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Fixed Point Results on Generalized $(\psi,\phi)_s$-Contractive Mappings in Rectangular b-Metric Spaces

Research Article

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Abstract. The aim of this paper is to present the definition of a weak altering distance function and new generalized contractive mapping in rectangular b-metric spaces. We discuss the fixed point result of such a mapping in rectangular b-metric spaces.

Keywords. Fixed point; Rectangular metric space; Rectangular b-metric space; Partially ordered set; Weak altering distance function

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1. Introduction

In nonlinear analysis, the Banach contraction mapping principle is introduced by Banach [3]. This is a classical and powerful tool to solve the existence and uniqueness of fixed points of certain self-maps of metric spaces. Afterwards, several authors have studied weak contraction mapping which is generalizations of the Banach contractions mapping. Some fixed point theorems of weak contractions in complete metric spaces were proved by many authors, see in [14]-[19]. In [6], [11]-[23], the partially ordered metric spaces extended the existence of a fixed point for weak contractions and generalized contractions. Some of them concern altering distance functions which were introduced by Khan *et al.* [14]. They discussed about fixed point theorems in complete and compact metric spaces.

In 2014, Su [28] introduced generalized contraction mappings concerning generalized altering distance functions. He proved a new fixed point theorem of generalized contraction mappings in a complete partially order metric spaces.

On the other hand, George *et al.* [7] introduced the concept of rectangular b-metric space. They proved an analogue of Banach contraction principle and Kannan's fixed point theorem in this space.

At the same time, Roshan *et al.* [22] also introduced almost generalized weakly contractive mappings which was constructed from an altering distance function. Some fixed point theorems with almost generalized weakly contractive mappings are established.

In this paper, we introduce a weak altering distance function and new generalized contractive mapping in rectangular b-metric spaces. The fixed point result of such a mapping in rectangular b-metric spaces are discussed.

2. Auxiliary Results

In this work, we focus rectangular b-metric spaces which was introduced by Roshan *et al.* [22]. In this space, Roshan *et al.* gave two lemmas which plays an important role to prove the fixed point result of new generalized contractive mapping in rectangular b-metric spaces.

Definition 2.1 ([22])**.** Let X be a nonempty set, $s \geq 1$ be a given real number and let $d : X \times X \to [0,\infty)$ be a mapping such that for all $x, y \in X$ and all distinct points $u, v \in X$, each distinct from x and y:

(i) $d(x,y) = 0$ if and only if $x = y$;

(ii) $d(x,y) = d(y,x)$;

(iii) $d(x,y) \leq s[d(x,u) + d(u,v) + d(v,y)]$ (b-rectangular inequality).

Then (X,d) is called a rectangular b-metric space or generalized b-metric space with parameter s.

Convergent and Cauchy sequences in rectangular b-metric spaces, completeness, we define as follows:

Definition 2.2 ([7]). Let (X,d) be a rectangular b-metric space, x_n be a sequence in X and $x \in X$:

(i) the sequence x_n is said to be converges to a point x in (X,d), if for every $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n,x) < \varepsilon$ for all $n > n_0$ and this fact is represented by $\lim\limits_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$;

(ii) the sequence x_n is said to be Cauchy sequence in (X,d) if for every $\varepsilon > 0$ there exists $n_0 \in \mathbb{N}$ such that $d(x_n,x_{n+p}) < \varepsilon$ for all $n > n_0$, $p > 0$ or equivalently, if $\lim\limits_{n\to\infty} d(x_n,x_{n+p}) = 0$ for all $p \in \mathbb{N}$;

(iii) (X,d) is said to be a complete rectangular b-metric space if every Cauchy sequence in X converges to some $x \in X$.

We define continuous mapping in rectangular b-metric spaces as follows:

Definition 2.3. A mapping f is called continuous mapping if $fx_n \to fx$ wherever $x_n \to x \in X$ as $n \to \infty$.

Next, the two lemmas concerning Cauchy sequence on rectangular b-metric spaces are stated here.

Lemma 2.4 ([22]). *Let (X,d) be a rectangular b-metric space and let $\{x_n\}$ be a Cauchy sequence in X such that $x_m \neq x_n$ whenever $m \neq n$. Then $\{x_n\}$ can converge to at most one point.*

Lemma 2.5 ([22]). *Let (X,d) be a rectangular b-metric space with coefficient $s \geq 1$. If $y \in X$ and $\{x_n\}$ is a Cauchy sequence in X with $x_n \neq x_m$ for infinitely many $m,n \in \mathbb{N}$, $n \neq m$, converging to x where $x \neq y$, then*

$$\frac{1}{s} d(x,y) \leq \liminf_{n\to\infty} d(x_n,y) \leq \limsup_{n\to\infty} d(x_n,y) \leq s d(x,y).$$

3. Main results

In this section, we will introduce definitions of weak altering distance function and generalized $(\psi,\phi)_s$-contractive mapping. With these definitions, we will establish the existence of fixed point results in rectangular b-metric spaces.

Definition 3.1. Let $s \geq 1$ be a given real number. A function $\psi : [0,\infty) \to [0,\infty)$ is called a s-weak altering distance function as following properties:

(i) if $\{x_n\}$ is a sequence in $[0,\infty)$ and $x \in [0,\infty)$ such that $\frac{x}{s} \leq \limsup\limits_{n\to\infty} x_n \leq sx$, then $\psi(\limsup\limits_{n\to\infty} x_n) \leq \limsup\limits_{n\to\infty} \psi(x_n)$;

(ii) ψ is non-decreasing;

(iii) $\psi(t) = 0$ if and only if $t = 0$.

Remark 3.2. It is easy to see that the class of s-weak altering distance functions is larger than the class of altering distance functions.

Example 3.3. *Let $\psi:[0,\infty)\to[0,\infty)$ define by*

$$\psi(t)=\begin{cases} t^2, & 0\le t\le \dfrac{1}{2},\\ t+\frac{1}{10}, & t>\dfrac{1}{2}. \end{cases}$$

Then ψ is a s-weak altering distance.

Proof. Let $\{x_n\}$ is a sequence in $[0,\infty)$ such that for all $s\ge 1$ there exist $x\in[0,\infty)$ with

$$\frac{x}{s}\le \limsup_{n\to\infty} x_n \le sx.$$

We want to show that

$$\psi(\limsup_{n\to\infty} x_n)\le \limsup_{n\to\infty}\psi(x_n).$$

Case 1: $0\le \limsup\limits_{n\to\infty} x_n\le \dfrac{1}{2}$, we get

$$\begin{aligned}\psi(\limsup_{n\to\infty} x_n) &= (\limsup_{n\to\infty} x_n)^2\\ &= \big(\lim_{n\to\infty}(\sup\{x_k;k\ge n\})\big)^2\\ &= \lim_{n\to\infty}((\sup\{x_k;k\ge n\})^2)\\ &= \lim_{n\to\infty}(\sup\{x_k^2;k\ge n\})\\ &= \limsup_{n\to\infty} x_n^2\end{aligned}$$

and

$$\begin{aligned}\limsup_{n\to\infty}\psi(x_n) &= \lim_{n\to\infty}(\sup\{\psi(x_k);k\ge n\})\\ &= \lim_{n\to\infty}(\sup\{x_k^2;k\ge n\})\\ &= \limsup_{n\to\infty} x_n^2.\end{aligned}$$

Case 2: $\limsup\limits_{n\to\infty} x_n>\frac{1}{2}$, we get

$$\psi(\limsup_{n\to\infty} x_n)=\limsup_{n\to\infty} x_n+\frac{1}{10}$$

and

$$\begin{aligned}\limsup_{n\to\infty}\psi(x_n) &= \lim_{n\to\infty}(\sup\{\psi(x_k);k\ge n\})\\ &= \lim_{n\to\infty}\left(\sup\left\{x_k+\frac{1}{10};k\ge n\right\}\right)\\ &= \lim_{n\to\infty}(\sup\{x_k;k\ge n\})+\frac{1}{10}\\ &= \limsup_{n\to\infty} x_n+\frac{1}{10}.\end{aligned}$$

Then ψ is a s-weak altering distance function but ψ is not an altering distance function because ψ is not continuous. □

Definition 3.4. Let $(X,\preceq,d)$ be a partially order rectangular b-metric spaces with parameter $s\geq 1$. A mapping $f:X\to X$ is said to be a *generalized* $(\psi,\phi)_s$*-contractive* if

$$\psi(sd(fx,fy))\leq\phi(d(x,y)) \tag{3.1}$$

for all $x,y\in X$ with $x\preceq y$, where $\psi:[0,\infty)\to[0,\infty)$ is a s-weak altering distance function and $\phi:[0,\infty)\to[0,\infty)$ is a function satisfies the following conditions:

(i) $\psi(t)>\phi(t)$ if and only if $t>0$,

(ii) if $\{x_n\}$ is a sequence in $[0,\infty)$ such that $\frac{x}{s}\leq\limsup\limits_{n\to\infty}x_n\leq sx$ for some $x\in[0,\infty)$, then $\limsup\limits_{n\to\infty}\phi(x_n)\leq\phi(x)$.

In our work, we will prove the fixed point result for generalized $(\psi,\phi)_s$-contractive mapping in partially ordered rectangular b-metric spaces. Before proving our main result we need the following Lemma:

Lemma 3.5. *Let $(X,\preceq,d)$ be a complete partially ordered rectangular b-metric spaces with parameter $s\geq 1$ and $f:X\to X$ be a generalized $(\psi,\phi)_s$-contractive mapping. If f is continuous non-decreasing with respect to $\preceq$ and there exist $x_0\in X$ such that $x_0\preceq fx_0$, then f has periodic point.*

Proof. Let $\{x_n\}$ be a sequence in X, such that $x_{n+1}=fx_n$, for all $n\in\mathbb{N}\cup\{0\}$. Since $x_0\preceq fx_0=x_1$ and f is non-decreasing, we have $x_1=fx_0\preceq fx_1=x_2$. Continuing this process, we get

$$x_0\preceq x_1\preceq\cdots\preceq x_n\preceq x_{n+1}\preceq\cdots$$

for all $n\in\mathbb{N}$. We will show that f has a periodic point, that is, there exist a positive integer p and a point $z\in X$ such that $z=f^pz$. Assume the contrary, that is, f has no periodic point. Then, all elements of the sequence $\{x_n\}$ are distinct, *i.e.* we can assume that $x_n\neq x_m$ for all $n\neq m$. Next, we want to show the $\{x_n\}$ is a Cauchy sequence in X. We suppose that $\{x_n\}$ is not a Cauchy sequence. Then there exists $\varepsilon>0$ for which we can find two subsequences $\{x_{m_i}\}$ and $\{x_{n_i}\}$ of $\{x_n\}$ such that n_i is the smallest index for which

$$n_i>m_i>i \quad\text{and}\quad d(x_{m_i},x_{n_i})\geq\varepsilon. \tag{3.2}$$

This means that

$$d(x_{m_i},x_{n_i-2})<\varepsilon. \tag{3.3}$$

Using (3.3) and taking the upper limit as $i\to\infty$, we get

$$\limsup_{i\to\infty}d(x_{m_i},x_{n_i-2})\leq\varepsilon. \tag{3.4}$$

On the other hand, we have

$$d(x_{m_i},x_{n_i})\leq sd(x_{m_i},x_{m_i+1})+sd(x_{m_i+1},x_{n_i-1})+sd(x_{n_i-1},x_{n_i}). \tag{3.5}$$

Next, we will show that $d(x_{m_i},x_{m_i+1})\to 0$ as $i\to\infty$. Since $x_{m_i-1}\neq x_{m_i}$, for all $m_i\in\mathbb{N}$, by (3.1), we have

$$\psi(d(x_{m_i},x_{m_i+1}))\leq\psi(sd(x_{m_i},x_{m_i+1}))$$

$$\begin{aligned}&\le \phi(d(x_{m_i-1},x_{m_i}))\\&< \psi(d(x_{m_i-1},x_{m_i})),\end{aligned} \tag{3.6}$$

for all $m_i \in \mathbb{N}$. This implies that

$$d(x_{m_i+1},x_{m_i}) < d(x_{m_i},x_{m_i-1}) \tag{3.7}$$

for all $m_i \in \mathbb{N}$. Hence, the sequence $\{d(x_{m_i+1},x_{m_i})\}$ is decreasing and bounded below. Consequently, there exists $r \ge 0$ such that

$$d(x_{m_i+1},x_{m_i}) \to r \quad \text{as} \quad i \to \infty. \tag{3.8}$$

This implies that $\lim_{i\to\infty} d(x_{m_i},x_{m_i-1}) = r$ and then

$$\limsup_{i\to\infty} d(x_{m_i},x_{m_i-1}) = \liminf_{i\to\infty} d(x_{m_i},x_{m_i-1}) = r.$$

Now we obtain that

$$\frac{r}{s} \le \limsup_{i\to\infty} d(x_{m_i},x_{m_i-1}) = r \le sr.$$

Consider the properties of ψ and ϕ, letting $i \to \infty$ in (3.6), we get

$$\psi(r) \le \limsup_{i\to\infty} \psi(d(x_{m_i+1},x_{m_i})) \le \limsup_{i\to\infty} \phi(d(x_{m_i},x_{m_i-1})) \le \phi(r).$$

By using the condition $\psi(t) > \phi(t)$ for all $t > 0$, we have $r = 0$, and hence

$$d(x_{m_i+1},x_{m_i}) \to 0 \quad \text{as} \quad i \to \infty. \tag{3.9}$$

Similarly, we have

$$d(x_{n_i-1},x_{n_i}) \to 0 \quad \text{as} \quad i \to \infty. \tag{3.10}$$

Therefore, by using (3.9) and (3.10), then the inequality (3.5) becomes

$$\limsup_{i\to\infty} d(x_{m_i},x_{n_i}) \le s(\limsup_{i\to\infty} d(x_{m_i+1},x_{n_i-1})). \tag{3.11}$$

Thus, by using (3.2) and (3.11), we have

$$\frac{\varepsilon}{s} \le \limsup_{i\to\infty} d(x_{m_i+1},x_{n_i-1}). \tag{3.12}$$

Using the b-rectangular inequality, so we have the following inequality:

$$d(x_{m_i+1},x_{n_i-1}) \le sd(x_{m_i+1},x_{m_i}) + sd(x_{m_i},x_{n_i-2}) + sd(x_{n_i-2},x_{n_i-1}). \tag{3.13}$$

Using the same argument as in (3.9), we have

$$d(x_{n_i-2},x_{n_i-1}) \to 0 \quad \text{as} \quad i \to \infty. \tag{3.14}$$

Therefore, by using (3.3), (3.9), (3.12) and (3.14), then the inequality (3.13) becomes

$$\frac{\varepsilon}{s} \le \limsup_{i\to\infty} d(x_{m_i+1},x_{n_i-1}) \le s\varepsilon. \tag{3.15}$$

Again using the b-rectangular inequality, so we have the following inequality:

$$d(x_{m_i},x_{n_i}) \le sd(x_{m_i},x_{n_i-2}) + sd(x_{n_i-2},x_{n_i-1}) + sd(x_{n_i-1},x_{n_i}). \tag{3.16}$$

Thus, by using (3.3), (3.4), (3.10), (3.14) and taking the upper limit as $i\to\infty$, we get

$$\frac{\varepsilon}{s}\le \limsup_{i\to\infty} d(x_{m_i},x_{n_i-2})\le s\varepsilon. \tag{3.17}$$

From (3.1), we have

$$\psi(sd(x_{m_i+1},x_{n_i-1}))\le \phi(d(x_{m_i},x_{n_i-2})). \tag{3.18}$$

Next, taking the upper limit as $i\to\infty$ in (3.18) and using (3.12), (3.15) and (3.17) we have

$$\begin{aligned}
\psi(\varepsilon) &= \psi\left(s\cdot\frac{\varepsilon}{s}\right)\\
&\le \psi(s\limsup_{i\to\infty} d(x_{m_i+1},x_{n_i-1}))\\
&= \psi(\limsup_{i\to\infty} sd(x_{m_i+1},x_{n_i-1}))\\
&\le \limsup_{i\to\infty}\psi(sd(x_{m_i+1},x_{n_i-1}))\\
&\le \limsup_{i\to\infty}\phi(d(x_{m_i},x_{n_i-2}))\\
&\le \phi(\varepsilon).
\end{aligned}$$

This contradicts to $\psi(t)>\phi(t)$ for all $t>0$. Thus, $\{x_{n+1}\}=\{fx_n\}$ is a Cauchy sequence in X. As X is complete, there exists $u\in X$ such that $x_n\to u$ as $n\to\infty$. By Lemma 2.4, we obtain that

$$u=\lim_{n\to\infty} x_{n+1}=\lim_{n\to\infty} fx_n=f(\lim_{n\to\infty} x_n)=fu.$$

This result contradicts the assumption that f has no periodic points. This completes the proof. □

Now, we are ready to prove the first main theorem.

Theorem 3.6. *Let $(X,\preceq,d)$ be a complete partially ordered rectangular b-metric spaces with parameter $s\ge 1$ and $f:X\to X$ be a generalized $(\psi,\phi)_s$-contractive mapping. Suppose that the following conditions hold:*

(i) *f is a continuous mapping;*
(ii) *f is non-decreasing with respect to $\preceq$;*
(iii) *there exists $x_0\in X$ such that $x_0\preceq fx_0$.*

Then f has a fixed point.

Proof. Since $x_0\preceq fx_0=x_1$ and f is non-decreasing, we have $x_1=fx_0\preceq fx_1=x_2$. Continuing this process, we get

$$x_0\preceq x_1\preceq\cdots\preceq x_n\preceq x_{n+1}\preceq\cdots$$

for all $n\in\mathbb{N}$. By Lemma 3.5, f has a periodic point. Thus, there exist $z\in X$ and a positive integer p such that $z=f^pz$. If $p=1$, then $z=fz$ so z is fixed point of f. Let $p>1$. We claim that the fixed point of f is $f^{p-1}z$. Suppose the contrary, that is, $f^{p-1}z\ne f(f^{p-1}z)$. Then $d(f^{p-1}z,f^pz)>0$ and so is $\phi(d(f^{p-1}z,f^pz))<\psi(d(f^{p-1}z,f^pz))$. Letting $x=f^{p-1}z$ and $y=f^pz$ in (3.1), we have

$$\psi(sd(z,fz))=\psi(sd(f^pz,f^{p+1}z))$$

$$
\begin{aligned}
&\leq \phi(d(f^{p-1}z, f^p z)) \\
&< \psi(d(f^{p-1}z, f^p z)), \\
&\leq \psi(sd(f^{p-1}z, f^p z)),
\end{aligned}
$$

and taking into account the fact that ψ is non-decreasing, we deduce

$$d(z, fz) < d(f^{p-1}z, f^p z).$$

Now, we write $x = f^{p-2}z$ and $y = f^{p-1}z$ in (3.1), we get

$$
\begin{aligned}
\psi(sd(f^{p-1}z, f^p z)) &\leq \phi(d(f^{p-2}z, f^{p-1}z)) \\
&< \psi(d(f^{p-2}z, f^{p-1}z)), \\
&\leq \psi(sd(f^{p-2}z, f^{p-1}z)),
\end{aligned}
$$

which implies $d(f^{p-1}z, f^p z) < d(f^{p-2}z, f^{p-1}z)$ since ψ is non-decreasing. We continue in this way and end up with the inequalities

$$d(z, fz) < d(f^{p-1}z, f^p z) < d(f^{p-2}z, f^{p-1}z) < \cdots < d(z, fz),$$

which is a contradiction. Therefore, the assumption $d(f^{p-1}z, f^p z) > 0$ is wrong, that is, $d(f^{p-1}z, f^p z) = 0$ and $f^{p-1}z$ is the fixed point of f. □

In the second Theorem, we replace the assumption that f is continuous by another condition, but we have the some conclusion as in Theorem 3.6.

Theorem 3.7. *Let $(X, \preceq, d)$ be a complete partially ordered rectangular b-metric spaces with parameter $s \geq 1$ and $f : X \to X$ be a generalized $(\psi,\phi)_s$-contractive mapping. Suppose that the following conditions hold:*

(i) *if $\{x_n\}$ is a non-decreasing sequence in X such that $x_n \to x \in X$, then $x_n \preceq x$ and $\limsup_{n\to\infty} \phi(d(x_n, x)) = 0$ for all $n \in \mathbb{N}$;*

(ii) *f is non-decreasing with respect to $\preceq$;*

(iii) *there exists $x_0 \in X$ such that $x_0 \preceq fx_0$.*

Then f has a fixed point.

Proof. Following similar arguments to those given in the proof of Lemma 3.5, we construct an increasing sequence $\{x_n\}$ in X such that $x_n \to u$, for some $u \in X$. Suppose that $fu \neq u$. Since $x_n \preceq u$ for all $n \in \mathbb{N}$, by (3.1), we have

$$\psi(sd(x_{n+1}, fu)) \leq \phi(d(x_n, u)). \tag{3.19}$$

By this inequality, using Lemma 2.5, first property in Definition 3.1 and inequality (3.1), we get

$$
\begin{aligned}
\psi(d(u, fu)) &= \psi\left(s \cdot \frac{1}{s} d(u, fu)\right) \\
&\leq \psi\left(s \limsup_{i\to\infty} d(x_{n+1}, fu)\right) \\
&\leq \limsup_{n\to\infty} \psi(sd(fx_n, fu))
\end{aligned}
$$

$$\leq \limsup_{n\to\infty} \phi(d(x_n,x))$$
$$= 0. \tag{3.20}$$

Since ψ is a weak altering distance function, we get $\psi(d(u,fu)) = 0$ and so $u = fu$. Hence, u is a fixed point of f. □

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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