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Research Article

Poisson Random Sums of Stochastic Integrals in Proactive Risk Management Operations

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Abstract. Random sums and stochastic integrals are generally adopted as fundamental probabilistic concepts with very valuable applicability in many disciplines strongly supporting formulation and interpretation of stochastic models. It is shown that random sums are suitable for investigating interconnections among equalities in distribution. Furthermore, the paper clarifies the importance of random sums in modelling.

Keywords. Random sum, Continuous discounting characteristic function, Stochastic integral

Mathematics Subject Classification (2020). 60E05, 60E10, 60H05, 90B50

1. Introduction

Random sums are considered as probabilistic concepts of extreme importance in a variety of research disciplines (Artikis and Artikis [2]). Positive random sums constitute strong analytical tools in formulating, investigating, and interpreting stochastic models strongly supporting decision making under uncertainty. It is valuable to recognize that positive random sums have been extensively incorporated as very useful principal components in a wide area of the discipline of stochastic modeling (Pidd [9], and Pinsky and Karlin [10]). The structural elements and the mathematical form of such random sums are the main factors supporting formulations of various stochastic models applicable in many practical disciplines. It is also known that these random sums have contributed to the establishment of theoretical results particularly useful for the theory of probability distributions (Artikis and Artikis [2]).

In addition, stochastic integrals are probabilistic concepts useful in theory and practice. More precisely, stochastic integrals are suitable for establishing theoretical results in the area of stochastic processes (Artikis and Artikis [1]). Moreover, such integrals strongly support decision making operations in the areas related to the structure and behavior of systems (Harrison [4]). It is important to mention that several research activities on the evaluations and investigations of characteristic functions of various stochastic integrals facilitate the applicability of stochastic integration methodologies (Harrison [4], Lukacs [7], and Riedel [11]).

Four purposes guide the contribution of the present paper. The first is the formulation of four Poisson random sums of two types of stochastic integrals. The introduction of an equality in distribution by making use of the formulated four random sums is the second purpose. The introduction of another equality in distribution by making use of a stochastic integral for continuous discounting is the third purpose. Finally, the establishment of an interrelationship, with theoretical and practical interest, between the first and second equality in distribution is the fourth purpose.

2. Random Sums of Equally Distributed Stochastic Integrals

It is shown that a concept of random sum and two concepts of stochastic integrals can be combined for establishing a theoretical result for the significant research area of equality in distribution (Hashorva *et al*. [5], and Mambo *et al*. [8]).

Sufficient conditions for the stochastic process $\{X(t), t \geq 0\}$, the r.v. $L = X(t+1) - X(t)$, and the c.f. $\varphi_L(u)$ required for defining the stochastic integral

$$\int_0^1 t^{\frac{1}{a}} dX(t) \tag{2.1}$$

and establishing that the corresponding c.f. of (2.1) has the form

$$\exp\left(\frac{a}{u^a}\int_0^u \log\varphi_L(w) w^{a-1} dw\right) \tag{2.2}$$

are known [7]. Moreover, the sequence $\{C_n, n = 1,2,\dots\}$ of independent r.v. distributed as the stochastic integral in (2.1) with c.f. (2.2), the Poisson random variable N is independent of $\{C_n, n = 1,2,\dots\}$, then

$$C_1 + C_2 + \cdots + C_N \tag{2.3}$$

is a Poisson random sum of stochastic integrals of the form (2.1) and the c.f. of the random sum (2.3) has the form

$$\exp\left\{\lambda\left[\exp\left(\frac{a}{u^a}\int_0^u \log\varphi_L(w) w^{a-1} dw\right) - 1\right]\right\}. \tag{2.4}$$

We also consider the stochastic process $\{Y(t), t \geq 0\}$, the r.v. $S = Y(t+1) - Y(t)$, the c.f. $\varphi_S(u)$, the stochastic integral

$$\int_0^1 t^{\frac{1}{a}} dY(t) \tag{2.5}$$

and the function

$$\exp\left(\frac{a}{u^a}\int_0^u \log\varphi_S(w) w^{a-1} dw\right) \tag{2.6}$$

is the corresponding characteristic function (Lukacs [7]). Similarly, the sequence $\{H_k, k = 1,2,\dots\}$ of independent r.v. distributed as the stochastic integral (2.5) with c.f. (2.2), the Poisson r.v. K is independent of $\{H_k, k = 1,2,\dots\}$, then

$$H_1 + H_2 + \cdots + H_K \tag{2.7}$$

is a Poisson random sum and (2.6) implies that the corresponding c.f. of (2.7) has the form

$$\exp\left\{\lambda\left[\exp\left(\frac{a}{u^a}\int_0^u \log\varphi_S(w)w^{a-1}dw\right)-1\right]\right\}. \tag{2.8}$$

Moreover, we consider the stochastic process $\{B(t), t \geq 0\}$, the r.v. $\Pi = B(t+1) - B(t)$, the c.f. $\varphi_\Pi(u)$, the stochastic integral

$$\int_0^1 t^{\frac{1}{a}} dB(t) \tag{2.9}$$

and the corresponding c.f. of (2.9) has the form

$$\exp\left(\frac{a}{u^a}\int_0^u \log\varphi_\Pi(w)w^{a-1}dw\right), \tag{2.10}$$

the sequence $\{J_n, n = 1,2,\dots\}$ of independent r.v. distributed as the stochastic integral (2.10), the Poisson r.v. N is independent of the $\{J_n, n = 1,2,\dots\}$, the Poisson random sum

$$J_1 + J_2 + \cdots + J_N, \tag{2.11}$$

and the corresponding c.f. of (2.11) has the form

$$\exp\left\{\lambda\left[\exp\left(\frac{a}{u^a}\int_0^u \log\varphi_\Pi(w)w^{a-1}dw\right)-1\right]\right\}. \tag{2.12}$$

In addition, we consider the stochastic process $\{Y(t), t \geq 0\}$, the positive r.v. $S = Y(t+1) - Y(t)$, the c.f. $\varphi_S(u)$ then the definition of the stochastic integral

$$\int_0^\infty e^{-\frac{t}{a}} dY(t), \tag{2.13}$$

and the evaluation of the corresponding c.f. of (2.13),

$$\exp\left(a\int_0^u \frac{\log\varphi_S(w)}{w}dw\right), \tag{2.14}$$

constitutes known result (Harrison [4]). In addition, the sequence $\{V_k, k = 1,2,\dots\}$ of independent r.v. distributed as the stochastic integral (2.13), the Poisson r.v. K which is independent of $\{V_k, k = 1,2,\dots\}$, the Poisson random sum

$$V_1 + V_2 + \cdots + V_K, \tag{2.15}$$

and the corresponding c.f. of (2.15) has the form

$$\exp\left\{\mu\left[\exp\left(a\int_0^u \frac{\log\varphi_S(w)}{w}dw\right)-1\right]\right\}. \tag{2.16}$$

The theoretical results of the preset section make use of the above mentioned stochastic integrals, random sums and their corresponding characteristic functions.

Theorem 2.1. *We suppose that the random variable* Π *is independent of the stochastic integral*

$$\int_0^\infty e^{-\frac{t}{a}} dY(t)$$

then the random sums

$$C_1 + C_2 + \cdots + C_N,$$

$$H_1+H_2+\cdots+H_K,$$
$$J_1+J_2+\cdots+J_N,$$
$$V_1+V_2+\cdots+V_K$$

satisfy the equality in distribution

$$C_1+C_2+\cdots+C_N+H_1+H_2+\cdots+H_K \stackrel{d}{=} J_1+J_2+\cdots+J_N+V_1+V_2+\cdots+V_K$$

if, and only if, the random variable L satisfies the equality in distribution

$$L \stackrel{d}{=} \Pi+\int_0^\infty e^{-\frac{t}{a}}dY(t). \tag{2.17}$$

Proof. We establish the sufficiency condition. The necessity condition can be demonstrated by inverting the argument. If we use (2.14) in (2.17) we get the equality

$$\varphi_L(u)=\varphi_\Pi(u)\exp\left(a\int_0^u \frac{\log\varphi_S(w)}{w}dw\right)$$

or equivalently

$$\log\varphi_L(u)=\log\varphi_\Pi(u)+a\int_0^u \frac{\log\varphi_S(w)}{w}dw. \tag{2.18}$$

From (2.18), we get

$$u^{a-1}\log\varphi_L(u)=u^{a-1}\log\varphi_\Pi(u)+au^{a-1}\int_0^u \frac{\log\varphi_S(w)}{w}dw. \tag{2.19}$$

It is obvious that (2.19) implies

$$\begin{aligned}&\frac{d}{du}\int_0^u w^{a-1}\log\varphi_L(w)dw\\&\quad=\frac{d}{du}\int_0^u w^{a-1}\log\varphi_\Pi(w)dw+\frac{d}{du}u^a\int_0^u \frac{\log\varphi_S(w)}{w}dw-u^{a-1}\log\varphi_S(w)\end{aligned}$$

which can be written

$$\begin{aligned}&\frac{d}{du}\int_0^u w^{a-1}\log\varphi_L(w)dw+u^{a-1}\log\varphi_S(w)\\&\quad=\frac{d}{du}\int_0^u w^{a-1}\log\varphi_\Pi(w)dw+\frac{d}{du}u^a\int_0^u \frac{\log\varphi_S(w)}{w}dw\end{aligned}$$

or equivalently

$$\begin{aligned}&\frac{d}{du}\int_0^u w^{a-1}\log\varphi_L(w)dw+\frac{d}{du}\int_0^u w^{a-1}\log\varphi_S(w)dw\\&\quad=\frac{d}{du}\int_0^u \log\varphi_\Pi(w)w^{a-1}dw+\frac{d}{du}u^a\int_0^u \frac{\log\varphi_S(w)}{w}dw.\end{aligned} \tag{2.20}$$

From (2.20) it follows that

$$\begin{aligned}&\int_0^u \log\varphi_L(w)w^{a-1}dw+\int_0^u \log\varphi_S(w)w^{a-1}dw\\&\quad=\int_0^u \log\varphi_\Pi(w)w^{a-1}dw+u^a\int_0^u \frac{\log\varphi_S(w)}{w}dw\end{aligned} \tag{2.21}$$

and from (2.21) it follows that

$$a\int_0^u \log\varphi_L(w)w^{a-1}dw+a\int_0^u \log\varphi_S(w)w^{a-1}dw$$

$$= \alpha \int_0^u \log \varphi_\Pi(w) w^{a-1} dw + \alpha u^a \int_0^u \frac{\log \varphi_S(w)}{w} dw. \tag{2.22}$$

It is easily seen that form (2.22), we get

$$\frac{a}{u^a} \int_0^u \log \varphi_L(w) w^{a-1} dw + \frac{a}{u^a} \int_0^u \log \varphi_S(w) w^{a-1} dw$$
$$= \frac{a}{u^a} \int_0^u \log \varphi_\Pi(w) w^{a-1} dw + \alpha \int_0^u \frac{\log \varphi_S(w)}{w} dw \tag{2.23}$$

for $u \neq 0$. From (2.4), (2.8), (2.12), (2.16) and (2.23) we get the integral equation

$$\left[\exp\left(\frac{a}{u^a} \int_0^u \log \varphi_L(w) w^{a-1} dw\right) - 1\right] \left[\exp\left(\frac{a}{u^a} \int_0^u \log \varphi_S(w) w^{a-1} dw\right) - 1\right]$$
$$= \left[\exp\left(\frac{a}{u^a} \int_0^u \log \varphi_\Pi(w) w^{a-1} dw\right) - 1\right] \left[\exp\left(\alpha \int_0^u \frac{\log \varphi_S(w)}{w} dw\right) - 1\right]$$

or equivalently

$$\exp\left\{\lambda \left[\exp\left(\frac{a}{u^a} \int_0^u \log \varphi_L(w) w^{a-1} dw\right) - 1\right]\right\} \exp\left\{\mu \left[\exp\left(\frac{a}{u^a} \int_0^u \log \varphi_S(w) w^{a-1} dw\right) - 1\right]\right\}$$
$$= \exp\left\{\lambda \left[\exp\left(\frac{a}{u^a} \int_0^u \log \varphi_\Pi(w) w^{a-1} dw\right) - 1\right]\right\} \exp\left\{\mu \left[\exp\left(\alpha \int_0^u \frac{\log \varphi_S(w)}{w} dw\right) - 1\right]\right\}. \tag{2.24}$$

If we use random variables in (2.24), we get

$$C_1 + C_2 + \cdots + C_N + H_1 + H_2 + \cdots + H_K \stackrel{d}{=} J_1 + J_2 + \cdots + J_N + V_1 + V_2 + \cdots + V_N. \tag{2.25}$$

□

It is readily understood that the theoretical results are strongly supported by a concept of Poisson random sum and concept of stochastic integral. These concepts are recognized as fundamental probabilistic tools for establishing and investigating theoretical results of significant applicability. More precisely, Poisson random sums of stochastic integrals are incorporated for the formulation of two stochastic models and the establishment of a useful interconnection between the formulated stochastic models. The practical interpretation of the theoretical results constitutes the contribution of the following section.

3. Stochastic Integrals in Implementing Proactivity

We assume that $Y(t)$ of the stochastic process $\{Y(t), t \geq 0\}$ denotes the income at time point t provided by an asset of indefinite life in the time interval $[0,t]$. It is known that

$$\int_0^\infty e^{-\frac{t}{a}} dY(t)$$

denotes the present value of the income produced by the asset during its indefinite life (Harrison [4]). In addition, the interpretation of Π as a cash flow arising at the time point 0 then (2.17) is readily adopted as a stochastic discounting model with $\frac{1}{a}$ denoting the force of interest. In consequence, the formulation of such a stochastic discounting model provides decision makers with information strongly supporting the incorporation of the concept of proactivity in the implementation of structural operation in a variety of disciplines. Risk management is easily understood as a practical discipline with extensive use of stochastic discounting models for introducing the concept of proactivity in risk identification and risk treatment operations. More precisely, global, catastrophic, and existential risks can be frequently managed by incorporating stochastic discounting models (Harrison [4], and Simpson and Hancock [12]). It is quite obvious that the establishment of the sufficiency condition (2.17) implies that (2.25) constitutes a stochastic model. In addition, the formulation of that stochastic model is readily recognized as a structural activity for investigation and treatment of proactivity in various practical disciplines. It constitutes a general adoption that the discipline of risk management is strongly influenced by the realistic manipulation of proactivity in strategic decision making under uncertainty. Consequently, the interpretations of the stochastic model (2.25) in various risk management operations are extremely useful. The incorporation of four Poisson random sums, involving two very significant stochastic integrals, in stochastic model (2.25) is easily considered as an activity substantially contributing to the interpretation of that stochastic model in various risk management operations.

The consideration of the necessity condition of the theoretical result of the second section appears to be of practical importance. More precisely, it is readily recognized the interpretation of (2.25) as a stochastic model for incorporating the concept of proactivity in decision making under uncertainty. The presence of the Poisson random sum

$$V_1+V_2+\cdots+V_K$$

of stochastic integrals of the form

$$\int_0^\infty e^{-\frac{t}{a}}dY(t)$$

in the equality (2.25) makes clear the suitability of that equality as a stochastic model supporting the implementations of proactive operations in various practical disciplines (Taha [13]). The general acceptance that risk management constitutes an extremely significant discipline incorporating thinking and acting strategically then (2.25) can be a useful analytical tool for proactive treatment of risks arising in a variety of human and natural activities (Harjule *et al*. [3], and Lin *et al*. [6]).

4. Conclusion

Proactivity in decision making and decision implementing constitutes an extremely significant concept for many practical disciplines. In particular, proactivity is easily adopted as a main structural factor for thinking and acting under conditions of uncertainty. Moreover, it is known that stochastic modeling strongly facilitates the incorporation of proactivity in strategic decision making. The formulation and applicability of stochastic discounting models are generally considered as extremely valuable operations in proactive treatment of risks threatening various systems. The present paper makes use of Poisson random sums of stochastic integrals for formulation of two stochastic models suitable for proactive treatment of structural risks. Also, the paper establishes an interconnection between the formulated stochastic discounting models. Moreover, the theoretical results are complemented by their practical interpretation in proactive decision making under uncertainty, providing an additional contribution of the present work.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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