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Artificial Bee Colony Algorithm for Solving Initial Value Problems

Research Article

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Abstract. A novel numerical differential equation method is presented to solve approximately an initial-value problem (IVP). The IVP is formulated as an optimization problem and the artificial bee colony algorithm (ABC) is used in order to find numerical solutions for this problem. Finally, we use an initial value problem example as illustration to testify the efficiency of the proposed method. The computational results showed that the proposed new method is quite promising, and the potential of the algorithm could be applied successfully in near future to other numerical method as well.

Keywords. Initial-value problem; Optimization problem; Artificial bee colony algorithm

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1. Introduction

Let $f = f(x,y)$ be a real-valued function of two real variables defined for $a \le x \le b$, where a and b are finite, and for all real values of y. The equations

$$\begin{cases} y' = f(x,y) \\ y(a) = y_0, \end{cases} \tag{1.1}$$

are called initial-value problem (IVP); they symbolize the following problem: To find a function $y(x)$, continuous and differentiable for $x \in [a,b]$ such that $y' = f(x,y)$ from $y(a) = y_0$ for all $x \in [a,b]$. Standard introductory texts are Ascher and Petzold [1], Gear [4], and Lambert [14]. Henrici [5] gives the details on some theoretical issues.

This problem possesses unique solution when: f is continuous on $[a,b] \times R$, and satisfies the Lipschitz condition; it exists a constant real $k > 0$, as $|f(x,\theta_1) - f(x,\theta_2)| \leq k|\theta_1 - \theta_2|$, for all $x \in [a,b]$ and all couple $(\theta_1,\theta_2) \in R \times R$.

Since there are relatively few IVP arising from practical problems for which analytical solutions are known, one must resort to numerical methods. In this situation it turns out that the numerical methods for each type of problem.

In practice, we always solve an initial-value problem (IVP) by using numerical methods. The solution is gotten with approximations: $y(x_0 + h), \ldots, y(x_0 + nh)$ where $a = x_0$ and $h = (b - a)/n$. Several methods are dedicated to the resolution of an IVP. These include Euler, Impproved Euler [6], Runge-Kutta methods [5] and extrapolation methods [3]. However, each of these numerical techniques has its own operational restrictions, which strictly confine their functioning domain. In addition, most of them are based on classical mathematical tools and not very accurate. To obtain a numerical solution with an acceptable accuracy, we have to use a very small step size h. A small step size h implies a larger number of steps, thus more computing time. It is desirable to develop methods that are more accurate than classical method.

Many different methods have been developed to solve IVP. However, the solution of IVP is still a challenge [11, 13]. Recently, artificial intelligence techniques are used to solve nonlinear differential equations and modeling engineering problems [17]. In this perspective, we are going to improve them while using Artificial bee colony (ABC) algorithm. The Artificial Bee Colony (ABC) algorithm was originally presented by Dervis Karaboga [7] under the inspiration of the collective behavior of honey bees. Because of its simplicity of implementation and capability to quickly converge to a reasonably good solution, the ABC has been successfully applied in solving many kinds of problems [8, 12, 15, 16, 18] beside numerical function optimization [2]. A good survey study on ABC algorithm can be found in [10].

The IVP is formulated as an optimization problem and the Artificial bee colony (ABC) algorithm may be used in order to find numerical solutions for this problem.

This paper is organized as follows. The formulation of the problem is revealed in Section 2; Section 3 provides basics on ABC algorithm and its main steps for finding an approximate solution of IVP. The Section 4 exposes an example to show how the ABC algorithm can lead to a satisfactory result for solving IVP. The comments and conclusion are made in Section 5.

2. Problem Formulation

2.1 Objective function

The variable x is discretized, say x_j for $j = 0, 1, 2, \dots, d$, then $y_j = y(x_j)$ for $j = 0, 1, 2, \dots, d$. The main idea behind the algorithm is to use the finite difference formula. For the derivative and equation (1.1) we obtain,

$$\frac{y(x_j) - y(x_{j-1})}{h} \approx f(x_{j-1}, y(x_{j-1})).$$

Thus,

$$\frac{y_j - y_{j-1}}{h} \approx f(x_{j-1}, y_{j-1}).$$

Consequently, we have to consider the error formula:

$$\left[\frac{y_j - y_{j-1}}{h} - f(x_{j-1}, y_{j-1}))\right]^2.$$

The objective function, associated to $Y = (y_1, y_2, \dots, y_d)$ will be:

$$F(Y) = \sum_{j=1}^{d} \left[\frac{y_j - y_{j-1}}{h} - f(x_{j-1}, y_{j-1})\right]^2. \tag{2.1}$$

2.2 Consistency

We are interested in the calculation of $Y = (y_1, y_2, \dots, y_d)$ which minimizes the objective function equation (2.1). We have from Taylor's Formula order 1;

$$y_j = y_{j-1} + h y'_{j-1} + O(h^2), \quad j = 1, \dots, d.$$

So,

$$\frac{y_j - y_{j-1}}{h} = y'_{j-1} + O(h).$$

If we subtract $f(x_{j-1}, y_{j-1})$ from both sides of last equation, we obtain

$$\frac{y_j - y_{j-1}}{h} - f(x_{j-1}, y_{j-1}) = y'_{j-1} - f(x_{j-1}, y_{j-1}) + O(h), \quad j = 1, \dots, d.$$

The last relation shows that the final value $Y = (y_1, y_2, \dots, y_d)$ is an approximate solution of IVP, for small value of h.

3. ABC Algorithm for Solving Initial Value Problems

The ABC algorithm imitates the behaviors of natural bees in searching food sources and sharing the information with other bees [7], the food search is collectively performed by three kinds of honey bees: employed bees, onlookers and scouts. The position of a food source represents a possible solution to the optimization problem and the nectar amount of a food source corresponds to the quality (fitness) of the associated solution. Each food source has just one employed bee. So, the number of employed bees and food source is equal. Onlooker bees and scout bees are described as unemployed bees.

The bee algorithm consists of an initialization step and a main search cycle which is iterated for a given number T of times, or until a solution of acceptable fitness is found. At the first step (initialization), a randomly distributed initial population (food source positions) is generated. After initialization, the population is subjected to repeat the cycles of the search processes of the employed, onlooker, and scout bees, respectively.

In this context, the function to be optimized is :

$$F(Y_i) = F(y_{1,i}, y_{2,i}, \dots, y_{j,i}, \dots, y_{d,i}) = \sum_{j=1}^{d} \left(\frac{y_{j,i} - y_{j-1,i}}{h} - f(x_{j-1,i}, y_{j-1,i}) \right)^2. \tag{3.1}$$

Where $y_{j,i} \in [y_{j,\min}; y_{j,\max}]$, $i \in \{1,2,\dots,n\}$ and $j \in \{1,2,\dots,d\}$, n is the number of employed bees, d is the dimension of the solution space.

The main process of the standard ABC for solving initial value problems is described as bellow:

Step 1: Initialize the initial swarm $Y_i = (y_{1,i}, y_{2,i}, \dots, y_{j,i}, \dots, y_{d,i})$ by using equation

$$y_{j,i} = y_{j,\min} + rand[0;1] * (y_{j,\max} - y_{j,\min}) \tag{3.2}$$

Calculate $F(Y_i)$ by using equation (3.1) and the fitness (fit_i) of each food source by using equation

$$fit_i = \begin{cases} \dfrac{1}{1+F(Y_i)} & \text{if } (F(Y_i) \geq 0) \\ 1 + abs(F(Y_i)) & \text{if } (F(Y_i) < 0). \end{cases} \tag{3.3}$$

Step 2: (*Move the employed bees*)

Calculate the new solution $y^*_{j,i}$ by using equation

$$y^*_{j,i} = y_{j,i} + rand[-1;1] * (y_{j,i} - y_{j,k}), \quad i \neq k \text{ and } i,k \in \{1,2,\dots,d\}. \tag{3.4}$$

Where j,k are selected randomly and $y_{j,k}$ is a neighbor bee of $y_{j,i}$. Calculate $F(Y^*_i)$ by using equation (3.1) and its fitness (fit_i) by using equation (3.3). After that we compare this fitness with its old one. If the new food source fitness has equal or better than the old fitness, the old one is replaced by the new one. Otherwise, the old one is retained.

Step 3: (*Move the onlookers*)

Calculate the probability p_i of selecting the food source i by

$$p_i = \frac{fit_i}{\sum_{j=1}^{d} fit_j}. \tag{3.5}$$

For improving the solution Y_i we use the main operations of Step 2.

Step 4: (*Move the Scouts*)

If the fitness values of the employed bees are not improved by a continuous predetermined number of iterations, which is called (Limit) those food sources are abandoned, and these employed bee become the scouts, and by using equation (3.2) generate a new solution for the employed bee.

Step 5: If the termination condition is met, the stop and the best food source is memorized; otherwise the algorithm returns to Step 2.

4. Results and Discussion

4.1 Example

Let us look at a simple IVP:

$$\begin{cases} \dfrac{dy}{dx} = y, & 0 \le x \le 1 \\ y(0) = 1\,. \end{cases}$$

For $d = 10$, $h = \frac{1-0}{10} = 0.1$, $x_0 = 0$, $y_0 = 1$, The exact solution is $y(x) = e^x$.

The objective function

$$F(y_1, y_2, \dots, y_{10}) = \sum_{j=1}^{10} \left(\frac{y_j - y_{j-1}}{h} - f(x_{j-1}, y_{j-1}) \right)^2$$
$$= \sum_{j=1}^{10} \left(\frac{y_j - y_{j-1}}{h} - y_{j-1} \right)^2 .$$

Parameters adopted for the ABC algorithm are given in Table 1.

Table 1. Parameters adopted for the ABC algorithm.

Max cycle	Swarm size (n)	Limit	Dimension (d)
3000	100	500	10

The obtained results, the comparison of the method to the exact solution and the incurred error in ABC's method are shown in Table 2.

Table 2. Exact/Numerical solution by ABC method of example for $d = 10$.

i	x_i	Y_{exact}	Y_{ABC}	Abs. err. (ABC)
0	0.0000	1.0000	1.0000	0.0000
1	0.1000	1.1052	1.1053	0.0001
2	0.2000	1.2214	1.2215	0.0001
3	0.3000	1.3499	1.3492	0.0007
4	0.4000	1.4918	1.4907	0.0011
5	0.5000	1.6487	1.6467	0.0020
6	0.6000	1.8221	1.8191	0.0030
7	0.7000	2.0138	2.0093	0.0045
8	0.8000	2.2255	2.2195	0.0060
9	0.9000	2.4596	2.4524	0.0072
10	1.0000	2.7183	2.7094	0.0089

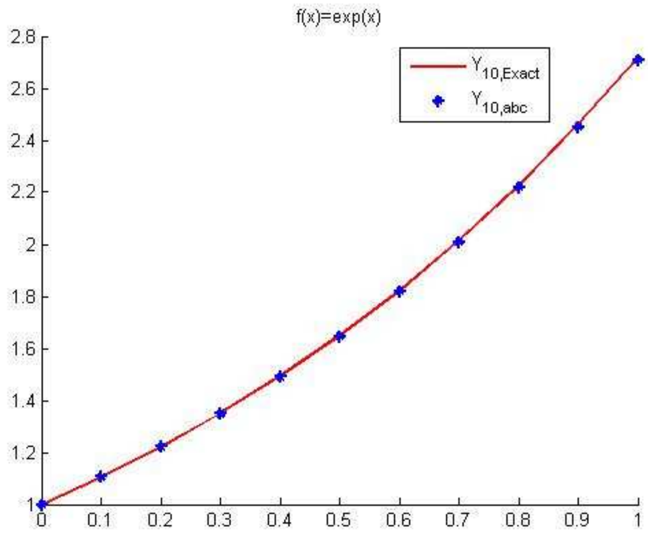

Figure 1. Numerical solutions by ABC method of example for $d = 10$.

5. Conclusion

In this study, the ABC algorithm is introduced to find an approximate solution of IVP. The conducted comparison between the exact solution and the algorithm outcomes in the investigated example showed that the algorithm yields satisfactorily precise approximation of the solution in its current preliminary formulation.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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