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A Conceptual Mathematical Examination of the Aerosols Loading over Abuja-Nigeria

Research Article

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Abstract. One of the urban cities in Nigeria known for elevated levels of atmospheric aerosol pollution is Abuja, been the fastest growing city in Nigeria. Due to the health problems associated with this fact, a study of aerosol loading must be prioritized so as to identify the preventive measures required. This study aims at estimating the aerosol loading and retention over Abuja. Statistical AOD analysis for thirteen years was obtained from the Multi-angle imaging spectro radiometer (MISR). Statistical tools, as well as analytically derived model for aerosols loading were used to obtain the aerosols retention and loading over the area. Observations shows that, the highest AOD analysis was found in 2005, the highest skew and kurtosis can be found in 2010 and the highest Kolmogorov-Smirnov stat can be found in 2004. This results shows that the lower atmosphere of Abuja may not be dynamic as cities in the southern Nigeria and proposes an inclusion of the attenuation due to moving aerosols layer into the ITU model which is significant via the atmospheric constants over Abuja. The aerosols retention peak in Abuja occurs every ten years.

Keywords. Aerosols; AOD analysis; Statistical tool; Analytical dispersion model

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1. Introduction

The atmosphere is a complex dynamic natural gaseous system that is essential to support life on planet Earth. Stratospheric ozone depletion due to air pollution has long been recognized as a threat to human health as well as to the Earth's ecosystems (Anderson, 2005). Aerosols are pollutants in form of solid particles, liquid droplets, or gases present in air which can cause harm to humans and the environment. They may be natural or man-made (Anderson, 2005).

Majorly, human activities causes an increase in aerosols globally. Some facts from researches shows that aerosol is causing a net cooling because it reflects incoming sunlight back to space as well as causes clouds to become brighter and more refective (Bian et al., 2009). Aerosols pollution reduction is a very important aspect because it is associated with health problems and reduced atmospheric visibility such as asthma, lung cancer, cardiovascular issues, birth defects, and premature death. It also causes stunted growth and mortality in some plant species(Cherian et al., 2010).

Abuja is one of the urban cities in Nigeria known for elevated levels of atmospheric aerosol pollution (Obioh et al., 2005) which can be emmitted from various sources within the Abuja metropolis. Population rate increases daily in The Federal Capital Territory (Abuja) and as the population increases pollution rate also increases. The adverse effects are intensified on the population living in urban environment influenced by high traffic density or industry (Dongarra et al., 2010). Since urban areas are influenced by multiple sources of aerosols, source-receptor relations is needed to establish source contribution, which helps to identify key sources important when designing pollution abatement strategy to enhance air quality management for the city.

Impact of aerosols is difficult to measure due to fact that they do not build up uniformly but only settle out of the atmosphere within a few days. (Cheng et al., 2008). Hence, this paper worked on statistical examination of the aerosols loading over Abuja-Nigeria. The satellite observation analysis was computed using coefficient of variation.

2. Theories: Statistical Formulation

The concept of variance is intrinsically connected with the effects of the difference between the monthly mean and the thirteen-years mean on the AOD performance in Abuja-Nigeria. The coefficient of variation is the measure of a normalized dispersion of probability distribution i.e. the thirteen years mean for each parameter used. In statistics, coefficient of variation is referred to as relative standard deviation and expressed in percentage. In recent times, the value of the coefficient of variation can exceed unity and therefore would create percentages larger than one hundred (Allison, 1978). Hence, the rationale for using measures such as the coefficient of variation is more substantive of its interaction effect between the standard deviation and the inverse of the mean. Coefficient of variation can be represented mathematically as

$$CV = \frac{\sigma}{\alpha}. \tag{1}$$

Here σ is the standard deviation and μ is the monthly mean. Therefore higher coefficient of variation (CV) is termed to be more dispersed than its lower value. The beauty of CV is its ability to compare distributions obtained with different units. It is based upon this important property; we can determine the aerosols retention between two consecutive years. Hence we propose as:

$$A = \left|\frac{G_r - G_P}{G_r}\right|^2 \times 100\%. \tag{2}$$

Here, the previous and current years are denoted as G_P and G_R respectively. The beauty of this formulation is the inclusion of the possibility of obtaining CV that is above unity. However, the possibility of obtaining above 100% is inevitable. Hence the proposition of the second equation:

$$A = \left| \frac{G_P}{G_P - G_r} \right|^2 \times 100\%. \tag{3}$$

Equation (3) is valid only if $G_P - G_r > G_P$. Upon this salient assumption, the aerosol retention can be controlled below unity or 100%.

3. Validation of Data source

Abuja is the fastest growing city in Nigeria and it is located on longitude 7.483 °E and latitude 9.067 °N in the Sahelian geographic region south of the Sahara (see Figure 1), hence, we expect a high impact of the north east winds alongside Sahara dust. Also, it is situated close to the Atlantic coast and is influenced by the local steppe climate. Abuja has average temperature and precipitation of 24 °C and 164 mm respectively. The distance of Abuja to the Sahara is about 2,512 km. In the past, no aerosols ground observation was available; hence, the satellite observation was adopted. Fourteen years satellite observation was obtained from the Multi-angle Imaging Spectro Radiometer (MISR). The MISR operates at various directions i.e. nine different angles (70.5°, 60°, 45.6°, 26.1°, 0°, 26.1°, 45.6°, 60°, 20.5°) and gathers data in four different spectral bands (blue, green, red, and near-infrared) of the solar spectrum. The blue band is at wavelength 443 nm, the green band is at wavelength 555 nm, the red band wavelength 670 nm and the infrared band is at wavelength 865 nm. MISR acquire images at two different levels of spatial resolution i.e. local and global mode. It gathers data at the local mode at 275 meter pixel size and 1.1 Km at the global mode. Typically, the blue band is to analyze coastal and aerosol studies. The green band is to analyze Bathymetric mapping and estimating peak vegetation. The red band analysis the variable vegetation slopes and the infrared band analysis the biomass content and shorelines.

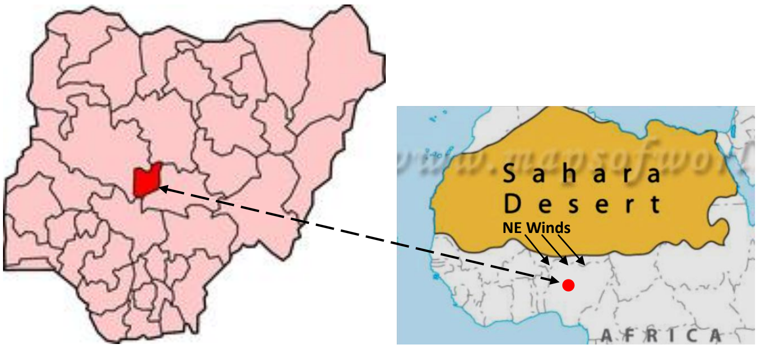

Figure 1. Map of Abuja and the Sahara influence

4. Methodology

The raw MISR dataset was processed using the Excel package. The mean for each month were calculated for each year. We tested the accuracy of the data by applying the aerosol dispersion model that was propounded by Emetere et al. (2015a). An extension of the dispersion model used is given as

$$\psi(\lambda) = a_1^2 \cos\left(\frac{n_1\pi\tau(\lambda)}{k_y} + \alpha\right)\cos\left(\frac{n_1\pi\tau(\lambda)}{k_z} + \alpha\right) + a_2^2 \cos\left(\frac{n_2\pi\tau(\lambda)}{k_y} + \beta\right)\cos\left(\frac{n_2\pi\tau(\lambda)}{k_z} + \beta\right) \tag{4}$$

Here α and β are the phase differences, k is the diffusivity, τ is the AOD, ψ is the concentration of contaminant, λ is the wavelength, a and n are atmospheric and tuning constants respectively.

The percentage of retention can be determined from the coefficient of variance for each year. This was done by considering the previous and current years which are denoted as G_P and G_r respectively. Hence we propound that the aerosols retention between two years as:

$$A = \left|\frac{G_P - G_r}{G_P}\right|^2 \times 100\%. \tag{5}$$

The aerosols retention can be calculated from Tables 1–2 to obtain Tables 3–4. Any statistical tool could be used to obtain the atmospheric aerosols retention. In this paper, the Matlab and the Excel package were used to obtain the results shown in the succeeding section.

5. Results and Discussion

The trending of the AOD distribution in Abuja is shown in Figures 2 to 4. The accurate curve fitting of the proposed model was used to derive the atmospheric constant as shown in Table 1. The data for March was far beyond the proposed model-showing the dynamism of the aerosols loading in March, 2008. However, the data set for March tallies with other years. The general performance of the AOD within 2000 to 2013 is shown in Figure 5. The dynamism of March and April keeps dropping per year. This certain AOD drop in March and April through 2000 to 2013 may be due to a reduced recirculation zone. The loadings for December increased in the past five years while January is unpredictably varying. The variation in January may be due to the influx of Sahara dust aerosol by the northeast winds. Scanty data was observed in June, July and September. No data was seen for August. Retrieving data using satellite sensor from June to September is difficult because of moisture contents (Adebiyi et al., 2015).

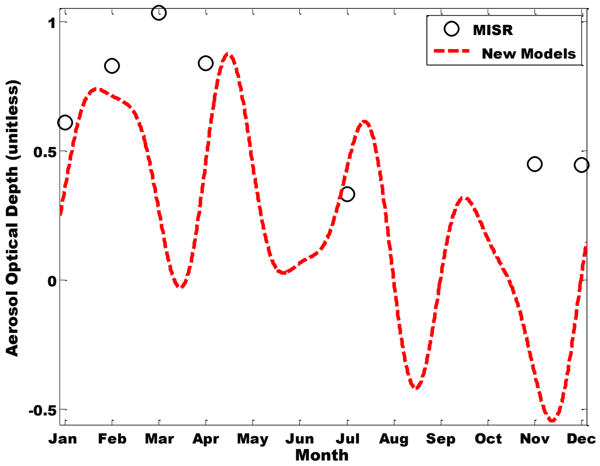

Figure 2. AOD for new model and MISR for the year 2004

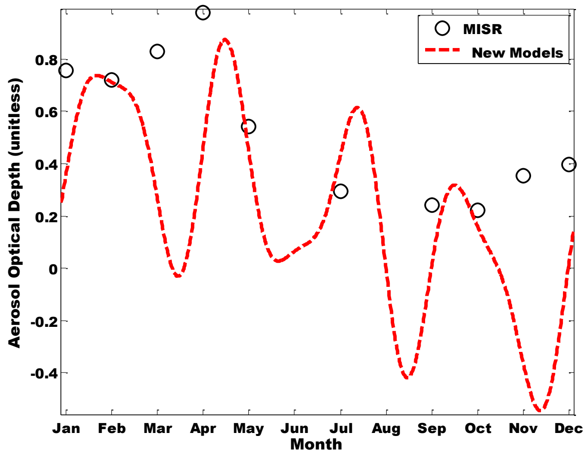

Figure 3. AOD for new model and MISR for the year 2008

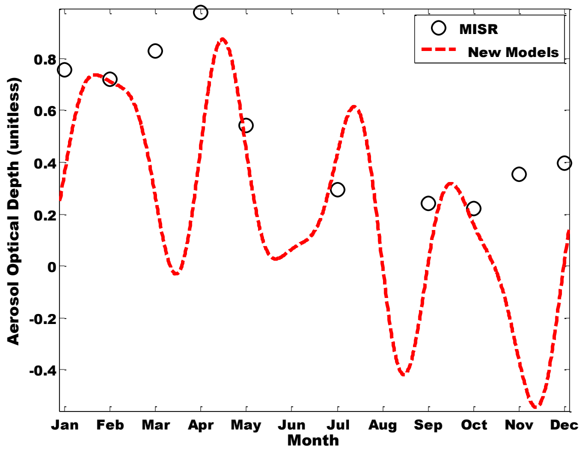

Figure 4. AOD for new model and MISR for the year 2012

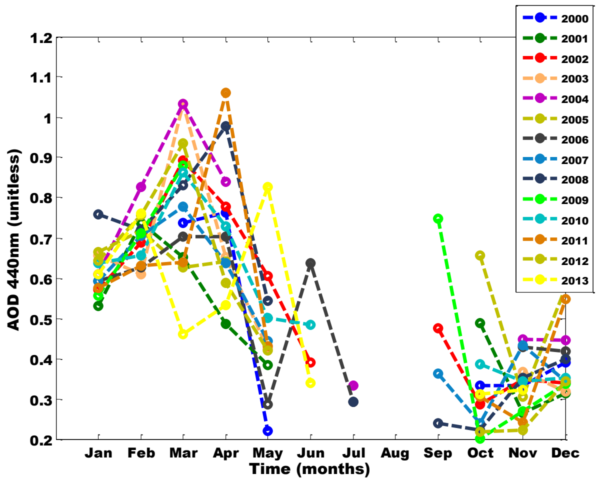

Figure 5. AOD pattern for Abuja 2000-2013

The AOD variation in Lagos (Figure 4) is unstable partly due to massive From Figure 2 to 5, the atmospheric constants, phase differences and tuning constants can be inferred from the Matlab curve fit tool and equation (4) as shown in Table 1 below.

Table 1. Atmospheric constants over Abuja

Location	a_1	a_2	n_1	n_2	α	β
Abuja	0.5421	0.8616	0.3271	0.5365	$\frac{\pi}{4}$	$\frac{\pi}{4}$

It is important to note that this paper has significant interest to the ITU model because it suggests an alteration in its known model shown in equation (7)

$$N = \frac{77.7P}{T} + 3.73 \times 10^5 \frac{e}{T^2} = N_{dry} + N_{wet}(N\text{-units}) \tag{6}$$

Where e is the water vapour pressure, P is the atmospheric pressure (hpa) and T is the absolute temperature (K). The mathematical relationship between relative humidity and water vapour pressure is expressed in the following equation:

$$e = \frac{RH}{100} a \exp \left| \frac{bT}{T+c} \right| \tag{7}$$

Here T is the temperature in the above equation is given in °C and the coefficients a, b and c takes the following values: $a = 6.1121$, $b = 17.502$, and $c = 240.97$. The implication of this research upon the understanding of the results from Leck and Svensson (2015) is that the determination of coefficients a, b and c are influenced by the optical state over a geographical location. This study proposes an inclusion of the attenuation due to moving aerosols layer into the ITU model which is significant via the atmospheric constants over Abuja. Upon this concept, we statistically examine the AOD distribution over Abuja as shown in Tables 2 and 3.

The highest AOD mean, 95% confidence interval, 99% confidence interval, variance, standard deviation and coefficient of variation was in 2000. The highest skew and kurtosis can be found in 2011. The highest Kolmogorov-Smirnov stat can be found in 2000. This results shows that the lower atmosphere of Abuja may not be dynamic as cities in the southern Nigeria (Emetere et al., 2015b). Hence we examine the atmospheric aerosol retention shown in Tables 5 and **??**.

Table 2. Statistical AOD analysis 2000-2006

Statistical Tool	2000	2001	2002	2003	2004	2005	2006
Mean	0.46	0.48	0.54	0.58	0.65	0.60	0.52
Standard error	0.09	0.06	0.06	0.09	0.10	0.05	0.06
95% confidence interval	0.24	0.13	0.15	0.22	0.24	0.13	0.13
99% confidence interval	0.38	0.20	0.21	0.34	0.36	0.19	0.19
Variance	0.05	0.03	0.04	0.06	0.07	0.02	0.03
Standard deviation	0.23	0.16	0.20	0.24	0.26	0.14	0.17
Coefficient of variation	0.49	0.33	0.38	0.42	0.40	0.23	0.32
Skew	0.71	0.26	0.46	1.01	0.31	−2.12	−0.40
Kurtosis	−1.74	−0.72	−0.97	1.20	−1.40	4.98	−1.64
Kolmogorov-Smirnov stat	0.29	0.14	0.16	0.24	0.21	0.30	0.22

Table 3. Statistical AOD analysis 2007-2013

Statistical Tool	2007	2008	2009	2010	2011	2012	2013
Mean	0.50	0.53	0.53	0.55	0.55	0.52	0.520
Standard error	0.06	0.09	0.10	0.06	0.09	0.09	0.070
95% confidence interval	0.14	0.19	0.24	0.14	0.21	0.22	0.167
99% confidence interval	0.20	0.28	0.37	0.20	0.31	0.32	0.246
Variance	0.03	0.07	0.07	0.03	0.06	0.07	0.040
Standard deviation	0.18	0.27	0.26	0.18	0.25	0.26	0.199
Coefficient of variation	0.36	0.51	0.50	0.33	0.45	0.50	0.383
Skew	0.15	0.37	−0.00	0.45	0.99	0.36	0.483
Kurtosis	−1.30	−1.40	−1.91	−0.91	1.84	−1.00	−1.290
Kolmogorov-Smirnov stat	0.18	0.19	0.2	0.16	0.24	0.15	0.192

Table 4. Atmospheric aerosols retention over Abuja 2001-2006

	2001	2002	2003	2004	2005	2006
Aerosols Retention	23.01	1.48	0.83	0.26	56.4	8.27

Table 5. Atmospheric aerosols retention over Abuja 2001-2006

	2007	2008	2009	2010	2011	2012	2013
Aerosols Retention	1.38	8.30	0.044	25.70	7.50	0.86	9.58

The year of highest atmospheric aerosols retention was found between 2001 and 2010. This shows that the skew and kurtosis are good indicators of atmospheric aerosols retention. Therefore, the aerosols retention peak in Abuja occurs every ten years. The significance of the atmospheric aerosols retention in a geographical region has great influence on aviation schedules (Gettelman and Chen, 2013), human health (Wyzga and Lawrence, 1995), measuring instruments, energy budget and meteorology (Emetere and Akinyemi, 2013).

6. Conclusion

In this study, a statistical examination of the aerosols loading over Abuja-Nigeria was observed and analytical dispersion model was used to estimate the aerosols loading for each month of the year while formulated aerosol dispersion model was used to test the accuracy of the data. From research, the Statistical AOD analysis varies over the years as follows; The dynamism of March and April keeps dropping per year, Scanty data was observed in June, July and September, no data was seen for August, the loadings for December increased in the past five years while January is unpredictable variation. The variation in January may be due to the influx of Sahara dust aerosol by the northeast winds while AOD drop in March and April through 2000 to 2013 may be due to a reduced recirculation zone. This study proposes an inclusion of the attenuation due to moving aerosols layer into the ITU model which is significant via the atmospheric constants over Abuja. Conclusively, The highest AOD mean, 95% confidence interval, 99% confidence interval, variance, standard deviation and coefficient of variation was in 2005. The highest skew and kurtosis can be found in 2010. The highest Kolmogorov-Smirnov stat can be found in 2004.

Competing Interests

The authors declare that he has no competing interests.

Author's Contributions

All the authors read and approved the final manuscript.

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