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Research Article

Exploring the Correlation Coefficient of Bipolar Complex Fuzzy Sets for Pattern Recognition

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Abstract. In pattern recognition, identifying the unknown pattern is a challenging process. Analysing features of the pattern is the initial phase of a pattern recognition process. Features that occasionally show bipolarity have been used for identification. Since *Bipolar Fuzzy Sets* (BCFSs) cover both the negative and positive features of a pattern in particular, they are utilized to address this bipolarity. BCFSs are one kind of bipolar fuzzy set that produces more accurate results than others. A novel method of pattern recognition is presented here by measuring the correlation coefficient of BCFSs, as BCFSs can handle complex fuzzy information more efficiently. In this study, a method of correlation coefficient of BCFSs is proposed. The pattern recognition method is proposed in a bipolar complex fuzzy environment to resolve uncertain and ambiguous information of an unknown pattern based on the aforementioned approach. A real-life example of the recognition of carbon allotrope is used to validate the efficacy and implementation of the proposed approach.

Keywords. Complex fuzzy sets, Bipolar complex fuzzy set, Complex-valued membership function, Amplitude term, Phase term, Information energy of BCFS, Covariance of BCFSs

Mathematics Subject Classification (2020). 62A86

1. Introduction

Decision-making is practically essential in every area of life. One of the various methods for handling decision-making issues is the fuzzy, set-based framework. The mathematical idea of *Fuzzy Set* (FS), a generalization of crisp set, was initiated by L. A. Zadeh [28]. It was developed to address the uncertainty, doubtfulness, and imprecision that comes from human judgment and data ambiguity in decision-making challenges. A large number of generalizations and extensions of fuzzy sets have been developed till today, viz., *Interval-Valued Fuzzy Set* (IVFS) (Zadeh [29]), *Intuitionistic Fuzzy Set* (IFS) (Atanassov [6]), *Interval-Valued Intuitionistic Fuzzy Set* (IVIFS) (Chen and Tsao [7]), *Hesitant Fuzzy Set* (HFS) (Torra [23]), *Pythagorean Fuzzy Set* (PFS) (Yager [26]), *Bipolar Fuzzy Set* (BFS), *Complex Fuzzy Set* (CFS), *Bipolar Complex Fuzzy Set* (BCFS) (Al-Husban *et al*. [1], Alqaraleh *et al*. [5], Ma *et al*. [14], Mahmood *et al*. [15], Ozlu [18], Ramot *et al*. [20,21], Yaqoob *et al*. [27], Zeeshan and Khan [30], Zhang [31]).

Human perception is frequently predicated on bipolar or dual-sided ideas in real-world scenarios. Some well-known instances that call for dual aspects of decision analysis are effects and side effects, profit and loss, and friendship and enmity. Al-Husban *et al*. [1] and Zhang [31], combined fuzziness and bipolarity to propose the idea of BFS. A positive membership degree from $[0,1]$ and a negative membership degree from $[-1,0]$ are assigned to each element by the BFS. This type of membership degree assignment shows how effectively an element fulfills both a property and its counter property.

Ramot *et al*. [21] first proposed the concept of a *Complex Fuzzy Set* (CFS). A CFS is the extension of a fuzzy set whose range is expanded from the closed interval $[0,1]$ to a unit disk in the complex plane. The CFS theory has the benefit of managing the ambiguity and intuitiveness that are common in time-periodic events. CFSs and their classes are essential to applications such as periodic event prediction and complex control systems. A CFS is closely resembles to a Fourier transform and finds applications across diverse fields, such as communication, astronomy, geology, optics, signals and systems, and more. CFS is the specialized form of the Fourier transform, obtained by limiting its scope to a complex unit disk. Consequently, a CFS can be used in various models, such as the Fourier transform (Ramot *et al*. [20]), pattern recognition, medical diagnosis, etc. In pattern recognition, an object can be represented by a set of data items and is viewed as a vector in a high-dimensional space. Particularly when fuzzy sets are involved, these multidimensional variables can be defined by using complex classes. The use of a CFS is quite beneficial for periodic events. Ma *et al*. [14] claimed that the phase component of a complex fuzzy set membership can be used to depict periodic or recurrent problems more appropriately. Zeeshan and Khan [30] introduced and formalized various algebraic operations on CFSs.

Although CFS plays an important role in addressing many real-life issues, certain problems involve both positive and negative aspects of CFS. Mahmood *et al*. [15], introduced the notion of a BCFs. In Ozlu [18] and Yaqoob [27] studied various operators of BCFSs with their real-life multi-attribute decision-making problems. By expanding the applications and methodologies, Alqaraleh *et al*. [5] combined the concept of bipolar complex fuzzy with soft sets for use in decision-making situations.

Pattern recognition is a method of data assessment that automatically finds patterns and regularities in data by using a machine learning algorithm. The data can be numeric, text, images, sound, etc. Pattern recognition systems can precisely and promptly identify familiar patterns. Additionally, they can distinguish patterns and objects, even if they are partially hidden, identify shapes, and classify and identify unknown objects. Pattern recognition has many applications, including speech recognition, fingerprint identification, picture processing, aerial photo interpretation, optical character recognition in scanned documents like agreements and photographs, and even medical imaging and diagnostics. Pattern recognition is also a tool for data analysis. For instance, the technique can be employed to guess the outcomes in the stock market. Pedrycz [19] proposed state-of-the art methodologies and fuzzy approaches in pattern recognition. A complex hesitant fuzzy sets-based similarity approach was employed for medical diagnosis and pattern recognition by Mahmood *et al*. [16]. Ali [3] used different similar measures based on CFSs for medical diagnosis and pattern recognition.

The correlation coefficient is a well-known notion of statistical analysis, that has been applied in many real-life problems of science, engineering, and technology. It measures the linear relationship between two random variables. Since crisp set theory cannot address the uncertainty, vagueness, imprecision, and ambiguity of real-life applications, the correlation coefficient is extended to fuzzy set theory for better results (Du [9], Riaz *et al*. [22], Wungreiphi and Mazarbhuiya [25]). Gerstenkorn and Mańko [10] proposed a method for correlation coefficient of IFSs whose values lie in the interval of $[0,1]$. Using the statistical approach, Hung [11] proposed to define a similar formula for the correlation coefficient of IFSs. Mazarbhuiya *et al*. [17] proposed the method of correlation coefficient of fuzzy sets that were defined in terms of an interval-valued membership function with reference to an interval-valued reference function and applied it in medical diagnosis and selection of appropriate medicines.

Various similarity measures of CFSs including correlation coefficients and their applications were studied in detail by Ali and Hadi [2], Ali *et al*. [4], Liu *et al*. [13], Wang *et al*. [24], and Zulqarnain *et al*. [32]. The Archimedean aggregation operators for CIFSs were efficiently applied in many problems of multi-attribute decision-making (Ali *et al*. [4]). Wang, *et al*. [24] not only proposed a couple of trigonometric functions-based similarities and weighted similarities on complex Pythagorean fuzzy sets but also applied them in pattern recognition and medical diagnosis. Ali and Hadi [2] studied the correlation coefficient of CFS along with its properties and applications. Zulqarnain *et al*. [32] introduced a weighted correlation formula aimed at calculating the correlation coefficient of interval-valued Pythagorean fuzzy hyper-soft sets, and illustrated the method through a real-life example related to the COVID-19 pandemic. Mahmood *et al*. [13] proposed a novel technique of correlation coefficient of complex dual type-2 hesitant fuzzy sets along with its application in a variety of fields like pattern recognition, clustering, and medical sciences. Although the fuzzy statistical parameters have made inroads in many real-life problems of decision-making and most of the approaches work very well in dealing with imprecision, uncertainty, vagueness etc., with certain limitations. For example, the fuzzy sets of some of their extensions cannot express the imprecision, vagueness, uncertainty, etc., conveniently as done by BCFSs.

Given the above limitation, we put forward a convenient approach for finding the correlation coefficient of the BCFSs. The value obtained by the proposed approach lies in $[0,1]$. The objective of the work is threefold. Firstly, the information energies of the BCFSs and the covariance of the BCFSs are defined using the magnitude and phase terms of both the positive and the negative memberships. Secondly, formulae for the correlation coefficient and weighted correlation coefficient for BCFSs are defined using the aforesaid information energy and the covariance. Finally, a real-life pattern recognition problem related to the mining industry is discussed in detail.

The structure of the article is as follows. The problem definition is described in Section 2. Section 3 discusses the suggested approach. In Section 4, proposed applications of the method along with numerical examples are discussed. Finally, we conclude our paper with a brief conclusion and lines for future work in Section 5.

2. Problem Definitions

Definition 2.1 ([28])**.** A fuzzy set A over universe of discourse U is characterized by,

$$A = \{(u, \mu_A(u)), u \in U\}, \tag{1}$$

where $\mu_A(u) \in [0,1]$ is the membership value of u on A.

Definition 2.2 ([28])**.** A fuzzy set A over a finite set of universe of discourse $U = \{u_1, u_2, \dots, u_n\}$ is given by

$$A = \{(u_j, \mu_A(u_j)), u_j \in U\}. \tag{2}$$

Definition 2.3 ([8])**.** The formula for calculating the fuzzy correlation coefficient is as follows: Let $\{u_1, u_2, \dots, u_n\}$ be a random sample from the universal set U, then the correlation coefficient between $A = \{(u_j, \mu_A(u_j)), u_j \in U\}$ and $B = \{(u_j, \mu_B(u_j)), u_j \in U\}$ is given by

$$\rho_{AB} = \frac{\frac{\sum_{j=1}^{n}(\mu_A(u_j)-\bar{\mu}_A)(\mu_B(u_j)-\bar{\mu}_B)}{n-1}}{\sqrt{\frac{\sum_{j=1}^{n}(\mu_A(u_j)-\bar{\mu}_A)^2}{n-1}\frac{\sum_{i=1}^{n}(\mu_B(u_j)-\bar{\mu}_B)^2}{n-1}}}, \tag{3}$$

where $\bar{\mu}_A = \frac{\sum_{j=1}^{n}\mu_A(u_j)}{n}$ and $\bar{\mu}_B = \frac{\sum_{j=1}^{n}\mu_B(u_j)}{n}$ are the sample means of the membership functions of A and B, respectively.

Definition 2.4 ([6])**.** An *Intuitionistic Fuzzy Set* (IFS) A defined on the universe of discourse U is characterized by

$$A = \{(u, \mu_A(u), \nu_A(u)), u \in U\}, \tag{4}$$

where $\mu_A(u) \in [0,1]$ and $\nu_A(u) \in [0,1]$ are respectively, the membership and nonmembership values of u on A such that $0 \le \mu_A(u) + \nu_A(u) \le 1$, for all $u \in U$. Also, $\pi_A(u) = 1 - \mu_A(u) - \nu_A(u)$ is the degree of hesitation of u to A.

Definition 2.5 ([10])**.** Let $A = \{(u_j, \mu_A(u_j), \nu_A(u_j)); u_j \in U\}$ and $B = \{(u_j, \mu_B(u_j), \nu_B(u_j)); u_j \in U\}$ be two IFSs defined on $U = \{u_1, u_2, \dots, u_n\}$, the correlation coefficient of IFSs is given by

$$\rho_{GM} = \frac{C_{GM}(A,B)}{\sqrt{T(A)\cdot T(B)}}, \tag{5}$$

where $C_{GM}(A,B) = \sum_{j=1}^{n}(\mu_A(u_j)\mu_B(u_j) + \nu_A(u_j)\nu_B(u_j))$, $T(A) = \sum_{j=1}^{n}(\mu_A(u_j)^2 + \nu_A(u_j)^2)$ and $T(B) = \sum_{j=1}^{n}(\mu_B(u_j)^2 + \nu_B(u_j)^2)$.

Definition 2.6 ([12])**.** Let $A = \{(u_j, \mu_A(u_j), \nu_A(u_j)); u_j \in U\}$ and $B = \{(u_j, \mu_B(u_j), \nu_B(u_j)); u_j \in X\}$ be two IFSs defined on $U = \{u_1, u_2, \dots, u_n\}$, [12] defined the correlation coefficient of the IFSs as

$$\rho_{HW} = \frac{C_{HW}(A,B)}{\sqrt{C_{HW}(A)\cdot C_{HW}(B)}}, \tag{6}$$

where $C_{HW}(A,B) = \sum_{j=1}^{n}(\mu_A(u_j)\mu_B(u_j) + \nu_A(u_j)\nu_B(u_j) + \pi_A(u_j)\pi_B(u_j))$, $\pi_A(u) = 1 - \mu_A(u) - \nu_A(u)$, and $\pi_B(u) = 1 - \mu_B(u) - \nu_B(u)$ are the degrees of hesitations of u on A and B, respectively.

Definition 2.7 ([25]). Let $A = \{(u_i, \mu_A(u_i), \nu_A(u_i)), u_i \in u; i = 1,2,\dots,n\}$ and $B = \{(u_i, \mu_B(u_i), \nu_B(u_i)), u_i \in U; i = 1,2,\dots,n\}$ two fuzzy sets with membership functions μ_A and μ_B, and reference functions ν_A and ν_B, then the correlation coefficient between A and B is given by

$$\rho_{AB} = \frac{\operatorname{cov}(A,B)}{\sqrt{\operatorname{cov}(A,A)\cdot\operatorname{cov}(B,B)}}, \tag{7}$$

where for the finite case (discrete universe of discourse U),

$$\operatorname{cov}(A,B) = \int_{i=0}^{n} (\mu_A(u_i) - \nu_A(u_i))(\mu_B(u_i) - \nu_B(u_i)),$$

$$\operatorname{cov}(A,A) = \int_{i=0}^{n} (\mu_A(u_i) - \nu_A(u_i))^2,$$

$$\operatorname{cov}(B,B) = \int_{i=0}^{n} (\mu_B(u_i) - \nu_B(u_i))^2,$$

and for $U \subseteq R$ (real-line),

$$\operatorname{cov}(A,B) = \int_U (\mu_A(u) - \nu_A(u))(\mu_B(u) - \nu_B(u))du,$$

$$\operatorname{cov}(A,A) = \int_U (\mu_A(u) - \nu_A(u))^2 du,$$

$$\operatorname{cov}(B,B) = \int_U (\mu_A(u) - \nu_A(u))^2 du.$$

Definition 2.8 ([20,21]). A *Complex Fuzzy Set* (CFS) A on U is defined by a membership function $\mu_A(u)$ that assigns a complex-valued membership grade in A to each element $u \in U$. By definition, all the membership values lie within the unit circle in the complex plane and are expressed as $\mu_A(u) = r_A(u)e^{i\theta_A(u)}$, where both $r_A(u)$ and $\theta_A(u)$ are real-valued and $0 \le r_A(u) \le 1$ and $0 \le \theta_A(u) \le 2\pi$. Therefore, a CFS A is represented by

$$A = \{(u, \mu_A(u)); u \in U\} = \{(u; r_A(u)\cdot e^{i\theta_A(u)});\ u \in U\}, \tag{8}$$

where $i = \sqrt{-1}$.
Obviously, $A \subset U \times D^2$, where $D^2 = \{r_A(u)\cdot e^{i\theta_A(u)} \mid r_A(u) \in [0,1]$ and $\theta_A(u) \in [0,2\pi]$.

Definition 2.9 ([1,15]). A *Bipolar Complex Fuzzy Set* (BCFS) on $U = \{u_1, u_2, \dots, u_n\}$ is defined below,

$$A = \{(r_A^+(u_i)e^{2\pi i\theta_A^+(u_i)}, r_A^-(u_i)e^{2\pi i\theta_A^-(u_i)}), u_i \in U\}, \tag{9}$$

where $r_A^+ : U \to [0,1]$ and $r_A^- : U \to [-1,0]$. $r_A^+(u_i)e^{2\pi i\theta_A^+(u_i)}$, the positive complex membership degree and $r_A^-(u_i)e^{2\pi i\theta_A^-(u_i)}$, the negative complex membership degree. Additionally, $\theta_A^+(u_i) \in [0,1]$, and $\theta_A^-(u_i) \in [-1,0]$.

3. Proposed Method

Definition 3.1 (Correlation Coefficient of BCFSs). Let $A = \{(r_A^+(u_i)e^{2\pi i\theta_A^+(u_i)}, r_A^-(u_i)e^{2\pi i\theta_A^-(u_i)}), u_i \in U\}$, and $B = \{(r_B^+(u_i)e^{2\pi i\theta_B^+(u_i)}, r_B^-(u_i)e^{2\pi i\theta_B^-(u_i)}), u_i \in U\}$ be two BCFSs over a discrete of universe of discourse $U = \{u_1, u_2, \dots, u_n\}$, then the information energies of A and B are given by

$$E(A) = \sum_{i=1}^{n}[(r_A^+(u_i))^2 + (r_A^-(u_i))^2 + (\theta_A^+(u_i))^2 + (\theta_A^-(u_i))^2],$$

$$E(B)=\sum_{i=1}^{n}[(r_B^+(u_i))^2+(r_B^-(u_i))^2+(\theta_B^+(u_i))^2+(\theta_B^-(u_i))^2]$$

and the covariance of A and B is given by

$$\text{cov}(A,B)=\sum_{i=1}^{n}[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)].$$

Then, the correlation coefficient $\kappa(A,B)$ of A and B can be expressed as follows:

$$\begin{aligned}\kappa(A,B)&=\frac{\text{cov}(A,B)}{\sqrt{E(A)\cdot E(B)}}\\&=\frac{\sum_{i=1}^{n}[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)]}{\sqrt{\begin{pmatrix}\sum_{i=1}^{n}[(r_A^+(u_i))^2+(r_A^-(u_i))^2+(\theta_A^+(u_i))^2+(\theta_A^-(u_i))^2]\\ \times\sum_{i=1}^{n}[(r_B^+(u_i))^2+(r_B^-(u_i))^2+(\theta_B^+(u_i))^2+(\theta_B^-(u_i))^2]\end{pmatrix}}}.\end{aligned}\tag{10}$$

Theorem 3.1. *If A and B are two BCFSs defined over universe of discourse U, then the correlation coefficient $\kappa(A,B)$ satisfies the following properties:*

(i) *If $A=B$, then $\kappa(A,B)=1$,*

(ii) *$\kappa(A,B)=\kappa(B,A)$,*

(iii) *$0\le\kappa(A,B)\le 1$.*

Proof. Let $A=\{(r_A^+(u_i)e^{2\pi i\theta_A^+(u_i)},r_A^-(u_i)e^{2\pi i\theta_A^-(u_i)}),\ u_i\in U\}$ and $B=\{(r_B^+(u_i)e^{2\pi i\theta_B^+(u_i)},r_B^-(u_i)e^{2\pi i\theta_B^-(u_i)}),\ u_i\in U\}$ be the two BCFSs over $U=\{u_1,u_2,\dots,u_n\}$, then the correlation coefficient $\kappa(A,B)$ of A and B can be expressed as

$$\kappa(A,B)=\frac{\text{cov}(A,B)}{\sqrt{E(A)\cdot E(B)}}=\frac{\sum_{i=1}^{n}[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)]}{\left(\sqrt{\begin{matrix}\sum_{i=1}^{n}[(r_A^+(u_i))^2+(r_A^-(u_i))^2+(\theta_A^+(u_i))^2+(\theta_A^-(u_i))^2]\\ \times\sum_{i=1}^{n}[(r_B^+(u_i))^2+(r_B^-(u_i))^2+(\theta_B^+(u_i))^2+(\theta_B^-(u_i))^2]\end{matrix}}\right)}.$$

(i) For, $A=B$,

$$\begin{aligned}\kappa(A,A)&=\frac{\text{cov}(A,A)}{\sqrt{\text{cov}(A,A)\cdot\text{cov}(A,A)}}\\&=\frac{\sum_{i=1}^{n}[r_A^+(u_i)r_A^+(u_i)+r_A^-(u_i)r_A^-(u_i)+\theta_A^+(u_i)\theta_A^+(u_i)+\theta_A^-(u_i)\theta_A^-(u_i)]}{\left(\sqrt{\begin{matrix}\sum_{i=1}^{n}[(r_A^+(u_i))^2+(r_A^-(u_i))^2+(\theta_A^+(u_i))^2+(\theta_A^-(u_i))^2]\\ \times\sum_{i=1}^{n}[(r_A^+(u_i))^2+(r_A^-(u_i))^2+(\theta_A^+(u_i))^2+(\theta_A^-(u_i))^2]\end{matrix}}\right)}\\ \Longrightarrow\quad\kappa(A,A)&=\frac{\left(\sum_{i=1}^{n}[(r_A^+(u_i))^2+(r_A^-(u_i))^2+(\theta_A^+(u_i))^2+(\theta_A^-(u_i))^2]\right)}{\sum_{i=1}^{n}[(r_A^+(u_i))^2+(r_A^-(u_i))^2+(\theta_A^+(u_i))^2+(\theta_A^-(u_i))^2]}=1.\end{aligned}$$

(ii) We have

$$\begin{aligned}\kappa(A,B)&=\frac{\text{cov}(A,B)}{\sqrt{E(A)\cdot E(B)}}\\&=\frac{\sum_{i=1}^{n}[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)]}{\left(\sqrt{\begin{matrix}\sum_{i=1}^{n}[(r_A^+(u_i))^2+(r_A^-(u_i))^2+(\theta_A^+(u_i))^2+(\theta_A^-(u_i))^2]\\ \times\sum_{i=1}^{n}[(r_B^+(u_i))^2+(r_B^-(u_i))^2+(\theta_B^+(u_i))^2+(\theta_B^-(u_i))^2]\end{matrix}}\right)}\end{aligned}$$

$$= \frac{\sum_{i=1}^{n}[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)]}{\left(\sqrt{\begin{array}{l}\sum_{i=1}^{n}[(r_B^+(u_i))^2+(r_B^-(u_i))^2+(\theta_B^+(u_i))^2+(\theta_B^-(u_i))^2]\\ \times\sum_{i=1}^{n}[(r_A^+(u_i))^2+(r_A^-(u_i))^2+(\theta_A^+(u_i))^2+(\theta_A^-(u_i))^2]\end{array}}\right)}$$

$$= \frac{\mathrm{cov}(B,A)}{\sqrt{\mathrm{cov}(B,B)\cdot\mathrm{cov}(A,A)}}$$

$$= \kappa(B,A).$$

(iii) To prove the property (iii), let us proceed as follow:

We have

$$\begin{aligned}
&[r_A^+(u_i)r_B^-(u_i)-r_A^-(u_i)r_B^+(u_i)]^2+[r_A^+(u_i)\theta_B^-(u_i)-r_A^-(u_i)\theta_B^+(u_i)]^2\\
&\quad+[r_B^-(u_i)\theta_A^+(u_i)-r_B^+(u_i)\theta_A^-(u_i)]^2+[\theta_A^+(u_i)\theta_B^-(u_i)-\theta_A^-(u_i)\theta_B^+(u_i)]^2\\
&\quad+[r_A^+(u_i)\theta_B^+(u_i)-r_B^-(u_i)\theta_A^-(u_i)]^2+[r_A^-(u_i)\theta_B^-(u_i)-r_B^+(u_i)\theta_A^+(u_i)]^2\geq 0\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\text{(sum of squared terms)}
\end{aligned}$$

$$\begin{aligned}
\implies\;& r_A^+(u_i)^2r_B^-(u_i)^2+r_A^-(u_i)^2r_B^+(u_i)^2+r_A^+(u_i)^2\theta_B^-(u_i)^2+r_A^-(u_i)^2\theta_B^+(u_i)^2+r_B^-(u_i)^2\theta_A^+(u_i)^2\\
&+r_B^+(u_i)^2\theta_A^-(u_i)^2+\theta_A^+(u_i)^2\theta_B^-(u_i)^2+\theta_A^-(u_i)^2\theta_B^+(u_i)^2+r_A^+(u_i)^2\theta_B^+(u_i)^2\\
&+r_B^-(u_i)^2\theta_A^-(u_i)^2+r_A^-(u_i)^2\theta_B^-(u_i)^2+r_B^+(u_i)^2\theta_A^+(u_i)^2\\
&\quad\geq 2r_A^+(u_i)r_B^-(u_i)r_A^-(u_i)r_B^+(u_i)+2r_A^+(u_i)\theta_B^-(u_i)r_A^-(u_i)\theta_B^+(u_i)\\
&\qquad+2r_B^-(u_i)\theta_A^+(u_i)r_B^+(u_i)\theta_A^-(u_i)+2\theta_A^+(u_i)\theta_B^-(u_i)\theta_A^-(u_i)\theta_B^+(u_i)\\
&\qquad+2r_A^+(u_i)\theta_B^+(u_i)r_B^-(u_i)\theta_A^-(u_i)+2r_A^-(u_i)\theta_B^-(u_i)r_B^+(u_i)\theta_A^+(u_i)\\
\implies\;& r_A^+(u_i)^2r_B^+(u_i)^2+r_A^+(u_i)^2r_B^-(u_i)^2+r_A^-(u_i)^2r_B^+(u_i)^2+r_A^-(u_i)^2r_B^-(u_i)^2+r_A^+(u_i)^2\theta_B^-(u_i)^2\\
&+r_A^-(u_i)^2\theta_B^+(u_i)^2+r_B^-(u_i)^2\theta_A^+(u_i)^2+r_B^+(u_i)^2\theta_A^-(u_i)^2+\theta_A^+(u_i)^2\theta_B^+(u_i)^2\\
&+\theta_A^+(u_i)^2\theta_B^-(u_i)^2+\theta_A^-(u_i)^2\theta_B^+(u_i)^2+r_A^+(u_i)^2\theta_B^+(u_i)^2+r_B^-(u_i)^2\theta_A^-(u_i)^2\\
&+r_A^-(u_i)^2\theta_B^-(u_i)^2+r_B^+(u_i)^2\theta_A^+(u_i)^2+\theta_A^-(u_i)^2\theta_B^-(u_i)^2\\
&\quad\geq r_A^+(u_i)^2r_B^+(u_i)^2+r_A^-(u_i)^2r_B^-(u_i)^2+\theta_A^+(u_i)^2\theta_B^+(u_i)^2+\theta_A^-(u_i)^2\theta_B^-(u_i)^2\\
&\qquad+2r_A^+(u_i)r_B^-(u_i)r_A^-(u_i)r_B^+(u_i)+2r_A^+(u_i)\theta_B^-(u_i)r_A^-(u_i)\theta_B^+(u_i)\\
&\qquad+2r_B^-(u_i)\theta_A^+(u_i)r_B^+(u_i)\theta_A^-(u_i)+2\theta_A^+(u_i)\theta_B^-(u_i)\theta_A^-(u_i)\theta_B^+(u_i)\\
&\qquad+2r_A^+(u_i)\theta_B^+(u_i)r_B^-(u_i)\theta_A^-(u_i)+2r_A^-(u_i)\theta_B^-(u_i)r_B^+(u_i)\theta_A^+(u_i)\\
\implies\;& r_A^+(u_i)^2[r_B^+(u_i)^2+r_B^-(u_i)^2+\theta_B^+(u_i)^2+\theta_B^-(u_i)^2]\\
&+r_A^-(u_i)^2[r_B^+(u_i)^2+r_B^-(u_i)^2+\theta_B^+(u_i)^2+\theta_B^-(u_i)^2]\\
&+\theta_A^+(u_i)^2[r_B^+(u_i)^2+r_B^-(u_i)^2+\theta_B^+(u_i)^2+\theta_B^-(u_i)^2]\\
&+\theta_A^-(u_i)^2[r_B^+(u_i)^2+r_B^-(u_i)^2+\theta_B^+(u_i)^2+\theta_B^-(u_i)^2]\\
&\quad\geq[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)]^2\\
\implies\;& [r_A^+(u_i)^2+r_A^-(u_i)^2+\theta_A^+(u_i)^2+\theta_A^-(u_i)^2][r_B^+(u_i)^2+r_B^-(u_i)^2+\theta_B^+(u_i)^2+\theta_B^-(u_i)^2]\\
&\geq[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)]^2
\end{aligned}$$

$$\Longrightarrow \sum_{i=1}^{n}[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)]^2$$
$$\leq \sum_{i=1}^{n}[r_A^+(u_i)^2+r_A^-(u_i)^2+\theta_A^+(u_i)^2+\theta_A^-(u_i)^2][r_B^+(u_i)^2+r_B^-(u_i)^2+\theta_B^+(u_i)^2+\theta_B^-(u_i)^2]$$
$$\leq \left(\sum_{i=1}^{n}[r_A^+(u_i)^2+r_A^-(u_i)^2+\theta_A^+(u_i)^2+\theta_A^-(u_i)^2]\right)^{\frac{1}{2}}$$
$$\times \left(\sum_{i=1}^{n}[r_B^+(u_i)^2+r_B^-(u_i)^2+\theta_B^+(u_i)^2+\theta_B^-(u_i)^2]\right)^{\frac{1}{2}} \quad \text{(using Schwarz inequality)}$$
$$\Longrightarrow \frac{\sum_{i=1}^{n}[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)]}{\left(\begin{array}{l}\left(\sum_{i=1}^{n}[r_A^+(u_i)^2+r_A^-(u_i)^2+\theta_A^+(u_i)^2+\theta_A^-(u_i)^2]\right)^{\frac{1}{2}}\\ \times\left(\sum_{i=1}^{n}[r_B^+(u_i)^2+r_B^-(u_i)^2+\theta_B^+(u_i)^2+\theta_B^-(u_i)^2]\right)^{\frac{1}{2}}\end{array}\right)}\leq 1$$
$$\Longrightarrow \frac{\sum_{i=1}^{n}[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)]}{\left(\sqrt{\begin{array}{l}\sum_{i=1}^{n}[r_A^+(u_i)^2+r_A^-(u_i)^2+\theta_A^+(u_i)^2+\theta_A^-(u_i)^2]\\ \times\sum_{i=1}^{n}[r_B^+(u_i)^2+r_B^-(u_i)^2+\theta_B^+(u_i)^2+\theta_B^-(u_i)^2]\end{array}}\right)}\leq 1$$
$$\Longrightarrow 0\leq \kappa(A,B)\leq 1 \quad \text{(using (10))}.$$

Hence proved. □

Definition 3.2 (Weighted Correlation Coefficient of BCFSs).
Let $A=\{(r_A^+(u_i)e^{2\pi i\theta_A^+(u_i)}, r_A^-(u_i)e^{2\pi i\theta_A^-(u_i)}), u_i\in U\}$, and $B=\{(r_B^+(u_i)e^{2\pi i\theta_B^+(u_i)}, r_B^-(u_i)e^{2\pi i\theta_B^-(u_i)}), u_i\in U\}$ be two BCFSs over a finite universe of discourse $U=\{u_1,u_2,\dots,u_n\}$. Also, let $W=\{w_1,w_2,\dots,w_n\}$ be the weight vector associated with u_i; $\{i=1,2,\dots,n\}$ such that $\sum_{i=1}^{n}w_i=1$, $0\leq w_i\leq 1$; $i=1,2,\dots n$, then the weighted information energies of A and B are given by

$$WE(A)=\sum_{i=1}^{n}w_i[(r_A^+(u_i))^2+(r_A^-(u_i))^2+(\theta_A^+(u_i))^2+(\theta_A^-(u_i))^2],$$
$$WE(B)=\sum_{i=1}^{n}w_i[(r_B^+(u_i))^2+(r_B^-(u_i))^2+(\theta_B^+(u_i))^2+(\theta_B^-(u_i))^2]$$

and the covariance of A and B is given by

$$W\mathrm{cov}(A,B)=\sum_{i=1}^{n}w_i[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)].$$

Then the weighted correlation coefficient $W\kappa(A,B)$ of A and B can be expressed as follows:

$$\begin{aligned}W\kappa(A,B)&=\frac{W\mathrm{cov}(A,B)}{\sqrt{WE(A)\cdot WE(B)}}\\&=\frac{\sum_{i=1}^{n}w_i[r_A^+(u_i)r_B^+(u_i)+r_A^-(u_i)r_B^-(u_i)+\theta_A^+(u_i)\theta_B^+(u_i)+\theta_A^-(u_i)\theta_B^-(u_i)]}{\left(\sqrt{\begin{array}{l}\sum_{i=1}^{n}w_i[(r_A^+(u_i))^2+(r_A^-(u_i))^2+(\theta_A^+(u_i))^2+(\theta_A^-(u_i))^2]\\ \times\sum_{i=1}^{n}w_i[(r_B^+(u_i))^2+(r_B^-(u_i))^2+(\theta_B^+(u_i))^2+(\theta_B^-(u_i))^2]\end{array}}\right)}.\end{aligned} \tag{11}$$

Theorem 3.2. *If A and B are two BCFSs defined over universe of discourse U, then the weighted correlation coefficient $W\kappa(A,B)$ satisfies the following properties:*

(i) *If $A = B$, then $W\kappa(A,B) = 1$,*

(ii) $W\kappa(A,B) = W\kappa(B,A)$,

(iii) $0 \le W\kappa(A,B) \le 1$.

Proof. The proof follows similar to Theorem 3.1. □

4. Application in Pattern Recognition

The correlation coefficient of BCFSs plays an active role in many decision-making problems. It may be used ideally for gathering data in a usual way, and then the collected data can be used to generalize findings to many real-life pattern recognition problems. These days, the pattern recognition systems are used to evaluate minerals, energy sources, and their patterns across various geographic areas. Let $M^{(i)}$; $i = 1,2,3,\dots,n$, be the BCFSs representing n types of known minerals. Let M be the BCFS representing an unknown mineral. It is required to categorize M based on its correlation coefficient with the n known patterns. An algorithm for pattern recognition is given below.

Algorithm

Step 1. Using domain experts, express each of the mineral categories with the help of BCFSs $\{M^{(i)}; i = 1,2,\dots,n\}$ of symptoms.

Step 2. Study the unknown pattern (M) carefully and represent it with the help of a BCFS of symptoms.

Step 3. Compute the correlation coefficient $\kappa(M^{(i)},M)$ (or weighted correlation coefficient $W\kappa(M^{(i)},M)$); $i = 1,2,\dots,n$ using the formula (10) or (11).

Step 4. If $\kappa(M^{(k)},M) \ge k(M^{(i)},M)$ (or $W\kappa(M^{(k)},M) \ge W\kappa(M^{(i)},M)$); $i \ne k$ then the pattern M is closest to $M^{(k)}$ and thus belong to pattern category $M^{(k)}$.

The flowchart of the proposed algorithm is given in Figure 1.

Below, a numeric example is discussed, to the demonstrate a potential use of the proposed method.

Example 4.1. Here, a solution to a pattern recognition problem is provided. Let's say a mining company is on a site and finds a new kind of carbon. The expert makes a detailed investigation based on the carbon's structure, properties, and smell and gives his opinion about its pattern. Based on the expert's opinion, let the pattern be represented by a BCFS over the set of the universe of discourse $X = \{x_1,x_2,x_3\}$ as follows:

$$M = \{(x_1, 0.9e^{i2\pi(0.3)}, -0.94e^{-i2\pi(0.26)}), (x_2, 0.79e^{i2\pi(0.1)}, -0.89e^{-i2\pi(0.11)}),$$
$$(x_3, 0.7e^{i2\pi(0.3)}, -0.94e^{-i2\pi(0.15)})\}.$$

Suppose there are three patterns (allotropes) of carbon—diamond (M^1), graphite (M^2), and fullerenes (M^3) — already known to the experts and the problem is to recognize an unknown pattern (M) among the given patterns $\{M^1,M^2,M^3\}$. For this purpose, the known allotropes are

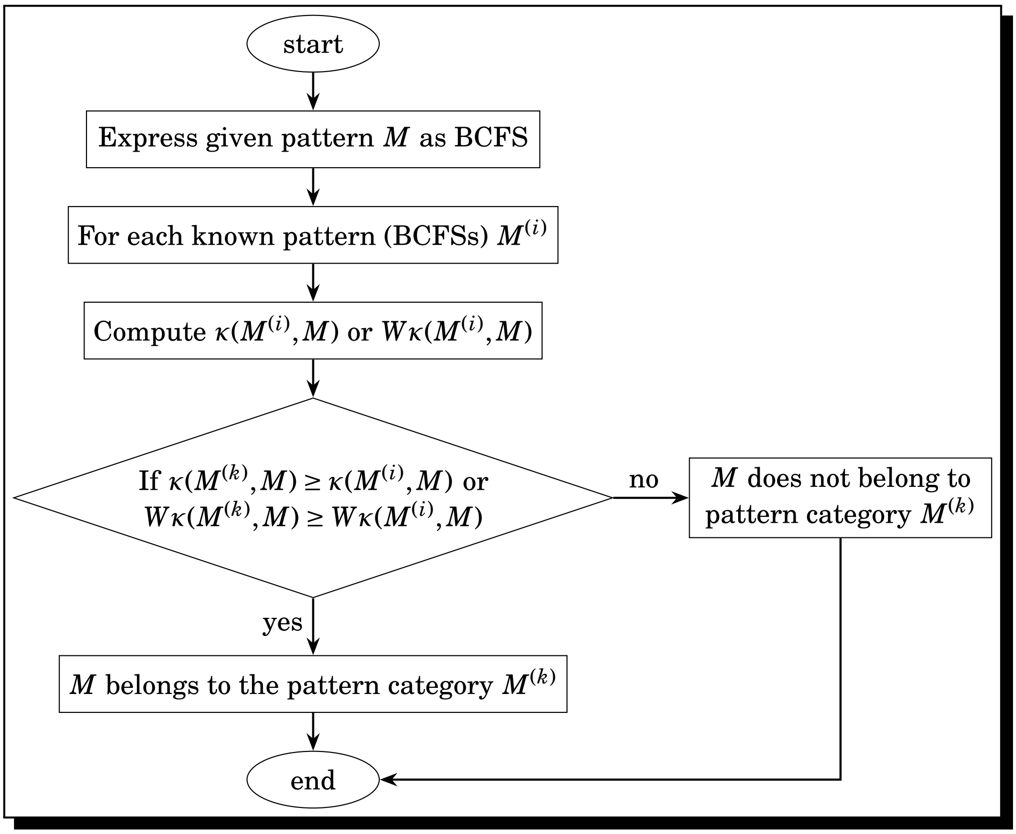

Figure 1. Flowchart of the proposed pattern recognition algorithm

reanalyzed and the similar BCFSs are assigned to each of the allotropes given in Table 1 as follows:

Table 1. BCFS representing the known patterns of allotropes

	x_1	x_2	x_3
$M^{(1)}$ (diamond)	$0.8e^{i2\pi(0.3)}$, $-0.6e^{-i2\pi(0.24)}$	$0.9e^{i2\pi(0.12)}$, $-0.9e^{-i2\pi(0.12)}$	$0.7e^{i2\pi(0.2)}$, $-0.5e^{-i2\pi(0.3)}$
$M^{(2)}$ (graphite)	$0.5e^{i2\pi(0.2)}$, $-0.6e^{-i2\pi(0.1)}$	$0.9e^{i2\pi(0.4)}$, $-0.6e^{-i2\pi(0.2)}$	$0.8e^{i2\pi(0.45)}$, $-0.7e^{-i2\pi(0.3)}$
$M^{(3)}$ (fullerenes)	$0.3e^{i2\pi(0.2)}$, $-0.2e^{-i2\pi(0.3)}$	$0.4e^{i2\pi(1)}$, $-0.4e^{-i2\pi(0.6)}$	$0.8e^{i2\pi(0.7)}$, $-0.7e^{-i2\pi(0.6)}$

Let the weights of the attributes x_1, x_2, and x_3 be 0.3, 0.4, and 0.3, respectively. Then, the correlation coefficients and weighted correlation coefficient of M with M_1, M_2, and M_3 are given in Table 2.

Table 2. Correlation and weighted coefficient of the unknown carbon pattern and known allotropes

	$\kappa(M, M^{(i)})$	$W\kappa(M, M^{(i)})$
$M^{(1)}$ (diamond)	0.9654707851	0.9710240698
$M^{(2)}$ (graphite)	0.9370739085	0.9172712567
$M^{(2)}$ (fullerenes)	0.6822748268	0.6756027157

From Table 2 and Figure 2, the following observations can be made.

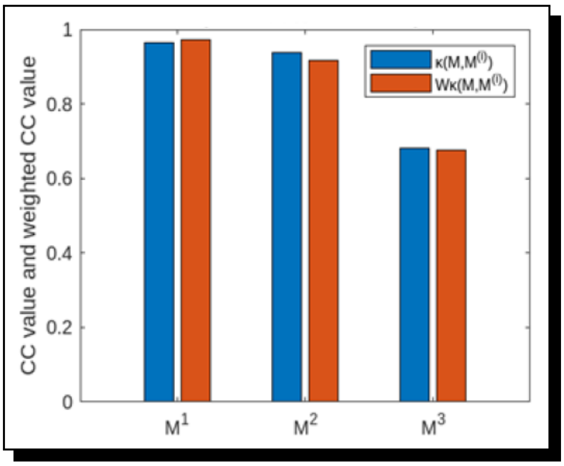

Figure 2. Correlation coefficient and weighted correlation coefficient between known and unknown patterns

From the obtained results, it is observed that the correlation coefficient of unknown pattern, a new type of carbon with known patterns such as diamond, graphite and fullerenes are in the following order:

$$\kappa(M, M^{(1)}) = 0.9654707851 > \kappa(M, M^{(2)}) = 0.9370739085 > \kappa(M, M^{(3)}) = 0.6822748268.$$

Further, the weighted correlation coefficients are in the following order:

$$W\kappa(M, M^{(1)}) = 0.9710240698 > W\kappa(M, M^{(2)}) = 0.9172712567 > W\kappa(M, M^{(3)}) = 0.6756027157.$$

From the above discussions, it can be established that the newly found carbon is most likely diamond.

In this article, the structures, properties and smell of the carbon allotropes are considered for pattern recognition. However, if additional features of carbon allotropes, such as colour, lustre, hardness, density, and transparency, etc., are considered, then the proposed formulae may produce superior results, and hence the pattern recognition will be more accurate.

5. Conclusion

In this article, new methods for calculating the correlation coefficient and weighted correlation coefficient for *bipolar complex fuzzy sets* (BCFSs) are proposed. First, the covariance and information energies of the BCFSs are defined with the help of both negative and positive membership values, which are then used to construct formulae for the correlation coefficient and weighted correlation coefficient. For the weighted correlation coefficient, each element of the universe of discourse is multiplied by its corresponding weight. The formulas yield similar results to most existing methods, meaning that the values of correlation coefficient and weighted correlation coefficient of BCFSs lie within the interval of $[0,1]$. Furthermore, a real-life example of pattern recognition for an unknown carbon allotrope is discussed in detail to validate the formulae's usefulness. Despite their effectiveness, the proposed methods for the correlation coefficient and weighted correlation coefficient have certain drawbacks. First, the correlation coefficient values may be dominated by certain features of the pattern, which could bias the approach toward the dominant features. Second, the non-membership component of fuzziness is not taken into consideration in the formula.

Potential avenues for future research include the following:

- The proposed approaches can be implemented using real data from the mining industry.
- The accuracy and dependability of the proposed approaches can be enriched in the future by appropriately modifying each input.
- The other types of statistical parameters can be incorporated to boost the approaches accuracy.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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