Communications in Mathematics and Applications
Vol. 17, No. 1, pp. 15–24, 2026
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v17i1.3445

http://www.rgnpublications.com

Research Article

Analyze the Radiating Effects on MHD Boundary Layer Flow

Vishal Patel[1] and Lalit Narsingani*[2]

[1] *School of Engineering and Technology, Gujarat Technological University, Chandkheda, Ahmedabad 382424, Gujarat, India*
[2] *Department of Science and Humanities, Government Engineering College, Modasa, Aravalli 383315, Gujarat, India*
*__Corresponding author:__ lalitgec@gmail.com

Received: November 4, 2025 **Revised:** January 12, 2026 **Accepted:** January 27, 2026

Abstract. This paper studies the effects of radiation on the flow near the two-dimensional stagnation point of a shrinking sheet immersed in a viscous and incompressible electrically conducting fluid in the presence of an applied constant magnetic field. The external velocity and the stretching velocity of the sheet are assumed to vary linearly with the distance from the stagnation point. The transformed governing differential equations are solved numerically by using the spline functions. The solutions are studied for a range of magnetic parameter, radiation parameter, Prandtl number, etc. The effects of these parameters are examined on the velocity and temperature distribution of the fluid, which are presented by graphs. The results exhibit that, by applying a strong magnetic field, the reverse flow can be reduced due to the shrinking sheet of shrinking sheet. The magnetic field increases shear stresses and decreases thermal boundary layer thickness. The thermal boundary layer becomes thinner with increasing values of the radiation parameter.

Keywords. MHD flow, Stagnation point, Radiation effect, Coupled nonlinear equations, Spline Collocation Method

Mathematics Subject Classification (2020). 65L10

1. Introduction

Heat transfer processes are integral parts of the science and technology. Its importance has been recognized within the recent past. The studies of heat transfer are increasingly intense concern in modern technology.

Many natural phenomena are susceptible to *magnetohydrodynamic* (MHD) analysis. Geophysicists encounter MHD phenomena in the interactions of conducting fluids and magnetic field that are present in and around heavenly bodies. MHD principles are applied in the design of heat exchanger, in space vehicle propulsion, thermal protection, etc. From technological point of view MHD free convection flow has great significance for the application in the field of stellar and planetary magnetospheres, aeronautics, etc. MHD (Magneto fluid-dynamics/hydrodynamics) is the branch which studies the dynamics of electrically conducting fluids. These include plasmas, liquid metals, salt water, etc. The field of *magnetohydrodynamic* (MHD) was initiated by H. Alfvén [1], for which he received the Nobel Prize in Physics in 1970.

MHD flows have many applications in manufacturing industries. These applications include textile and paper industries, plastic sheet, and blood flow problems, etc. More applications of MHD are in fusion research, in MHD accelerator and power generator and in causing delay in the transition from laminar to turbulent flow. Recently, MHD has been the subject of intensive study and their importance of the study has been extended to many other kinds of associated problems, even in missile rockets.

Nazar *et al*. [7] analyzed the behavior of unsteady two-dimensional stagnation point flow of an incompressible viscous fluid over a flat deformable sheet. Similarity transformation and the Keller-box method were used to find the numerical solution of the unsteady boundary layer equations. Ishak *et al*. [3] studied steady stagnation point flow on vertical surface through porous medium. Sharma and Singh [9] described the effects of variable thermal conductivity and heat source/sink on the flow of viscous incompressible electro conducting fluid by solving ordinary differential equations using shooting method. Kumaran *et al*. [4] obtained a solution for a boundary layer flow of an electrically conducting fluid past a linearly permeable sheet. Hayat *et al*. [2] studied about the two-dimensional *magnetohydrodynamic* (MHD) stagnation-point flow by *homotopy analysis method* (HAM). The influence of various pertinent parameters on the velocity, micro rotation and skin-friction was discussed. Kuo [5] investigated the velocity and temperature field in the thermal boundary layer over a semi-infinite flat plate by differential transformation method. Muhammad and Shehzad [6] studied Radiation effect on stagnation point flow towards heated shrinking sheet. Srivastav and RamReddy [10] analyze fluid flow with thermal radiation. Shaheen *et al*. [8] discuss the applications of magnetic field and porous medium for Jeffrey (non-Newtonian) fluid.

In the present study, two-dimensional MHD boundary layer stagnation point flow with radiation effects towards a shrinking sheet sank in an electrically conducting viscous incompressible fluid in the presence of a transverse magnetic field is examined numerically using spline collocation method.

Nomenclature

Symbols	*Meaning*
β_0	Strength of magnetic field
v	Velocity field
ρ	Density of the fluid
p	Pressure
σ_e	Electric conductivity

Symbols	*Meaning*
U	Free stream velocity
T	Temperature
k_0	Thermal conductivity
c_p	Specific heat capacity
q_r	Radiative heat flux
k	Stefan-Boltzmann constant
σ	Mean absorption coefficient
Nr	Radiation parameter
T_0	Heated surface temperature
T_∞	Temperature of fluid at outside the boundary
η	Similarity parameter
p_0	Stagnation pressure
a	Strength of the stagnation point
M	Magnetic parameter
Pr	Prandtl number

2. Mathematical Formulation

The two-dimensional MHD stagnation point flow of an influence of an electrically conducting fluid on a heated shrinking sheet is considered as shown in Figure 1.

It is assumed that

- the flow is steady, laminar, viscous and incompressible;
- the uniform stationary magnetic field of strength β_0 is perpendicular to the velocity field v;
- the magnetic Reynolds number is small so that the induced magnetic field can be neglected in comparison to the imposed field;
- the electric field is zero as there is no applied polarization voltage;
- the Boussinesq and boundary layer approximations are valid.

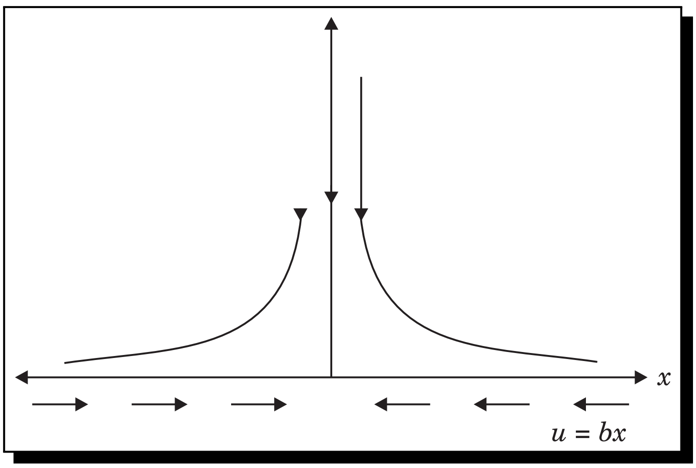

Figure 1. A sketch of the physical problem

Two-dimensional electro conducting fluid's motion equations are written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{\sigma_e B_0^2}{\rho}(U-u). \tag{2}$$

The equation of temperature is

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_0\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}. \tag{3}$$

The Radiative heat flux is simplified as

$$q_r=-\frac{4\sigma}{3k}\frac{\partial T^4}{\partial y}. \tag{4}$$

The term T^4 may be expressed as a linear function of temperature,

$$T^4\cong 4T_\infty^3 T-3T_\infty^4. \tag{5}$$

From eqs. (4) and (5), eq. (3) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\frac{k_0}{\rho c_p}+\frac{16\sigma T_\infty^3}{3\rho c_p k}\right)\frac{\partial^2 T}{\partial y^2}. \tag{6}$$

Take $\mathrm{Nr}=\frac{kk_0}{4\sigma T_\infty^3}$ as the radiation parameter, eq. (6) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\alpha_0}{k}\frac{\partial^2 T}{\partial y^2}, \tag{7}$$

where $k=\frac{3\mathrm{Nr}}{3\mathrm{Nr}+4}$.

The boundary conditions are

$$\left.\begin{aligned}&u(x,0)=bx,\ v(x,0)=0,\ u(x,\infty)=U=ax,\\ &T(x,0)=T_0,\ T(x,\infty)=T_\infty.\end{aligned}\right\} \tag{8}$$

To obtain the velocity and temperature field for the problem, eqs. (1), (2) and (7) subject to boundary conditions (8) are to be solved. Use the following similarity transformations (Muhammad and Shehzad [6]) in eqs. (2), (7) and (8),

$$\left.\begin{aligned}&\eta=\sqrt{\tfrac{a}{v}}\,y,\ p(x,\infty)=p_0-\tfrac{\rho a^2}{2}(x^2+y^2),\\ &u(x,y)=axf'(\eta),\ v(x,y)=-\sqrt{av}\,f(\eta),\\ &\theta(\eta)=\tfrac{T-T_\infty}{T_0-T_\infty}.\end{aligned}\right\} \tag{9}$$

We obtain

$$f'''+M^2(1-f')+1=f'^2-ff'', \tag{10}$$

$$\theta''+\mathrm{Pr}\,kf\theta'=0 \tag{11}$$

with initial and boundary conditions

$$\left.\begin{aligned}&f(0)=0,\ f'(0)=B,\ f'(\infty)=1,\\ &\theta(0)=1,\ \theta(\infty)=0,\end{aligned}\right\} \tag{12}$$

where $M=\sqrt{\frac{\sigma_e B_0^2}{\rho a}}$, $k=\frac{3\mathrm{Nr}}{3\mathrm{Nr}+4}$ and $\mathrm{Pr}=\frac{\mu c_p}{k_0}$.

3. Basic Numerical Method

Spline Collocation Method

Consider $s_i(\eta)$ be a quartic spline in $[\eta_{i-1},\eta_i]$. The third derivative of spline must be linear in $[\eta_{i-1},\eta_i]$,

$$s_i'''(\eta_i)=\frac{1}{h_i}[(\eta_i-\eta)f_{i-1}'''+(\eta-\eta_{i-1})f_i'''], \tag{13}$$

where $h_i=\eta_i-\eta_{i-1}$ and $s'''(\eta_i)=f_i'''$.
Integrate (13), twice with respect to x,

$$s_i'(\eta)=\frac{1}{h_i}\left[\frac{(\eta_i-\eta)^3}{6}f_{i-1}'''+\frac{(\eta-\eta_{i-1})^3}{6}f_i'''\right]+c_i(\eta_i-\eta)+d_i(\eta-\eta_{i-1}). \tag{14}$$

Using $s_i'(\eta_{i-1})=f_{i-1}'$ and $s_i'(\eta_i)=f_i'$ to derive c_i and d_i, and substituting in (14),

$$s_i'(\eta)=\frac{1}{h_i}\left[\frac{(\eta_i-\eta)^3}{6}f_{i-1}'''+\frac{(\eta-\eta_{i-1})^3}{6}f_i'''\right]+\frac{1}{h_i}\left(f_{i-1}'-\frac{h_i^2}{6}f_{i-1}'''\right)(\eta_i-\eta)$$
$$+\frac{1}{h_i}\left(f_i'-\frac{h_i^2}{6}fi'''\right)(\eta-\eta_{i-1}). \tag{15}$$

Integrate (15), once with respect to x,

$$s_i(\eta)=\frac{1}{h_i}\left[-\frac{(\eta_i-\eta)^4}{24}f_{i-1}'''+\frac{(\eta-\eta_{i-1})^4}{24}f_i'''\right]-\frac{1}{h_i}\left(f_{i-1}'-\frac{h_i^2}{6}f_{i-1}'''\right)\frac{(\eta_i-\eta)^2}{2}$$
$$+\frac{1}{h_i}\left(f_i'-\frac{h_i^2}{6}fi'''\right)\frac{(\eta-\eta_{i-1})^2}{2}+e_i. \tag{16}$$

Again take $s_i(\eta_{i-1})=f_{i-1}$ and obtain constant $e_i=f_{i-1}-\frac{h^3}{8}f_{i-1}'''+\frac{h}{2}f_{i-1}'$.
Substitute e_i in (16), for $i=1,2,\dots,n$, we get

$$s_i(\eta)=\frac{1}{h_i}\left[-\frac{(\eta_i-\eta)^4}{24}f_{i-1}'''+\frac{(\eta-\eta_{i-1})^4}{24}f_i'''\right]-\frac{1}{h_i}\left(f_{i-1}'-\frac{h_i^2}{6}f_{i-1}'''\right)\frac{(\eta_i-\eta)^2}{2},$$
$$+\frac{1}{h_i}\left(f_i'-\frac{h_i^2}{6}f_i'''\right)\frac{(\eta-\eta_{i-1})^2}{2}+f_{i-1}-\frac{h^3}{8}f_{i-1}'''+\frac{h}{2}f_{i-1}', \tag{17}$$

for $i=1,2,\dots,n-1$, the recurrence relations are:
For equal mesh size,

$$s_i''(\eta_i^-)=s_{i+1}''(\eta_i^+),\ i=1,2,,\dots,n-1:\quad f_{i+1}'-2f_i'+f_{i-1}'=\frac{h^2}{6}(f_{i+1}'''+4f_i'''+f_{i-1}'''), \tag{18}$$

$$s_i(\eta_i^-)=s_{i+1}(\eta_i^+),\ i=1,2,\dots,n:\quad f_{i-1}-f_i=-\frac{h}{2}(f_i'+f_{i-1}')+\frac{h^3}{24}(3f_{i-1}'''-f_i'''). \tag{19}$$

Similarly, for cubic splines:

$$\text{For } i=1,2,\dots,n-1:\quad f_{i+1}-2f_i+f_{i-1}=\frac{h^2}{6}(f_{i+1}''+4f_i''+f_{i-1}''). \tag{20}$$

4. Solution by Using Spline Collocation Method

The governing nonlinear eqs. (10) and (11) are highly nonlinear and coupled. To find their solutions along with boundary conditions (12) use eqs. (18), (19) and (20). For that, we begin with initial curves $f(\eta) = 0.25\eta^2 + 0.5\eta$ and $\theta(\eta) = -\eta + 1$ for f and θ, respectively, which satisfy given boundary condition (12). Take $h = 0.2$, use (18) and (19) to get the values of f_i' for $i = 1,2,3,4$, and the final solutions f_i for $i = 1,2,3,4,5$.

Cubic spline collocation is used to solve (11). For that, substitute the values of f_i in (11) to find θ_i'' and then use (20) to get solutions for θ. Continue this iterative process until the required accuracy is obtained. The patterns of graphs are matched with the available graphs obtained by Muhammad and Shehzad [6].

5. Results and Discussion

This section is intended to present our results in graphical form. To develop a better understanding, the effects of the magnetic parameter, the radiation parameter and the Prandtl number on the flow and temperature profiles are analyzed (Figures 2-13).

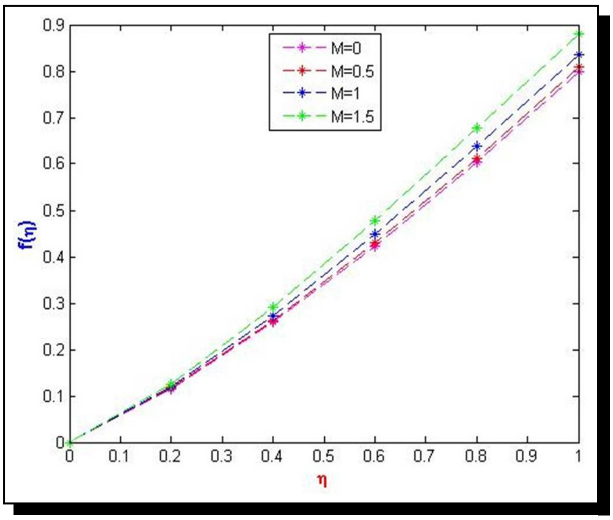

Figure 2. Normal velocity profile for $B = 0.5$, $\mathrm{Nr} = 3$, $\mathrm{Pr} = 0.7$ and various values of M

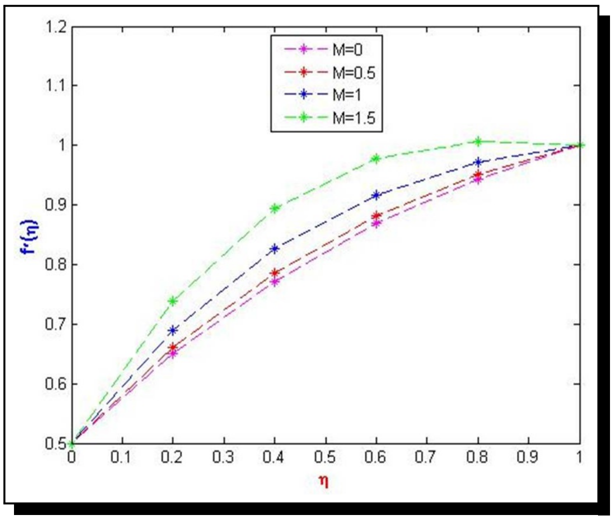

Figure 3. Horizontal velocity profile for $B = 0.5$, $\mathrm{Nr} = 3$, $\mathrm{Pr} = 0.7$ and various values of M

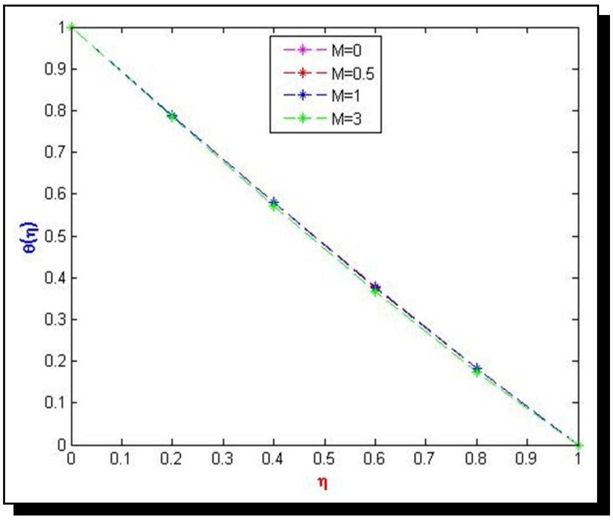

Figure 4. Temperature profile for $B = 0.5$, $\mathrm{Nr} = 3$, $\mathrm{Pr} = 0.7$ and various values of M

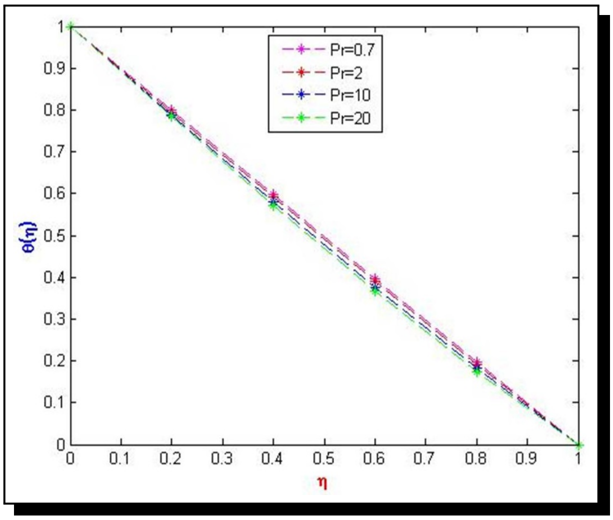

Figure 5. Temperature profile for $B = 0.5$, $\mathrm{Nr} = 3$, $M = 1$ and various values of Pr

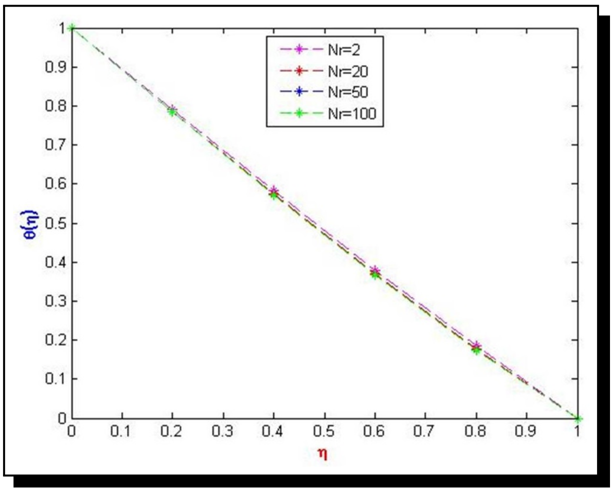

Figure 6. Temperature profile for $B = 0.5$, $\mathrm{Pr} = 0.7$, $M = 1$ and various values of Nr

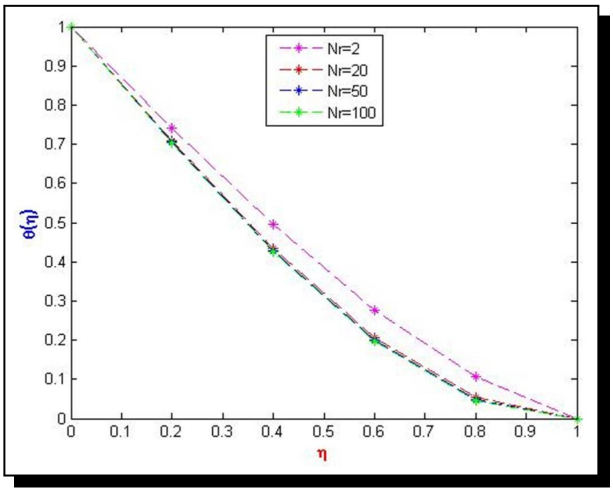

Figure 7. Temperature profile for $B = 0.5$, $\mathrm{Pr} = 0.7$, $M = 10$ and various values of Nr

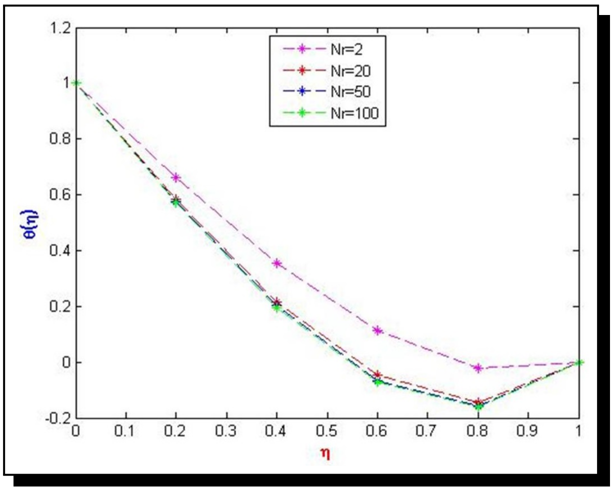

Figure 8. Temperature profile for $B = 0.5$, $\mathrm{Pr} = 10$, $M = 1$ and various values of Nr

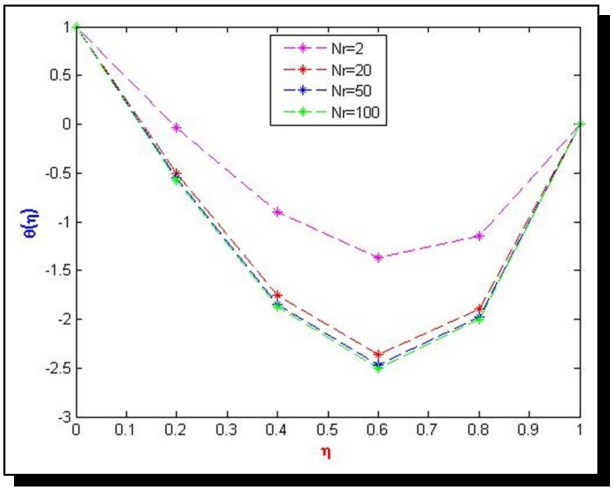

Figure 9. Temperature profile for $B = 0.5$, $\mathrm{Pr} = 10$, $M = 10$ and various values of Nr

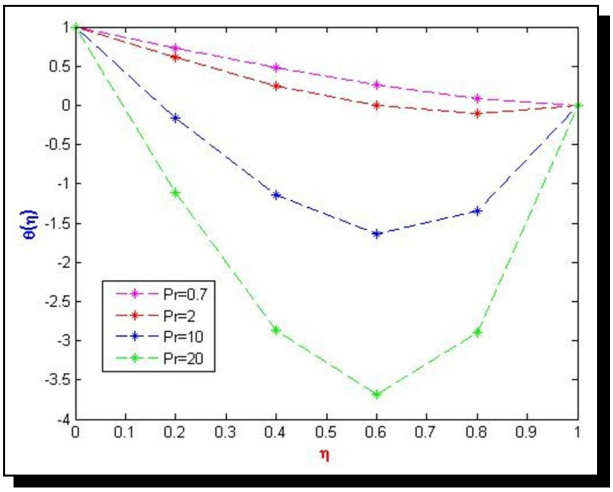

Figure 10. Temperature profile for $B = 0.5$, $\mathrm{Nr} = 3$, $M = 10$ and various values of Pr

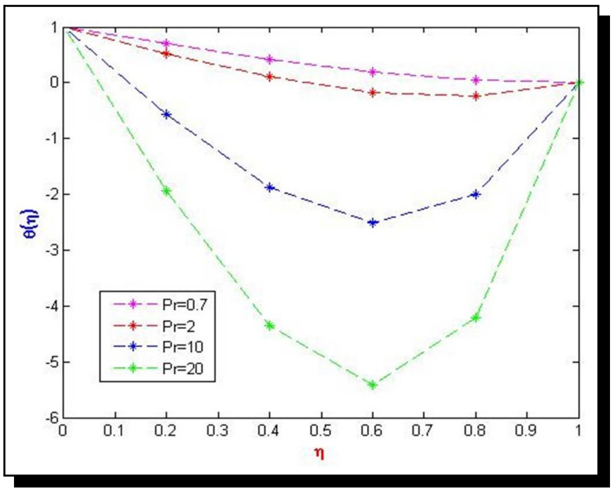

Figure 11. Temperature profile for $B = 0.5$, $\mathrm{Nr} = 100$, $M = 10$ and various values of Pr

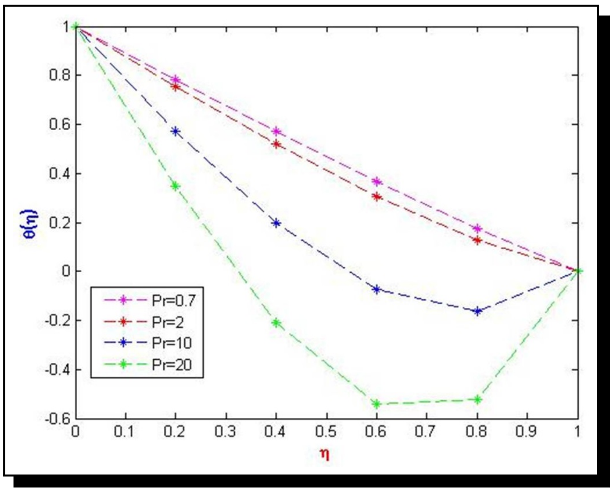

Figure 12. Temperature profile for $B = 0.5$, $\mathrm{Nr} = 100$, $M = 1$ and various values of Pr

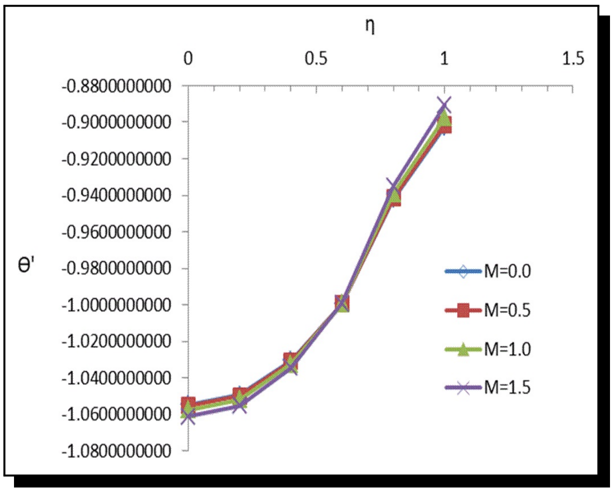

Figure 13. Heat Transfer rate for $B = 0.5$, $\mathrm{Nr} = 3$, $\mathrm{Pr} = 0$ and various values of M

- From Figures 2 and 3, it is clear that, as magnetic parameter increases, the normal and horizontal velocity profiles increase. As the transverse magnetic field assist the magnetic flow and so, reduces the flow reversal.
- The hydrodynamic and thermal boundary layer decrease and become thinner as the magnetic parameter increases as shown in Figure 4.
- From Figure 5, it concludes that as increase in Prandtl number, the temperature at point decrease. It means that, the thermal boundary layer become thinner as increase in Prandtl number.
- The influence of radiation parameter on temperature filed shows in Figure 6. The temperature profile decrease as increase in radiation parameter. Because of this thinning rate increase as increase in radiation parameter.
- From Figure 7, it concludes that if we increase the magnetic parameter and radiation parameter, temperature falls down rapidly. Also, when Prandtl number increases, temperature decreases faster as compared with increase in magnetic parameter, which is reflected in Figure 8.
- In Figure 9 and 10, we can observe that increase in magnetic parameter and radiation parameters both; drastically change in temperature profile.
- Similarly, from Figure 10, 11 and 12, we conclude that higher value of radiation and magnetic parameter show rapid decrement in temperature flow as increase in Prandtl number. As increase in both parameters, Prandtl number creates more effect on temperature field as compared to radiation parameter.
- Figure 13 predicts the influence of applied magnetic field on the heat transfer rate. The heat transfer rate from the surface increase with an increase in the magnetic parameter M.

6. Conclusion

Concluding remarks are as follows:

- The thermal boundary layer decreases as the magnetic parameter increases, Prandtl number and Radiation parameter. Thus, we can say that all these parameters are inversely proportional to temperature. The horizontal and vertical velocity increase as increase in magnetic parameter. The heat transfer rate also increases as increase in magnetic parameter.
- The cubic spline collocation method can be generalized for solving third order boundary value problems and successfully applied to the problem of MHD flow.
- It is noted that, the nonlinear coupled differential equation can be easily solved without converting to linear one using this method. More accuracy can be obtained by reducing the mesh size.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] H. Alfvén, Existence of electromagnetic-hydrodynamic waves, *Nature* **150** (1942), 405 – 406, DOI: 10.1038/150405d0.

[2] T. Hayat, T. Javed and Z. Abbas, MHD flow of a micropolar fluid near a stagnation-point towards a non-linear stretching surface, *Nonlinear Analysis: Real World Applications* **10**(3) (2009), 1514 – 1526, DOI: 10.1016/j.nonrwa.2008.01.019.

[3] A. Ishak, K. Jafar, R. Nazar and I. Pop, MHD stagnation point flow towards a stretching sheet, *Physica A: Statistical Mechanics and its Applications* **388**(17) (2009), 3377 – 3383, DOI: 10.1016/j.physa.2009.05.026.

[4] V. Kumaran, A. K. Banerjee, A. V. Kumar and K. Vajravelu, MHD flow past a stretching permeable sheet, *Applied Mathematics and Computation* **210**(1) (2009), 26 – 32, DOI: 10.1016/j.amc.2008.10.025.

[5] B.-L. Kuo, Thermal boundary-layer problems in a semi-infinite flat plate by the differential transformation method, *Applied Mathematics and Computation* **150**(2) (2004), 303 – 320, DOI: 10.1016/S0096-3003(03)00233-9.

[6] A. Muhammad and A. Shehzad, Radiation effects on MHD boundary layer stagnation point flow towards a heated shrinking sheet, *World Applied Sciences Journal* **13**(7) (2011), 1748 – 1756, URL: https://idosi.org/wasj/wasj13(7)/27.pdf.

[7] R. Nazar, N. Amin, D. Filip and I. Pop, Unsteady boundary layer flow in the region of the stagnation point on a stretching sheet, *International Journal of Engineering Science* **42** (11-12) (2004), 1241 – 1253, DOI: 10.1016/j.ijengsci.2003.12.002.

[8] M. Shaheen, A. Raza, H. Khursheed, M. Jameel, I. Tlili, S. U. Khan, S. A. Althobaiti, M. Gupta and M. I. Khan, Applications of magnetic field and porous medium for Jeffrey (non-Newtonian) fluid by using Laplace simulations, *Journal of Radiation Research and Applied Sciences* **17**(4) (2024), 101176, DOI: 10.1016/j.jrras.2024.101176.

[9] P. R. Sharma and G. Singh, Effects of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet, *Journal of Applied Fluid Mechanics* **2**(1) (2009), 13 – 21, DOI: 10.36884/jafm.2.01.11851.

[10] A. Srivastav and C. RamReddy, Numerical estimations in a power-law fluid flow with thermal radiation: A complete case study, *Radiation Effects and Defects in Solids* **178**(3-4) (2023), 429 – 441, DOI: 10.1080/10420150.2022.2148253.