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Hybrid Uncertainties Modeling for Production Planning Problems

Research Article

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Abstract. The formulated mathematical model needs pre-determined and precise model parametersto find a solution. However, the model parameters such as coefficient value are usually not precisely known. Coefficient plays a pivotal role sincethe coefficientcouldprovide important information in relationship between algebraic and linguistic expression. Existing method which is commonly used to generate the precise parametric valuesis unable to handle the coexistence of fuzzy information. Moreover, selecting real numbers for coefficients in random process increases the complexity inprogramming process. Hence, we proposed a fuzzy random regression method in this paper to estimate the precise coefficient values which contains fuzzy random information. An illustrative numerical example is provided to deduce coefficient values from different data representation which included the fuzziness and randomness.The coefficients were treated based on the property of fuzzy random regression. The approach results show that we have the significant capabilities to estimate the coefficient value and improve the model which retain the simultaneous uncertainties and set up in production planning problem.

Keywords. Production planning; Coefficient estimation; Hybrid uncertainties; Fuzzy random variable; Fuzzy random regression

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1. Introduction

Mathematical programming is one of the common solutions to model real world problem which employs a mathematical model to assist decision making. The formation of mathematical model requires coefficient value. Principally, coefficient is a numerical quantity which characterizes the rate of change of one variable in an algebraic expression. The coefficient value plays an important role to describe the relation between the dependent and independent variables. One of the applications that used this coefficient value is in the development of production planning model. The relationship between dependent and independent variables in the production planning model is presented using linear program.

However, in real life situation, it is difficult to determine the exact value of model's coefficient. Difficulties in determining coefficient occurred if relevant data are not available in time it is needed. In fact, the decision might contain imprecise and vague information. If the coefficient is determined by the human expert, the variation between one decision maker to another raises multiple occurrence of uncertainties. Hence, such uncertainties must be handled sufficiently. Extracting coefficient values from the historical data can be a solution to determine the approximate coefficient.

An appropriate method is necessary to approximate the coefficient value when the historical data are available. Regression analysis is one possible method which is able to approximate variables coefficient value [13]. This technique is supported by effective statistical analysis dealing with numerical data. That is, the obtained statistical model does not include the uncertainty in the data. However, in real life application, there exist uncertainty elements such as fuzzy data which is observed from subjective human estimation. Therefore, a variant of fuzzy regression models was introduced to handle with the fuzzy uncertainty in the input-output data [15, 18–20].

On the other hand, simultaneous uncertainties happen in real life i.e. fuzziness and randomness. Fuzziness occurs mainly from the imprecise information and lingual reasoning made by human, and meanwhile, the randomness occurred when the observed data is collected at random and unknown event. Fuzzy random situation is explained as follows. Let us assume that an expert is responsible to assess the quality of n sample of products. Randomness is characterized by the mechanism of unknown outcome (i.e. it is not known which product may produce higher quality, and vice versa). Meanwhile, fuzziness arises from the lingual notion which is used by the expert to express his evaluation, such as 'sufficiently high quality, 'is about' and so on. The existence of uncertainties (e.g. fuzzy and random information) in the real data could not be neglected as it is important to handle the uncertainties sufficiently in the modeling of real situation to avoid misleading result. These uncertainties should be treated properly in the modeling of problem solving solution.

Motivated by the abovementioned problems, a fuzzy random regression method is employed in the production planning problem to approximate the model's coefficient value in the fuzzy random environment. The solution presented in this study is able to assist coefficient values approximation from statistical data where fuzziness and randomness are co-existing. This solution is beneficial as the fuzzy random regression method can facilitate the decision maker to decide the coefficient value, and to formula the linear program to find a solution. Thus, the complexity to decide the value of model coefficient can be reduced and uncertainties situation can be treated adequately. The remainder of this paper is organized as follows. Section 2 describes the background of fuzzy random regression model and Section 3 explainsfuzzy random regression method. Section 4 demonstrates the solution method in numerical experiments and the result is analyzed in Section 5. Section 6 draws conclusions.

2. Preliminary Studies and Definitions

This section explains a Fuzzy Random Regression analysis, statistical analysis, and production planning as a ground studies for this work.

2.1 Fuzzy Random Regression Analysis

Regression analysis is useful for its capability which able to find coefficient for the model's variables. Effective statistical analysis in the regression method can deal with precise crisp data. Regression analysis is utilized in assessing the functional relationship between the dependent and independent variables [24]. Numerous applications of regression can be found in almost every field such as economic, industrial, engineering, science, and many others.

Regression method works with precise numerical values. It makes the method incompetent to handle imprecise values which are common in real life situation. Employment of fuzzy theory in regression improves the capability of fuzzy regression to deal with imprecise data and uncertain evaluation. Imprecise data are basically observed from fuzzy environments due to human subjective evaluation and estimation [1, 24]. Fuzzy regression analysis has remark the success through many applications. Another expansion of fuzzy regression namely fuzzy random regression enhances fuzzy regression with random variables. This method works well in the data which contains hybrid uncertainties; fuzziness and randomness [22]. This method has been applied in fuzzy multi attribute decision making to find the importance weight [16] of independent variable in the model. A fuzzy random regression approximates attribute importance (weight) in total evaluation within the bound of hybrid uncertainty [3].

Statistical data that are observed from real world situation may contain imprecise lingual or vague value. Normal random variable is incapable to handle such data due to the presence of stochastic and fuzzy uncertainty [17]. Hence fuzzy random variable is integrated in the regression technique to cope with the fuzzy random data. Statistical analysis with fuzzy random data may require transformation of the vague parameter while making decision, whereby the fuzziness are transferred into coefficient value [15]. Confidence interval and expected value are used to describe fuzzy random in regression model. The interval is valuable to estimates a population parameter hence indicate the reliability of an estimate. The detail explanation of fuzzy random variable and fuzzy random regression are given elsewhere [14, 23]. This method works efficiently with the data containing simultaneous fuzzy random uncertainties.

The data format (dependent and independent variable) for fuzzy random regression is characterized by triangular fuzzy number (TFN). Fuzzy random data Y_j (dependent) and X_{jk} (independent) for all $j = 1,\dots,N$ and $k = 1,\dots,K$ are defined as $Y_j = \bigcup_{t=1}^{M_{Y_j}} \{(Y_j^t, Y_j^{t,l}, Y_j^{t,r})_\Delta, p_j^t\}$, and $X_{jk} = \bigcup_{t=1}^{M_{X_{jk}}} \{(X_j^t, X_j^{t,l}, X_j^{t,r})_\Delta, q_{jk}^t\}$, respectively. p_j^t and q_{jk}^t demonstrate the probability of the event happens in $j = 1,\dots,N$, $k = 1,\dots,K$ and $t = 1,\dots,K$.

Using $\underset{FR}{\supset}$ is a fuzzy random inclusion relation [14], the fuzzy regression model is written as $Y_j^* = A_j^* X_{j1} + \dots + A_K^* X_{jK} \underset{FR}{\supset} Y_i$, $j = 1,\dots,n$. This model is solvable using traditional linear program of simplex method.

The fuzzy random regression model with σ-confidence intervals [14] is described as follows:

$$\begin{aligned}
&\min_A \ J(A) = \sum_{k=1}^{K} (A_k^r - A_k^l) \\
&\qquad A_k^r \geq A_k^l, \\
&\qquad Y_j^* = A_j^* I[e_{X_{j1}}, \sigma_{X_{j1}}] + \dots + A_K^* I[e_{X_{jK}}, \sigma_{X_{jK}}] \supset_{\tilde{h}} I[e_{Y_j}, \sigma_{Y_j}] \\
&\qquad j = 1,\dots,N;\ k = 1,\dots,K,
\end{aligned} \tag{2.1}$$

where $e_{X_{j1}}$, $\sigma_{X_{j1}}$ is the expected value and variance, respectively. The σ-confidence interval is defined as $I[e_X, \sigma_X] \triangleq [E(X) - \sqrt{\mathrm{var}(X)}, E(X) + \sqrt{\mathrm{var}(X)}]$. The confidence level shows us the frequency of an observed interval contains the parameter.

The data should be followed the statistical property of central limit theorem [7]. We give the definition in the following subsection.

2.2 Statistical Analysis

Let $X_1,\dots,X_{n1}$ be a sequence of random variables. If that X_i are independent and have the same distribution, X_i are independently identically distributed (i.i.d).

The total and average is then denoted as $W_n = \sum_{i=1}^{n} X_i$ and $\overline{X}_n = \frac{w_n}{n}$ respectively, of the nX's. The Central Limit Theorem in statistics is as follows:

Theorem 1 (Central Limit Theorem)**.** *Let $X_1,\dots,X_{n1}$ be i.i.d. random variables with mean m and variance σ^2. Let $Z_n = \frac{n^{1/2}\bar{X}_n - m}{\sigma}$. Then, Z_n converges in distribution to Z as $n \to \infty$. We denote $Z_n \to Z \sim N(0,1)$ as where Z distributes according to a standard normal distribution function $N(0,1)$.*

Note that m denotes mean and μ expresses a membership function.

In the following, we define a concept [6] which could let us know that absolute percentage error of coefficient value between *Fuzzy Random Regression Model* (FRRM) and *Fuzzy Regression Model* (FRM) in this experiment.

Definition 1 (Absolute Percentage Error for Coefficient of Fuzzy Random Regression Model (APEC-FRRM))**.** The absolute percentage error for is a measure of accuracy of a method for constructing fitted values in statistics, specifically in trend estimation. Here, we compare our coefficient value with FRM (actual coefficient value). The *Absolute Percentages Error for Coefficient* (APEC) is defined by the following formulation:

$$\mathrm{APEC}_{\mathrm{FRRM}} = \left|\frac{\mathrm{FRM}-\mathrm{FRRM}}{\mathrm{FRM}}\right| \times 100\%, \tag{2.2}$$

where FRM is coefficient values from fuzzy regression model and FRRM is coefficient values from fuzzy random regression model.

The absolute value in this calculation is summed for every fitted or estimated point and multiplying by 100 makes a percentage error.

2.3 Production Planning

Production planning models describe decisions on production and inventory quantities. Setting target and resource acquisition and allocation decision are important element in formulating production planning model [5]. In simple word, production planning is a procedure to set the production goals and to estimate the resources which are required to achieve the targeted goals. In the manufacturing environment, production planning and scheduling engages different complicated task which deals with a hierarchy of decision making problem. Such situation requires cooperation among multiple function unit such as production, accounting and marketing in an organization [4].

Production planning model is an important strategy yet efficient to achieve the goals of any production process and also help the decision maker to deal with forecasting, planning, scheduling and executing the required production work [21]. Production planning has play an important role in production management due to dynamic market conditions and customer demand for shorter delivery times, lower prices and better quality of services [12]. Common goal in production planning relies in maximizing production rate while reducing the cost of production. Thus, selecting appropriate methods and raw materials with conditional constraint requires effective planning [21].

Production planning influenced many industrial domain and numerous methods are applied to improve the efficiency of this strategy. For example an optimization-based model for production planning is proposed to optimize the production planning process [21]. Production planning strategy is extended from optimizing single objective (goal) to the multiple objectives. i.e. the model is able to minimize the cost and manufacturing time, while maximizing production rate based on the available system constraints. The application of production planning in real world industry also could not get away from uncertainty. Inspired by the situation of real world where there exist uncertainty, study on production planning under uncertainty are also been done.

Given the above information, all prior knowledge for dealing with a production planning problem with fuzzy random regression is presented.

3. Solution Model

The solution model in this study relies on the application of fuzzy random regression in building production planning model. Fuzzy random regression method to approximate the coefficient value is as follows:

Step 1: *Data preparation*: Transfer the data into fuzzy random data format in a form of triangular fuzzy number with its probability $X_{jk} = \bigcup_{t=1}^{M_{X_{jk}}} \{(X_j^t, X_j^{t,l}, X_j^{t,r})_\Delta, q_{jk}^t\}$.

Step 2: *Build the confidence interval*: Confidence interval $I[e_X, \sigma_X]$ of each fuzzy random is inferred by computing the expected value and variance of fuzzy random variable.

2.1. Calculate the expected value $E[X]$ of triangular fuzzy random variable X as Equation (3.1):

$$E[X] = \int_\Omega \left[\int_0^\infty \left(\frac{1}{2} \left[1 + \sup_{t \geq r} \mu_{Z(\omega)^{(t)}} - \sup_{t<r} \mu_{Z(\omega)^{(t)}} \right] \right) dr - \int_{-\infty}^0 \left(\frac{1}{2} \left[1 + \sup_{t \leq r} \mu_{Z(\omega)^{(t)}} - \sup_{t>r} \mu_{Z(\omega)^{(t)}} \right] \right) dr \right] \Pr(d\omega). \tag{3.1}$$

2.2. Calculate the variance of X based on Equation (3.2).

$$\mathrm{var}[X] = E\left[(X-e)^2\right], \tag{3.2}$$

where $e = E[X]$ is given by Equation (3.1).

Step 3: *Approximate the coefficient*: Build fuzzy random regression model to estimate the coefficient. Let A_k denote an attribute for $k = 1, \ldots, K$ and Y_j is the total evaluation for $j = 1, \ldots, N$. A fuzzy random regression model is written as follows:

$$\begin{aligned} \min_A \quad & J(A) = \sum_{k=1}^K (\bar{a}_k - \underline{a}_k) \\ \text{subject to } \; & \bar{a}_k \geq \underline{a}_k, \\ & y_j^r + (e_{Y_j} + \sigma_{Y_j}) \leq \sum_{k=1}^K \bar{a}_k (e_{X_{j1}} + \sigma_{X_{j1}}), \\ & y_j^l - (e_{Y_j} - \sigma_{Y_j}) \geq \sum_{k=1}^K \underline{a}_k (e_{X_{j1}} - \sigma_{X_{j1}}), \\ & j = 1, \ldots, N; \;\; k = 1, \ldots, K. \end{aligned} \tag{3.3}$$

Step 4: *Analysis*: The coefficients $[\underline{a}_k, \bar{a}_k]$ are attained from fuzzy random regression model (3.3). The lower boundary $\underline{a}_k$ and upper boundary $\bar{a}_k$ explains the flexibility of the coefficient values which is included during approximation.

The approximated coefficient which is derived by fuzzy random regression can be employed for several purposes including developing a mathematical model for real-world problem.

4. Numerical Experiment

The application of the solution method is expressed in a case of rubber production. In Malaysia, one of the important economic contributors is natural rubber industry. In 2012, the rubber industry contributed approximately RM36.4 billion in export earnings [2]. Malaysia's total exports for manufacturing products accounts 3.9% from rubber product. There are 500 surplus manufacturers included in the Malaysian rubber product industry which produces latex products, tyre-related products, and general rubber products [11]. It makes Malaysia a world's leading producer and exporter of catheters, latex thread and natural rubber medical gloves [8]. At present there are 120 companies in the tyre and tyre-related products sub sector. The continual demand from rubber-based industry would sustain the industry and lead Malaysia to become the world's largest consumer of natural rubber latex.

In this study, 13 years of statistical rubber data has been gathered from the year 2000 until 2012 [10]. Table 1 tabulates the sample of dataset. The dataset of rubber production contain two main products from the rubber plant, which are *dry* and *latex*. The two types of rubbers have different applications and market outlets. The dry rubber is used mainly for general rubber product, while latex is used in producing gloves, catheters, latex thread and others. In the Table 1, no explicit relationship is shown in the statistical data. However, the relationship and its analysis are important to determine the most dependent variable and to what extent the independent variable contributes to the total evaluation of dependent variable (output).

In this study, the independent variables are *latex* and *dry*, and are used to determine the total production (dependent) of the respective industry. From this general relationship, it is useful to determine which product is significant to the total production. This can be done by knowing the coefficient of the independent variable. Regression can be employed to approximate coefficient values. However classical regression is unable to deal with fuzzy information. Consequently, fuzzy regression is introduced to address the limitation of handling fuzziness in the data. The rubber statistical data in this experiment however contain element of fuzziness and randomness. As for that problem, the classical regression technique and fuzzy regression is incapable to deal with the data. Therefore, fuzzy random regression is proposed to approximate the coefficient based on the statistical data with the existence of these hybrid uncertainties.

Table 1. Data of rubber production

Year	Rubber Production		
	Dry	Latex	Total production
2000	774,248.00	153,360	927,608
2001	761,594.00	120,473	882,067
2002	775,334.00	114,498	889,832
2003	854,619.00	131,028	985,647
2004	960,841.00	207,894	1,168,735
2005	935,529.00	190,494	1,126,023
2006	1,073,698.00	209,934	1,283,632
2007	1,023,190.00	176,363	1,199,553
2008	918,656.00	153,709	1,072,365
2009	746,106.00	110,913	857,019
2010	846,813.00	92,428	939,241
2011	916,270.00	79,940	996,210
2012	846,813.00	75,985	922,798

In this experiment the coefficient values are approximated for the two products of rubber, which are the independent variable of the model. The approximated value can further been use for setting the production planning model. Fuzzy data arises from the expert preferences that contain fuzzy value or lingual notion. Meanwhile the unknown outcome and the difference of the expert's evaluation are regarded as random.

Table 2 shows the model's name and are further used in development of production planning as constraint model. Table 3 and Table 4 shows the sample of statistical data used to determine the coefficient value for planted area, and employment.

Table 2. Rubber production requirement

Item	Model
Target Constraint	1. Natural rubber production 2. Planted area 3. Employment in the rubber sector 4. Number of companies provided 5. Natural rubber consumption

Table 3. Data of planted area

Year	Planted area		
	Estate	Smallholding	Total Planted Area
2000	123.78	1,306.90	1,430.68
2001	95.52	1,293.80	1,389.32
2002	84.81	1,264.00	1,348.81
2003	78.46	1,247.41	1,325.60
2004	64.42	1,214.41	1,278.83
2005	57.37	1,213.93	1,271.30
2006	54.15	1,209.44	1,263.59
2007	53.35	1,194.69	1,248.04
2008	61.1	1,185.93	1,247.03
2009	61.1	967.14	1,028.24
2010	64.2	956.18	1,020.38
2011	64.2	962.84	1,027.04
2012	64.2	977.34	1,041.54

Table 4. Data of number of employment and labour

Year	Employment		
	Dry	Latex	Total
2000	37875.00	25250.00	63,125
2001	37152.00	24768.00	61,920
2002	35521.20	23680.80	59,202
2003	36585.00	24390.00	60,975
2004	37264.80	24843.20	62,108
2005	35654.40	23769.60	59,424
2006	36573.60	24382.40	60,956
2007	36301.20	24200.80	60,502
2008	35641.20	23760.80	59,402
2009	33885.60	22590.40	56,476
2010	36677.40	24451.60	61,129
2011	43217.40	28811.60	72,029
2012	50707.20	33804.80	84512

4.1 Data Preparation for Coefficient Approximation

This step engages data preparation to approximate the coefficients by using a fuzzy random regression model. The derived input data in this study were based on the previous actual data for production. Sample data preparation is as shown in Table 2 – Table 4 in which tabulates the total value for rubber production, planted area and employment, respectively. $\underline{a}_i$ and $\overline{a}_i$ denotes the assumed $\pm 5\%$ variations in the data from the actual data. The probability is observed as the proportional production of different mills distributed in the states of Malaysia and are denoted as Pr_i. The value of the probability shows the randomness. Table 5 tabulates the data for total value for production. The same data format is prepared for other criteria of *planted area* and *employment and labour*.

Table 5. The total production

$\underline{a}_i$	Pr_i	a_i	Pr_i	$\overline{a}_i$	Pr_i
774,247.95	0.33	774,248	0.33	774,248.05	0.33
761,593.95	0.33	761,594	0.33	761,594.05	0.33
775,333.95	0.33	775,334	0.33	775,334.05	0.33
854,618.95	0.33	854,619	0.33	854,619.05	0.33
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
746,105.95	0.33	746,106	0.33	746,106.05	0.33
846,812.95	0.33	846,813	0.33	846,813.05	0.33
916,269.95	0.33	916,270	0.33	916,270.05	0.33
846,812.95	0.33	846,813	0.33	846,813.05	0.33

4.2 Estimating the Coefficient

This section demonstrates the linear programming for rubber production in which the coefficient is estimated. The model is developed based on the algorithm for fuzzy random regression as explained in Section 3. The confidence interval which is necessary to treat fuzzy random uncertainties is calculated beforehand. The linear programming for rubber production is shown in equation (4.1) respectively. Regression model of model (4.1) is solved by using simplex method of linear program.

$$\begin{aligned}
&\min_{\bar{A}_{\text{production}}} \quad J(\bar{A}_{\text{production}}) = \sum_{k=1}^{2} (\bar{A}_k^r - \bar{A}_k^l) \\
&\text{subject to} \quad \bar{A}_1^r \ge \bar{A}_1^l \ge 0, \\
&\qquad \bar{A}_2^r \ge \bar{A}_2^l \ge 0, \\
&\qquad (7.7042 \times 10^5)\bar{A}_1^l + (1.5260 \times 10^5)\bar{A}_2^l \le 9.2302 \times 10^5, \\
&\qquad (7.5782 \times 10^5)\bar{A}_1^l + (1.1988 \times 10^5)\bar{A}_2^l \le 8.7770 \times 10^5, \\
&\qquad (7.7150 \times 10^5)\bar{A}_1^l + (1.1393 \times 10^5)\bar{A}_2^l \le 8.8543 \times 10^5, \\
&\qquad (8.5039 \times 10^5)\bar{A}_1^l + (1.3038 \times 10^5)\bar{A}_2^l \le 9.8077 \times 10^5, \\
&\qquad (9.1173 \times 10^5)\bar{A}_1^l + (7.9544 \times 10^4)\bar{A}_2^l \le 9.3454 \times 10^5, \\
&\qquad (9.4016 \times 10^5)\bar{A}_1^r + (1.9144 \times 10^5)\bar{A}_2^r \ge 11.3160 \times 10^5, \\
&\qquad (10.7901 \times 10^5)\bar{A}_1^r + (2.1097 \times 10^5)\bar{A}_2^r \ge 12.8999 \times 10^5, \\
&\qquad (10.2825 \times 10^5)\bar{A}_1^r + (1.7724 \times 10^5)\bar{A}_2^r \ge 12.0549 \times 10^5, \\
&\qquad (9.2320 \times 10^5)\bar{A}_1^r + (1.5447 \times 10^5)\bar{A}_2^r \ge 10.7767 \times 10^5.
\end{aligned} \tag{4.1}$$

Table 6. Approximated coefficient result for the rubber production

Item	Attributes	FRRM		FRM		Analysis of APEC
		coefficient	width	coefficient	width	
Production	x_1	1.01	0.01	0.99	0.00	2.0%
	x_2	1.00	0.00	1.00	0.00	0.0%
Planted area	x_1	0.97	0.08	0.99	0.00	2.0%
	x_2	0.99	0.00	0.99	0.00	0.0%
Employment	x_1	0.99	0.02	1.00	0.00	1.0%
	x_2	0.98	0.015	0.99	0.00	1.0%
Companies	x_1	1.03	0.00	1.00	0.00	3.0%
	x_2	0.98	0.015	0.99	0.00	1.0%
Consumption	x_1	1.01	0.005	0.99	0.00	2.0%
	x_2	0.99	0.00	1.00	0.00	1.0%

Keys:

FRRM : Fuzzy Random Regression Model x_1 : Dry

FRM : Fuzzy Regression Model x_2 : Latex

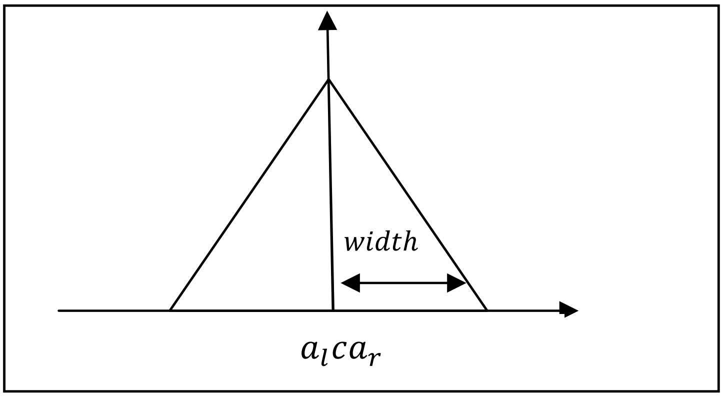

Figure 1. Triangular fuzzy number

5. Results and Discussions

This section gives explanation to the result obtained from the experiment. The linear program of fuzzy random regression model is solved. The comparison was made by the result with those obtained by the comparable method of fuzzy regression.

5.1 Coefficient Result

The fuzzy random regression model (4.1) is developed based on computation of confidence interval for rubber production data and other criteria, respectively. Table 6 shows the coefficient value which is obtained by Fuzzy Random Regression and Fuzzy Regression method. The coefficient value is represented in the form of centre value and its spread or width. Figure 1 illustrates the triangular fuzzy number where c is the center value of the coefficient, and $\mathrm{a_l}$ and $\mathrm{a_r}$ are the left and right boundaries. The width value is the spread from centre to the left and right boundaries, as shown in Figure 1.

Table 6 shows the approximated coefficient resulting for the rubber production. The coefficients are approximated using fuzzy random regression and fuzzy regression methods. The evaluation of independent variable x_1 and x_2 indicate that both variables are significant to the total evaluation. Meanwhile, the evaluation of decision factor x_1 and x_2 for *planted area* is $(0.97, 0.99)$, *employment* is $(0.99, 0.98)$, *companies* is $(1.03, 0.98)$ and *consumption* is $(1.01, 0.99)$. From the coefficient value, the independent variable x_1 has significant contribution to the total evaluation. In general, both fuzzy regression and fuzzy random regression is capable of dealing such data to estimate the coefficient values. However fuzzy regression is incapable to treat both fuzzy and random uncertainties at the same time.

The decision results are shown in the form of coefficient and its width, of x_1. The fuzzy random regression model produced a broader coefficient width because of the consideration of the confidence interval in its evaluation. The broader width shows that more information can be included under fuzzy valuation. The proposed APEC can be utilised to compare the result from FRRM and FRM to check the sensitivity of the results. From the analysis tabulated in Table VI, the weight value analysis through APEC for x_1 is 1.0% and x_2 is 0.0% for *production* model. This analysis indicates that the estimated weight obtained by the proposed FRRM has nearly same value to the one yielded by the FRM method. The APEC analysis shows the risk which decision maker should consider in this problem. Further study can be made to examine the width of triangular fuzzy number which may affect the obtained result.

The fuzzy random regression model with confidence interval for the rubber industry data were then defined as follows:

Rubber Production Model:

$$\begin{aligned} Y &= (A_i^{L,R})_T\, I[e_{X_i}, \sigma_{X_i}] \\ &= (0.99, 1.01)_T\, I[e_{X_i}, \sigma_{X_i}] + (1.00,\ 1.00)_T\, I[e_{X_i}, \sigma_{X_i}] \end{aligned} \tag{5.1}$$

Planted Area of Rubber Plant Model:

$$\begin{aligned} L &= (A_i^{L,R})_T\, I[e_{X_i}, \sigma_{X_i}] \\ &= (0.89,\ 1.05)_T\, I[e_{X_i}, \sigma_{X_i}] + (0.99,\ 0.99)_T\, I[e_{X_i}, \sigma_{X_i}] \end{aligned} \tag{5.2}$$

Employment and Labor Model for Rubber Production:

$$\begin{aligned} P &= (A_i^{L,R})_T\, I[e_{X_i}, \sigma_{X_i}] \\ &= (0.97, 1.01)_T\, I[e_{X_i}, \sigma_{X_i}] + (0.96,\ 0.99)_T\, I[e_{X_i}, \sigma_{X_i}] \end{aligned} \tag{5.3}$$

Required Companies Model for Rubber Production:

$$\begin{aligned} C &= (A_i^{L,R})_T\ I[e_{X_i},\sigma_{X_i}] \\ &= (0.98,\ 0.98)_T\ I[e_{X_i},\sigma_{X_i}] + (0.69,\ 0.72)_T\ I[e_{X_i},\sigma_{X_i}] \end{aligned} \tag{5.4}$$

Consumption Model for Rubber Production:

$$\begin{aligned} D &= (A_i^{L,R})_T\ I[e_{X_i},\sigma_{X_i}] \\ &= (1.00,\ 1.01)_T\ I[e_{X_i},\sigma_{X_i}] + (0.99,\ 0.99)_T\ I[e_{X_i},\sigma_{X_i}] \end{aligned} \tag{5.5}$$

The models (5.1)–(5.5) are then used to develop production planning model.

5.2 Development of Production Planning

The production planning model is developed based on the estimated coefficient deduced by the fuzzy random regression model (4.1). The following notations are defined to formulate the production planning problem.

- G_i : the total production (tonnes)
- x_1 : the quantity of dry rubber product to be produced (tonnes)
- x_2 : the quantity of latex rubber product to be produced (tonnes)
- P_j : the cultivated area (hectares)
- L_j : the number of employment
- C_j : the number of companies
- D_j : the consumption rate (tonnes)

For experimentation, production planning problems with one functional objective is investigated under four system constraints. The functional objective is *production* while the four system constraints are *cultivated are, employment, companies* and *consumption rate.* Note that all the coefficients used to develop the production model are deduced from the FRRM method; shows that the hybrid uncertainties exist in the statistical data are treated.

The production planning model for the problem is defined as follows.

$$\begin{array}{ll} \text{find} & x \\ \text{to satisfy:} & a_1x_1 + a_2x_2 \tilde{\geqq} g_1 \\ \text{subject to:} & L_1x_1 + L_2x_2 \le l \\ & P_1x_1 + P_2x_2 \le P \\ & C_1x_1 + C_2x_2 \tilde{\geqq} C \\ & D_1x_1 + D_2x_2 \tilde{\geqq} D\,. \end{array} \tag{5.6}$$

Based on the model shown in Equation (5.6), the coefficient values for the model are unknown. Occasionally, the decision makers are assumed to provide the values of these coefficients. However, sometimes the coefficient information is unavailable and estimating such values is difficult. Thus, in this study, the historical data available to this industry is utilized whereby fuzzy random regression and fuzzy regression are used to estimate the coefficient values.

Using the estimated coefficient deduced by the fuzzy random regression and the model in equation (4.1), the production planning model (5.6) is set as follows:

$$\begin{array}{ll} \text{find} & x \\ \text{to satisfy:} & 0.99x_1 + 1.00x_2 \gtrsim g_1 \\ \text{subject to:} & 1.00x_1 + 0.99x_2 \leq l \\ & 0.34x_1 + 1.99x_2 \leq P \\ & 1.00x_1 + 0.99x_2 \gtrsim C \\ & 1.00x_1 + 0.99x_2 \gtrsim D\,. \end{array} \tag{5.7}$$

Model (5.7) can be expanded to include more objective functions and constraints depending on the requirements. Solution to the multi-objective is required if the functional objectives consider more than one objective.

The work described in this study reveals that fuzzy random-based evaluations in a problem modeling can be used to better facilitate the decision-making process, specifically to extract important coefficients from among decision criteria and to appropriately address hybrid uncertainties (fuzzy random) in data.

6. Conclusion

This analysis allows the approximation of the coefficient values using historical data for the cases in which the data contained fuzziness and randomness. The results demonstrate that the proposed method can determine the important coefficient values and treats uncertainty in data. The experiment results shows that fuzzy random regression can be used to facilitate the speed for making decision, especiallyin extracting important coefficients among variables and fuzzy random information. The deduced coefficient from the fuzzy random regression is useful for decision maker (manager) to use the coefficient values as a guide for the production planning development. This production planning can be utilized to improve industry productivity in order to sustain its competitive edge. Moreover, heuristic approach of algorithm can be implemented to address the limitation of combinatorial problem in vertices method. In particular, this analysis permits the approximation of the weight of model attributes using historical data instead of the weight directly obtained from human experts, facilitating problem solving despite the existence of hybrid fuzzy random data.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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