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Research Article

Task Block in a Multistage Flowshop Scheduling Problem Under Uncertainty Reduces the Waiting Time of Tasks

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Abstract. The current paper investigates the scheduling problem for multiple-stations in a task-block-based model, particularly under conditions of uncertainty in processing times. We propose a framework for task block scheduling that adaptively schedules tasks across multiple stages with the objective of minimizing the total waiting time of tasks in a fuzzy environment. The algorithm is formulated and analyzed using MATLAB and its performance is supported through a numerical example.

Keywords. Flowshop scheduling, Uncertain processing periods, Task block, Complete waiting time of tasks

Mathematics Subject Classification (2020). 90B35, 90C70, 90C27

1. Introduction

Scheduling of tasks plays a crucial role in improving efficiency of modern manufacturing and service systems such as automated production lines, semiconductor fabrication and other industrial processes, as it directly impacts overall system utilization and performance. In a flowshop scheduling system consisting of multiple stations, tasks are needed to run across these several stations in a fixed order. Each task must be fully processed at one station before proceeding to the next, resulting in strong interdependence among stages. It has been widely noticed that the order (sequence) in which tasks are processed significantly affects their movement and waiting period between all stations. Even a minor change in task sequence can create a major difference in the idle time of stations and overall system production. Therefore, determining an optimal sequence of tasks is a challenging and important problem in production scheduling. The complexity of the problem further increases when some tasks are categorized in a block based on priority or operational requirements, as commonly observed in industries such as textile manufacturing, chemical processing and assembly systems. In such cases, maintaining task blocks while optimizing the sequence adds an additional layer of difficulty to the scheduling process. Moreover, in real-world environments, the assumption of deterministic processing times is often unrealistic. In practical scenarios such as healthcare systems and cloud computing and logistics networks, processing times are considered uncertain due to fluctuations in workstation efficiency, human factors or environmental conditions. Fuzzy set theory offers a useful way to model these kinds of uncertainties, which leads to the creation of more flexible, reliable sequences and strong scheduling strategies. Therefore, there is a need to develop an efficient scheduling approach that integrates task block criteria with uncertainty considerations in a multi-stage flowshop environment. The primary aim is to reduce the overall waiting time of tasks while maintaining system efficiency under uncertain conditions. This motivation builds the foundation of the current research (Goyal *et al*. [6]).

Most of the existing scheduling models mainly focus on minimizing the total elapsed time of the system. However, they do not give proper attention to the complete waiting time of tasks between different stages. In practical industrial environments, reducing the waiting time of tasks is very important, as it helps in better utilization of resources and can improve overall profit. Although scheduling models for two- and three-stage systems have been well studied, there is still a lack of work for multi-stage systems, especially under uncertain processing conditions. In addition, task flow aspects such as task block formation are generally not considered. In contrast, the proposed approach introduces a task-block strategy that groups interrelated tasks and schedules them collectively, which helps in reducing unnecessary leisure times and improves flow continuity, especially under uncertain processing environments. Compared to existing methods that treat tasks independently and assume deterministic environments, the proposed model provides a more robust and realistic scheduling framework. Therefore, this study focuses on developing a scheduling approach for multi-stage systems by considering uncertain processing times and task block formation. The main objective is to minimize the total waiting time of tasks while also maintaining an efficient overall schedule. The main contributions of this work include the integration of task block scheduling with uncertainty in a multi-stage flowshop environment and the explicit focus on minimizing waiting time of tasks, which has not been adequately addressed in existing studies (Goyal and Kaur [7]).

2. Literature Review

Task scheduling has been widely studied in the context of flowshop environments due to its significant impact on system performance and efficiency. The foundational work by Johnson [11] introduced optimal scheduling for two- and three-stage production systems with setup times. Subsequently, Wagner [22] developed an integer linear programming model for machine scheduling problems, while Graham *et al*. [8] provided a comprehensive survey on optimization and approximation techniques in scheduling theory. Early research primarily focused on deterministic environments. Nawaz *et al*. [19] proposed a heuristic approach for multi-machine flowshop scheduling, whereas Ho *et al*. [9] investigated an $O(n \log n)$-time algorithm with time-dependent processing. Kim [13] introduced a branch-and-bound framework to reduce mean tardiness for a two-stage problem. Similarly, Yang and co-authors [23, 24] examined flowshop problems with waiting time constraints and processing times dependent on job waiting, highlighting the importance of sequencing decisions in system performance. Further studies addressed additional practical constraints such as deterioration effects and waiting time variability. Huang and Wang [10] analyzed position-dependent deterioration in scheduling, while Yu *et al*. [26] focused on minimizing waiting time variation in two-stage systems. Miao [17] determined the complexity of individual station problems with deteriorating tasks having non-zero release dates. In recent years, research has increasingly shifted towards incorporating uncertainty and real-world complexities. Goyal *et al*. [6, 7] introduced structured flowshop models considering task blocks and fuzzy processing times to minimize waiting time. Lee *et al*. [14] studied overlapping waiting time constraints and Behnamian and Ghomi [2] addressed multi-objective scheduling problems in industrial applications. Allahverdi and Allahverdi [1] investigated a four-machine problem into a single machine problem to minimize the total elapsed period with uncertain processing periods, while Castaneda *et al*. [3] discovered the permutation flow shop problem with both stochastic and fuzzy processing periods to minimize complete makespan. Chen and Luo [4] extended scheduling models to multi-factory systems with transportation considerations. Panwalkar and Koulamas [20] developed an asymptotic solution for the two-stage no-wait problem aimed at reducing absolute deviation of task completion period, whereas Kazemi *et al*. [12] addressed multi-factory assembly scheduling problems. Ying *et al*. [25] introduced two-agent flowshop models with deadline constraints. Lv and Wang [16] incorporated learning effects and release dates into scheduling decisions. Luo and Yan [15] applied hybrid optimization techniques to stochastic distributed flowshop problems. Pei *et al*. [21] proposed robust scheduling approaches for hybrid flowshops under uncertainty. Furthermore, Dehnavi *et al*. [5] and Mokhtari-Moghadam *et al*. [18] incorporated energy-efficient and sustainability considerations into modern flowshop scheduling models. From the existing studies, the approaches have addressed two- and three- stage systems, limited work has been carried out for multi-stage environments, particularly under uncertain processing conditions. Likewise, important performance measures such as complete waiting time of tasks and their impact on resource consumption have not been adequately explored. In this context, the present study considers a task block-based scheduling approach under uncertainty with the objective of minimizing the complete waiting time of tasks.

Notations

Symbol	*Description*
n	number of tasks
m	number of stations
C_{im}	Processing period of ith task on mth station under certainty
C_{im}	Processing period of ith task on mth station under uncertainty
B_i	Processing period of ith task on first fictitious station
Y_i	Processing period of ith task on second fictitious station
ϕ	Indicator representing that certain tasks are grouped into a task block
W_a	Complete waiting time of tasks

3. Assumptions

- The system involves a finite number of tasks that are processed sequentially across multiple stations in a fixed order.
- Some tasks are grouped in predefined task block and all tasks in task block are utilized consequently without discontinuity.
- The processing times of tasks are considered to be uncertain and represented as triangular fuzzy numbers.
- Fuzzy processing times are converted into crisp periods through defuzzification method.
- A station can process only one task at a time.
- Stations are considered to be continuously available throughout the span without any disturbance or failure of system.
- The internal order of tasks that are grouped in a task block remains unchanged.
- Setup period and transportation period are assumed to be negligible.

4. Problem formulation

Consider a finite number of ith tasks processing sequentially over a set of stations, i.e., $C_1, C_2, C_3, \dots, C_m$ in the same pattern. Every task has to run on each station without jumping to another and the processing times of these tasks are taken as fuzzy numbers, represented by $C'_{im} = (q_{im}, v_{im}, s_{im})$ where q, v and s represent the deterministic processing times respectively. A set of tasks are categorized in a task block, represented as $\phi = (g, h)$, where $g, h \in \{1, 2, \dots, n\}$ as given in Table 1.

Table 1. Uncertain processing times of tasks

Tasks	Station C'_1	Station C'_2	Station C'_3	...	Station C'_m
i	(q_{i1}, v_{i1}, s_{i1})	(q_{i2}, v_{i2}, s_{i2})	(q_{i3}, v_{i3}, s_{i3})	...	(q_{im}, v_{im}, s_{im})
1	(q_{11}, v_{11}, s_{11})	(q_{12}, v_{12}, s_{12})	(q_{13}, v_{13}, s_{13})	...	(q_{1m}, v_{1m}, s_{1m})
2	(q_{21}, v_{21}, s_{21})	(q_{22}, v_{22}, s_{22})	(q_{23}, v_{23}, s_{23})	...	(q_{2m}, v_{2m}, s_{2m})
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	(q_{n1}, v_{n1}, s_{n1})	(q_{n2}, v_{n2}, s_{n2})	(q_{n3}, v_{n3}, s_{n3})	...	(q_{nm}, v_{nm}, s_{nm})

The proposed study targets to determine an optimal sequence of tasks including task block to minimize the complete awaiting duration of all the tasks under uncertain environment.

5. Practical Situations

Consider a textile dyeing and finishing industry as shown in Figure 1, where different fabrics are assumed to be a finite number of tasks that are sequentially working on different stations named as pre-treatment unit, dyeing unit, washing unit, drying unit and finishing unit. The different qualities of fabrics are processed in a pre-treatment unit that eliminates the impurities before dyeing, i.e., cleans the fabric properly, then moves to a dyeing station that adds desirable color to

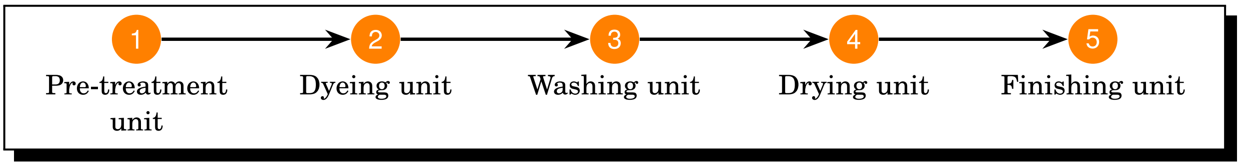

Figure 1. Textile manufacturing industry

the fabric, then shifts to a washing unit that extracts the excess dye and chemicals after dyeing and then comes to drying station where it dries the wet fabrics and last reaches a finishing station where it enhances the quality of the final product. In a real production system, similar fabric types or the same color fabrics are often categorized into task blocks in order to improve the operational efficiency, keeping in mind that all the fabrics in the task block are processed consecutively without disturbance. Sometimes, the production system faces many challenges like variation in fabric thickness, rate of dye absorption, station temperature fluctuations and operator skill levels. Thus, the processing times cannot be accurately defined and must be represented as fuzzy numbers. When a task completes the dying stage prior but the washing stage is already preoccupied with another task, then that task has to wait, thereby increasing its waiting duration. Therefore, improper scheduling of tasks in an order leads to an elevated total waiting duration. As a result, the current paper aims to discover an optimal sequence of tasks that reduces the overall waiting time of all fabrics (tasks) even under fuzzy processing conditions.

6. Algorithm

Step 1: The uncertain processing times are transformed into deterministic processing times for all the tasks performing on the multiple stations through the *Average High Ranking* (AHR) formula:

$(C)=\frac{3v+s-q}{3}$, where q, v and s represent the optimistic, most likely, and pessimistic processing times respectively.

Step 2: Study the structural relationship as below:

$$\min(C_{ie})\geq\max(C_{i(e+1)}),\quad e=(1,2,\dots,m-2)$$

or

$$\min(C_{i(e+1)})\geq\max(C_{ie}),\quad e=(2,3,\dots,m-1).$$

Proceed to the subsequent step if either or both of the above conditions are fulfilled.

Step 3: Every station naming $C_{i1},C_{i2},C_{i3},\dots$ upto C_{im} are modified into two fictitious stations named as B and Y with the processing period B_i and Y_i of ith task are expressed accordingly:

$$B_i=C_{i1}+C_{i2}+\dots+C_{i(m-1)}\ \text{ and }\ Y_i=C_{i2}+C_{i3}+\dots+C_{im}.$$

Step 4: Examine the structural condition

$$\max B_i\leq\min Y_i.$$

Step 5: Now take into account that some tasks are grouped into one block, i.e., task block $\phi=(g,h)$, where $g,h\in\{1,2,\dots,n\}$, then calculate the processing times of this equivalent task utilizing on stations by the formula addressed below:

$$B_\phi = B_g + B_h - \min(B_h, Y_g) \text{ and } Y_\phi = Y_g + Y_h - \min(B_h, Y_g).$$

Step 6: Formulate the table as shown in Table 2 and calculate the $p_i = Y_i - B_i$ and then organize the tasks in increasing order in a sequence created as $\langle \delta_1, \delta_2, \dots, \delta_n \rangle$ and the possible sequences from this created sequence will be $\langle \delta_1, \delta_2, \dots, \delta_n \rangle, \langle \delta_2, \delta_1, \dots, \delta_n \rangle, \dots, \langle \delta_n, \delta_1, \dots, \delta_{n-1} \rangle$.

Table 2. Tasks containing task-block

Task	Station B	Station Y	
i	B_i	Y_i	$p_i = Y_i - B_i$
1	B_1	Y_1	p_1
2	B_2	Y_2	p_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$
ϕ	B_ϕ	Y_ϕ	p_ϕ
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	B_n	Y_n	p_n

Step 7: Discover $\min B_i$.

(a) If $B_{\delta_1} = \min B_i$, therefore, the sequence obtained from *Step* 6 is consider the final optimal sequence.

(b) If $B_{\delta_1} \neq \min B_i$, continue to the subsequent step.

Step 8: Calculate $j_{if} = (n-f)p_i$ for all the actual tasks as illustrated in Table 3:

Table 3. Matrix table

Task	Station B	Station Y		$j_{if} = (n-f)p_i$				
i	B_i	Y_i	$p_i = Y_i - B_i$	$f = 1$	$f = 2$	$f = 3$	$\dots$	$f = (n-1)$
1	B_1	Y_1	p_1	f_{11}	f_{12}	f_{13}	$\dots$	$f_{1(n-1)}$
2	B_2	Y_2	p_2	f_{21}	f_{22}	f_{23}	$\dots$	$f_{2(n-1)}$
3	B_3	Y_3	p_3	f_{31}	f_{32}	f_{33}	$\dots$	$f_{3(n-1)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	B_n	Y_n	p_n	f_{n1}	f_{n2}	f_{n3}	$\dots$	$f_{n(n-1)}$

Step 9: Calculate the complete awaiting duration of tasks, $W_a = nB_{\delta 1} + \sum_{f=1}^{n-1}(n-f)p_i - \sum_{i=1}^{n} B_i$ for all the possible sequences:

$$\langle \delta_1, \delta_2, \dots, \delta_n \rangle, \langle \delta_2, \delta_1, \dots, \delta_n \rangle, \dots, \langle \delta_n, \delta_1, \dots, \delta_{n-1} \rangle.$$

Step 10: Thus, among all the possible sequences, the final optimal is the one with the least overall awaiting duration of tasks.

7. Numerical

Consider five tasks are working on these five stations whose processing times are mentioned in fuzzy numbers and two tasks are grouped in one task block named $\phi = (3,5)$ shown in Table 4:

Table 4. Imprecise processing times of tasks

Tasks	Station C_1'	Station C_2'	Station C_3'	Station C_4'	Station C_5'
1	(35,38,39)	(19,20,21)	(10,12,13)	(7,9,10)	(48,49,51)
2	(34,35,37)	(23,25,27)	(18,19,20)	(8,10,11)	(45,47,48)
3	(33,36,38)	(22,23,25)	(13,15,16)	(8,9,10)	(41,43,44)
4	(37,39,40)	(21,24,27)	(13,16,17)	(5,7,9)	(43,44,45)
5	(30,33,34)	(19,21,23)	(9,10,12)	(7,9,10)	(48,49,50)

The proposed paper focuses on determining an optimal sequence of the tasks that reduces the overall awaiting duration of all tasks under uncertainty using task block approach.

Solution.
Step 1: The uncertain processing times C' (q_{nm}, v_{nm}, s_{nm}) are transformed into deterministic processing times C for all the tasks performing on the multiple stations through the *Average High Ranking* (AHR) formula, displayed in Table 5:

Table 5. Crispifies the processing period

Tasks	Station C_1	Station C_2	Station C_3	Station C_4	Station C_5
1	39.33	20.66	13.00	10.00	50.00
2	36.00	26.33	19.66	11.00	48.00
3	37.66	24.00	16.00	9.66	44.00
4	40.00	26.00	17.33	8.33	44.66
5	34.33	22.33	11.00	10.00	49.66

Step 2: Assess the structural relationship:

$$\min C_1 \geq \max C_2$$
$$34.3 > 26.33\ ;$$
$$\min C_2 \geq \max C_3$$
$$20.66 > 19.66\ ;$$
$$\min C_3 \geq \max C_4$$
$$11 = 11$$

Since, the condition is satisfied, continue with following step.

Step 3: Convert five stations into two fictitious stations named as B and Y with the processing period B_i and Y_i of ith task are expressed accordingly in Table 6:

$$B_i = C_1 + C_2 + \ldots + C_4,$$
$$Y_i = C_2 + C_3 + \ldots + C_5.$$

Table 6. Modified processing times

Tasks	Station B_i	Station Y_i
1	82.99	93.66
2	92.99	104.99
3	87.32	93.66
4	91.66	96.32
5	77.66	92.99

Step 4: Examine the structural condition

$$\max B_i \le \min Y_i$$

$$92.99 = 92.99$$

Step 5: Now take into account that two tasks are grouped into one block, i.e., task block $\phi = (3,5)$ then calculate the processing times of this equivalent task utilizing on stations B_i and Y_i by the formula addressed below:

$$B_\phi = B_3 + B_5 - \min(B_5, Y_3) = 164.98 - 77.66 = 87.32, \quad \text{and}$$

$$Y_\phi = Y_3 + Y_5 - \min(B_5, Y_3) = 186.65 - 77.66 = 108.99.$$

Step 6: Formulate the table and calculate the $p_i = Y_i - B_i$ and then organize the tasks in increasing order in a sequence which we obtained as $\langle 4,1,2,\phi\rangle$ and the possible sequences from this sequence will be $F_1 = \langle 4,1,2,\phi\rangle$, $F_2 = \langle 1,4,2,\phi\rangle$, $F_3 = \langle 2,4,1,\phi\rangle$, $F_4 = \langle \phi,4,1,2\rangle$ as shown in Table 7.

Table 7. Table containing task-block

Task	B_i	Y_i	p_i
1	82.99	93.66	10.67
2	92.99	104.99	12.00
ϕ	87.32	108.99	21.67
4	91.66	96.32	4.66

Step 7: Discover $\min B_i$.
Since, $B_4 \ne \min B_i$,
$91.66 \ne 82.99$ then continue to the subsequent step.

Step 8: Calculate $j_{if} = (n-f)p_i$ for all the actual tasks as illustrated in Table 8:

Table 8. Matrix table

Task	Station B	Station Y		$j_{if} = (n-f)p_i$			
i	B_i	Y_i	$p_i = Y_i - B_i$	$f = 1$	$f = 2$	$f = 3$	$f = 4$
1	82.99	93.66	10.67	42.68	32.01	21.34	10.67
2	92.99	104.99	12.00	48.00	36.00	24.00	12.00
3	87.32	93.66	6.34	25.36	19.02	12.68	6.34
4	91.66	96.32	4.66	18.64	13.98	9.32	4.66
5	77.66	92.99	15.33	61.32	45.99	30.66	15.33

Step 9: Calculate the complete awaiting duration of tasks,

$$W_a = nB_{\sigma 1} + \sum_{f=1}^{n-1}(n-f)p_i - \sum_{i=1}^{n} B_i \text{ for all the possible sequences:}$$

For sequence $F_1 = \langle 4,1,2,\phi\rangle$ or $\langle 4,1,2,3,5\rangle$,

$$W_a = 5 \times 91.66 + 18.64 + 32.01 + 24 + 6.34 - 432.62 = 106.67.$$

For sequence $F_2 = \langle 1,4,2,\phi\rangle$ or $\langle 1,4,2,3,5\rangle$,

$$W_a = 5 \times 82.99 + 42.68 + 13.98 + 24 + 6.34 - 432.62 = 69.33 = \text{Optimal}.$$

For sequence $F_3 = \langle 2,4,1,\phi\rangle$ or $\langle 2,4,1,3,5\rangle$,

$$W_a = 5 \times 92.99 + 48 + 13.98 + 21.34 + 6.34 - 432.62 = 121.99.$$

For sequence $F_4 = \langle \phi,4,1,2\rangle$ or $\langle 3,5,4,1,2\rangle$,

$$W_a = 5 \times 87.32 + 25.36 + 45.99 + 9.32 + 10.67 - 432.62 = 95.32.$$

Step 10: Thus, among all the possible sequences, the final optimal is the F_2 sequence with the least overall awaiting duration of tasks.

8. Computational Analysis

To investigate the performance and reliability of the presented framework under uncertainty, it was comparatively examined by the classical Johnson's scheduling model (Johnson [11]). The seven sets containing the numerous examples with task sizes of 5, 10, 15, 20, 30, 40 and 50 are consider for analysis in Table 9. The graph in Figure 2 represents a clear contrast of the average awaiting duration of tasks and Johnson's method which indicates that the current approach performs better.

Table 9. Evaluation of total awaiting duration of tasks

Number of tasks	Johnson's method	Proposed method
5	93.99	83.19
10	412.33	381.26
15	989.73	905.66
20	1854.06	1666.13
30	4420.40	3933.33
40	7816.33	6861.06
50	12401.40	10921.59

9. Conclusion and Future Scope

The present study represents a novel approach for the multi-stage flowshop scheduling system to reduce the overall complete waiting time of tasks, explicitly including the task block with uncertain processing periods. By structuring the scheduling model under a fuzzy environment, the developed model effectively captures real-world situations that occur due to fluctuations and uncertainties. The numerical results demonstrate the better outcome of the proposed model relative to existing scheduling methods and also confirms that it is a more practical solution for sophisticated production systems. Future research can expand this model by addressing dynamic

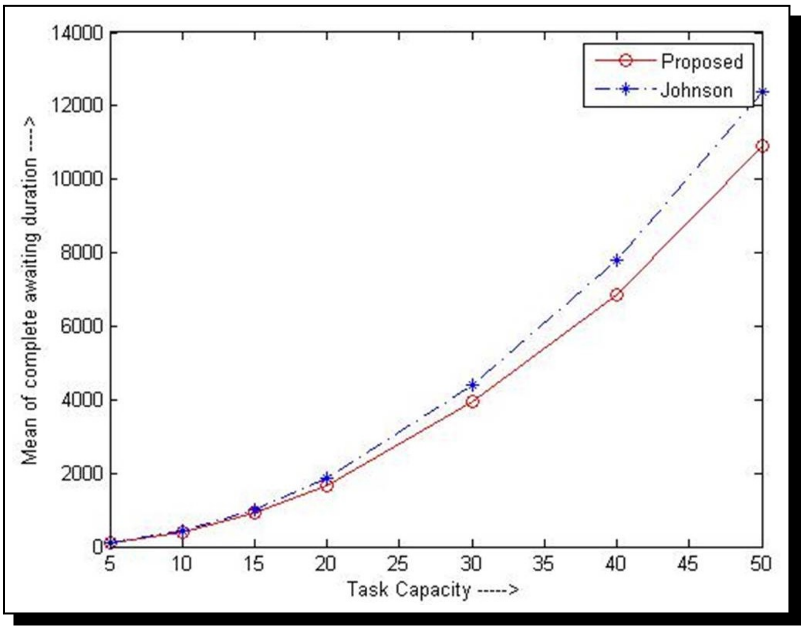

Figure 2. Comparison of computational results

task arrival, random disruptions or advanced optimization techniques for improving scheduling performance. Details of research grants, etc. In future work, the proposed task-block scheduling framework for multiple stations can be extended to more complex production environments, such as open shop and hybrid scheduling systems, to further evaluate its effectiveness in handling uncertainty in processing times and large-scale multi-stage problems.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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