n-absorbing.

Next, let R be a commutative ring. Clearly, every semi-n-absorbing ideal of R is ϕ-semi-n-absorbing but the converse does not necessarily hold. In Theorems 3.4, 3.5 and Corollary 3.2 provide some conditions under which a ϕ-semi-n-absorbing ideal is semi-n-absorbing.

Next, let R_i be a commutative ring with identity. Then $R_1 \times R_2$ is a commutative ring with identity and each ideal of $R_1 \times R_2$ is of the form $I_1 \times I_2$ for some ideals I_1 of R_1 and I_2 of R_2. Next we show that, if I_1 is a $(\psi_1)_0$-semi-n-absorbing ideal of R_1, then $I_1 \times R_2$ is a ϕ-semi-n-absorbing ideal if $\{0\}\times R_2 \subseteq \phi(I_1\times R_2)$. First, we would like to show that, I_1 is a ψ_1-semi-n-absorbing ideal of R_1 if $I_1\times R_2$ is a ϕ-semi-n-absorbing ideal of $R_1\times R_2$.

Theorem 3.2. *Let $\psi_i : \Im(R_i) \to \Im(R_i)\cup\{\emptyset\}$ be a function with $\phi = \psi_1\times\psi_2$. If $I_1\times R_2$ is a ϕ-semi-n-absorbing ideal of $R_1\times R_2$, then I_1 is a ψ_1-semi-n-absorbing ideal of R_1.*

Proof. Let $a\in R_1$ such that $a^{n+1}\in I_1-\psi_1(I_1)$. Clearly, $(a,0)^{n+1}=(a^{n+1},0)\in I_1\times R_2-\psi_1(I_1)\times\psi_2(R_2)=I_1\times R_2-\phi(I_1\times R_2)$. By Definition 3.1, $(a^n,0)=(a,0)^n\in I_1\times R_2$. Therefore $a^n\in I_1$. Hence I_1 is a ψ_1-semi-n-absorbing ideal of R_1. □

Lemma 3.1. *Let $\psi_i : \Im(R_i) \to \Im(R_i)\cup\{\emptyset\}$ be a function with $\phi = \psi_1\times\psi_2$. If I_1 is a $(\psi_1)_0$-semi-n-absorbing ideal of R_1 such that $\{0\}\times R_2\subseteq\phi(I_1\times R_2)$, then $I_1\times R_2$ is a ϕ-semi-n-absorbing ideal of $R_1\times R_2$.*

Proof. Let $(a,b)\in R_1\times R_2$ such that $(a,b)^{n+1}\in I_1\times R_2-\phi(I_1\times R_2)$. Since $\{0\}\times R_2\subseteq\phi(I_1\times R_2)$, we have $(a^{n+1},b^{n+1})=(a,b)^{n+1}\notin\{0\}\times R_2$. Clearly, $a^{n+1}\neq 0$. Thus $a^{n+1}\in I_1-(\psi_1)_0(I_1)$. By Definition 3.1, $a^n\in I_1$. Therefore $(a,b)^n\in I_1\times R_2$. Consequently $I_1\times R_2$ is a ϕ-semi-n-absorbing ideal of $R_1\times R_2$. □

Corollary 3.1. *Let $\psi_i : \Im(R_i) \to \Im(R_i)\cup\{\emptyset\}$ be a function with $\phi = \psi_1\times\psi_2$. If I_2 is a $(\psi_2)_0$-semi-n-absorbing ideal of R_2 such that $R_2\times\{0\}\subseteq\phi(R_2\times I_1)$, then $R_1\times I_2$ is a ϕ-semi-n-absorbing ideal of $R_1\times R_2$.*

Proof. Similar to the proof of Lemma 3.1. □

Theorem 3.3. *Let $\psi_i : \Im(R_i) \to \Im(R_i)\cup\{\emptyset\}$ be a function with $\phi=\psi_1\times\ldots\times\psi_k$ and $R_1\times\ldots\times R_{i-1}\times\{0\}\times R_{i+1}\times\ldots\times R_k\subseteq\phi(R_1\times\ldots\times R_{i-1}\times I_i\times R_{i+1}\times\ldots\times R_k)$. Then I_i is a $(\psi_i)_0$-semi-n-absorbing ideal of R_i if and only if $R_1\times R_2\times\ldots\times R_{i-1}\times I_i\times R_{i+1}\times\ldots\times R_k$ is a ϕ-semi-n-absorbing ideal of $R_1\times R_2\times\ldots\times R_k$.*

Proof. The proof of the following result is similar to the proof of Lemma 3.1 and Corollary 3.1. □

Theorem 3.4. *Let $\psi_i : \Im(R_i) \to \Im(R_i)\cup\{\emptyset\}$ be a function with $\phi = \psi_1\times\psi_2$. Then the following conditions are equivalent:*

(1) *I_1 is a semi-n-absorbing ideal of R_1.*

(2) *$I_1\times R_2$ is a semi-n-absorbing ideal of $R_1\times R_2$.*

(3) *$I_1\times R_2$ is a ϕ-semi-n-absorbing ideal of $R_1\times R_2$ where $\psi_2(R_2)\neq R_2$.*

Proof. (1) $\Rightarrow$ (2): Let $(a,b)\in R_1\times R_2$ such that $(a^{n+1},b)=(a,b)^{n+1}\in I_1\times R_2$. Clearly, $a^{n+1}\in I_1$. By Definition 3.1, $a^n\in I_1$. Thus $(a,b)^n\in I_1\times R_2$. Hence $I_1\times R_2$ is a semi-n-absorbing ideal of R.
(2) $\Rightarrow$ (3): It is obvious.
(3) $\Rightarrow$ (1): Let $a\in R_1$ such that $a^{n+1}\in I_1$. Since $(a,1)^{n+1}=(a^{n+1},1)\notin I_1\times\psi_2(R_2)$ and $\psi_1\times\psi_2(I_1\times R_2)\subseteq I_1\times\psi_2(R_2)$, we have $(a,1)^{n+1}\notin\phi(I_1\times R_2)$. Clearly, $(a,1)^{n+1}\in I_1\times R_2-\phi(I_1\times R_2)$.

By Definition 3.1, $(a,1)^n \in I_1 \times R_2$. Therefore $a^n \in I_1$. Hence I_1 is a semi-n-absorbing ideal of R_1. □

Corollary 3.2. *Let $\psi_i : \mathfrak{I}(R_i) \to \mathfrak{I}(R_i) \cup \{\emptyset\}$ be a function with $\phi = \psi_1 \times \psi_2$. Then the following conditions are equivalent:*

(1) *I_2 is a semi-n-absorbing ideal of R_2.*

(2) *$R_1 \times I_2$ is a semi-n-absorbing ideal of $R_1 \times R_2$.*

(3) *$R_1 \times I_2$ is a ϕ-semi-n-absorbing ideal of $R_1 \times R_2$ where $\psi_1(R_1) \neq R_1$.*

Proof. Similar to the proof of Theorem 3.4. □

Theorem 3.5. *Let $\psi_i : \mathfrak{I}(R_i) \to \mathfrak{I}(R_i) \cup \{\emptyset\}$ be a function with $\phi = \psi_1 \times \ldots \times \psi_k$. Then the following conditions are equivalent:*

(1) *I_i is a semi-n-absorbing ideal of R_i.*

(2) *$R_1 \times R_2 \times \ldots \times R_{i-1} \times I_i \times R_{i+1} \times \ldots \times R_k$ is a semi-n-absorbing ideal of $R_1 \times \ldots \times R_k$.*

(3) *$R_1 \times R_2 \times \ldots \times R_{i-1} \times I_i \times R_{i+1} \times \ldots \times R_k$ is a ϕ-semi-n-absorbing ideal of $R_1 \times \ldots \times R_2$ with $\psi_j(R_j) \neq R_j$.*

Proof. Similar to the proof of Theorem 3.4 and Corollary 3.2. □

Theorem 3.6. *Let $\psi_i : \mathfrak{I}(R_i) \to \mathfrak{I}(R_i) \cup \{\emptyset\}$ be a function with $\psi_2(R_2) = R_2$ and $\phi = \psi_1 \times \psi_2$. Then $I_1 \times R_2$ is a ϕ-semi-n-absorbing ideal of $R_1 \times R_2$ if and only if I_1 is a ψ_1-semi-n-absorbing ideal of R_1.*

Proof. ($\Rightarrow$): This follows from Theorem 3.2.
($\Leftarrow$): Let $(a,b) \in R_1 \times R_2$ such that $(a^{n+1}, b^{n+1}) = (a,b)^{n+1} \in I_1 \times R_2 - \phi(I_1 \times R_2) = I_1 \times R_2 - \psi_1(I_1) \times R_2$. Clearly, $a^{n+1} \in I_1 - \psi_1(I_1)$. By Definition 3.1, $a^n \in I_1$. Hence $I_1 \times R_2$ is a ϕ-semi-n-absorbing ideal of $R_1 \times R_2$. □

Corollary 3.3. *Let $\psi_i : \mathfrak{I}(R_i) \to \mathfrak{I}(R_i) \cup \{\emptyset\}$ be a function with $\psi_1(R_1) = R_1$ and $\phi = \psi_1 \times \psi_2$. Then $R_1 \times I_2$ is a ϕ-semi-n-absorbing ideal of $R_1 \times R_2$ if and only if I_2 is a ψ_2-semi-n-absorbing ideal of R_2.*

Proof. Similar to the proof of Theorem 3.6. □

Theorem 3.7. *Let $\psi_i : \mathfrak{I}(R_i) \to \mathfrak{I}(R_i) \cup \{\emptyset\}$ be a function with $\psi_j(R_j) = R_j$ and $\phi = \psi_1 \times \ldots \times \psi_k$. Then $R_1 \times R_2 \times \ldots \times R_{i-1} \times I_i \times R_{i+1} \times \ldots \times R_k$ is a ϕ-semi-n-absorbing ideal of $R_1 \times \ldots \times R_k$ if and only if I_i is a ψ_i-semi-n-absorbing ideal of R_i.*

Proof. Similar to the proof of Theorem 3.6 and Corollary 3.3. □

4. Conclusion

We introduced the concept of a semi-n-absorbing of a commutative ring. The aim of this paper is to investigate the concept of ϕ-semi-n-absorbing, ϕ_0-semi-n-absorbing, $\phi_\emptyset$-semi-n-absorbing, $\phi_{n\geq 2}$-semi-n-absorbing and ϕ_ω-semi-n-absorbing ideals of commutative rings. A number of results concerning relationships between ϕ-semi-n-absorbing, ϕ_0-semi-n-absorbing, $\phi_\emptyset$-semi-n-absorbing and $\phi_{n\geq 2}$-semi-n-absorbing ideals of commutative rings.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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