Communications in Mathematics and Applications
Vol. 17, No. 1, pp. 175–190, 2026
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v17i1.3438

http://www.rgnpublications.com

Research Article

Detection of a Randomly Hidden Target in Constrained Domains: A Model with Discounted Effort-Driven Incentives

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Received: October 27, 2025 **Revised:** December 9, 2025 **Accepted:** December 22, 2025

Abstract. In this work, a search region is a bounded interval on the line. This interval is divided into a number of small subintervals. The target probability in each subinterval is determined from the internal truncation method of the double truncated distribution of the target position, where the sub-intervals that had little chance of containing the hidden target were eliminated. Due to this uncertainty principle, we can apply the discount effort-reward search parameter in the detection probability function. We solve this discrete problem to determine the least amount of effort needed to detect a target, where this effort is constrained by a normal distribution. Furthermore, we determine the target detection probability's maximum value and examine the stability of the minimal search effort. We provide an example to demonstrate the usefulness and relevance of our model.

Keywords. Detection model, Internal truncated distribution, Stability of the minimum search effort, Detection probability, Normal distribution

Mathematics Subject Classification (2020). 90B40, 62E15, 90C30

1. Introduction

Despite the close relationship between planet Earth and the natural disasters that strike it from time to time, humans find themselves in successive predicaments on planet Earth. These natural disasters take many forms, including hurricanes, floods, volcanoes, earthquakes, and others. These disasters often claim huge numbers of lives, while survivors are given a new lease of life, some of whom remain trapped between life and death under the rubble of collapsed buildings, waiting to see the sun again. Rescuing survivors from under the rubble as a result of any disaster, or earthquakes in particular, is a top priority for governmental and relief organizations. There are specialized teams called "search and rescue teams" that oversee this complex process of saving survivors from certain death. Hardly a year goes by without a natural disaster striking the Earth, sending people to their fates, and history is full of such horrific disasters.

Natural disaster researchers have indicated that the time factor plays a fundamental role in the first moments of the rescue operation; the survival rate of those trapped under the rubble reaches about 80% if they are pulled out on the first day. However, this rate drops significantly to 30% the next day and decreases to between 15 and 10% on the third day. This does not necessarily mean that the survival rate is zero later, as studies have shown that a person is able to stay alive for a period of time that may reach 40 days without food, provided that he does not abstain from water, and in the event of being cut off from water, the reasonable limit for his survival is 3 days, depending on the physical and psychological state as well. The probability of survival increases in winter more than in summer due to the temperature, as happened in the recent earthquake in Turkey, where survivors were found after more than 8 days. On average, water constitutes 60-70% of the human body, and within the vital processes, the body gets rid of a quantity of fluids through waste, sweating, and breathing, and therefore, the body needs to constantly replace the lost fluids. When the body is severely dehydrated, sodium levels in the blood rise, condensing and becoming more concentrated, which means the heart and blood vessels have to work harder to maintain blood pressure in the body. In addition to the need for water, there must be a source of oxygen for those trapped under the rubble.

The question here is: What are the approved methods for searching for the trapped, and how do search and rescue teams achieve their efforts? Is there a more efficient mechanism that can be used in the future? Searching for survivors is a risky process, as rescue crew members place themselves in dangerous places among the rubble that may collapse under any impact. The first task of rescuers is to collect information related to the disaster by responding, moving quickly, and conducting reconnaissance. The second stage is known as "surface rescue", which means urgent response to any victims who can be seen, felt, or heard in the affected area, and they are often trapped in places that are easy to reach or see with the naked eye. In the third stage, the collapsed buildings are searched for gaps between the rubble, or as they are called, "empty spaces in which to survive", which are natural spaces that occur as a result of the collapse of walls on each other and where the trapped are often found and remain. This process includes a great ability and quick response from the rescue team to identify these places and reach them by breaking down insulators and obstacles, creating narrow passages, and crawling through them. Therefore, the probability of finding the target varies from one area to another, as there are areas with high probability and others with low probability. Going through these stages, the effort expended and necessary to discover the target becomes very costly. Based on the available information, rescue teams develop quick and inexpensive search plans that depend on the nature of the search area to help discover the target. This plan should be characterized by reducing the time to discover the target as much as possible by excluding some areas where the probability of the target is weak.

From a probability theory perspective, the best way to eliminate these regions is to divide the search area into a set of small regions. One of the most important of these methods was presented by Hong *et al*. [12, 13], which is to divide the search area into a set of identical hexagonal cells. Their goal was to find the optimal search path inside these cells such that the randomly moving target would have the maximum detection probability. Furthermore, a different partition of the search zone based on a known number of square cells (similar or not) was presented by El-Hadidy [7]. In order to get the ideal distribution of the search effort that maximizes the detection probability of a Markovian target, he had to solve a challenging discrete stochastic optimization problem. Various search strategies have been studied in the past to find lost targets in the plane and space; El-Hadidy and Alfreedi [8] is one such example. Their common goal was to reduce the expected value of the detection time. Several analytical techniques that demonstrated the existence of these optimal search strategies in the case of a randomly moving target have been examined in numerous studies; for instance, see El-Hadidy [6]. These optimal search strategies' primary goal is to find the minimum amount of time that is expected to be needed to reach the target. However, El-Hadidy and Alzulaibani [9], Angelopoulos and Lidbetter [3], and Lidbetter [15, 16] covered more intriguing and unique search strategies. These works aim to explore the optimal search strategies for reducing the expected value of the first meeting time between the randomly moving target and the searcher.

On a bounded interval on the line, Alamri and El-Hadidy [1] presented an interesting model to detect a hidden target on one of some sub-intervals on this interval in order to examine the same form of the discrete search problem as El-Hadidy [7]. The target existence has a high probability in these sub-intervals. We should distribute the goal probability throughout these intervals in order to find this probability in each interval. El-Hadidy [5] explored a new truncation approach that we need in order to remove certain sub-intervals from the original interval. The target probability will therefore be dispersed throughout recognized intervals. Three types of target distribution truncations-uneven, commensurate, and symmetric were presented by Fakharany *et al*. [11]. When the coordinated search technique is taken into consideration, all of these categories are used to calculate the expected value of the cost to detect the hidden target, as in Reyniers [18]. In order to attain the optimal expected cost value, Fakharany *et al*. [11] also gave Newton's method for figuring out the ideal values of the sub-interval boundaries where the truncation should be done. For further details on contemporary probabilistic models addressing the detection of a lost target (hidden or randomly moving), readers are referred to recent works by El-Hadidy *et al*. [10].

One method alone may not be sufficient for the desired purpose, so search and rescue teams may rely on several methods to carry out their tasks, as some research shows in studying dual methods that combine sound, heat, and chemical sensing to search for those trapped under rubble. The time factor remains the most decisive factor in this regard, in addition to the high cost. In this work, we seek to reduce the cost of the search while increasing the probability of discovering the target. We consider the model that has been studied in Alamri and El-Hadidy [1], where the target is hidden on a known interval in the line. They obtained the optimal distribution effort (bounded by a *normal* random variable) that optimizes the detection probability in order to find the target. Due to the uncertainty in this model, we can apply the discount effort-reward search parameter, which was discussed in Lanillos *et al*. [14], to the detection probability function. This will give the minimum search effort to detect the target with maximum probability.

This paper is organized as follows: In Section 2, the model's formulation is explained. In Section 3, the detection probability is calculated, which is dependent on the sub-interval boundaries. Furthermore, this section demonstrates how the probability detection function incorporates the discounted effort-reward parameter in order to reduce the detection cost. This section also provides the optimal solution, taking into account the discounted effort-reward parameter, to a discrete optimization problem that yields both a maximum detection probability and a minimum search effort simultaneously. Furthermore, it offers the conditions demonstrating the stability of the minimum amount of search effort. Our model's applicability and efficacy are presented in Section 4. Finally, we discuss some concluding remarks and future work.

2. Problem Formulation

Since the probability of the target's presence varies from one area to another, as there are areas with high probability and others with low probability. Thus, It is assumed that the target is concealed within one of the K sub-intervals of the search space (interval $[a,b]$), and that the probability distribution of its position is known. We face a problem of choosing these sub-intervals due to the availability of some information about the target. Therefore, we remove N sub-intervals from $[a,b]$ (all potential sub-intervals that we do not want to study) using El-Hadidy's [5] definition of multiple truncations of the target probability distribution. As a result, we need to find K sub-intervals so that $K+N$ provides all feasible sub-intervals on $[a,b]$. By doing this, the probability p_i and ξ_i, $i=1,2,\dots,K$ will be maximized, resulting in the lowest possible search costs. If we let the target's position on the interval $[a,b]$ is presented by a random variable Y with a known probability distribution, its double truncated probability density and cumulative distribution functions are given by,

$$q_Y(y)=\begin{cases}\frac{g_Y(y)}{G(b)-G(a)}, & \text{if } a\le y\le b,\\ 0, & \text{otherwise,}\end{cases} \tag{1}$$

$$Q_Y(y)=\begin{cases}\frac{G_Y(y)-G(a)}{G(b)-G(a)}, & \text{if } a\le h\le b,\\ 0, & \text{otherwise,}\end{cases} \tag{2}$$

where $g(\cdot)$ and $G(\cdot)$ are the original probability density and cumulative distribution functions see, Ali and Nadarajah [2], then applying the definition in El-Hadidy [5] we get a new random variable X with multiple truncated probability distribution its probability density and cumulative distribution functions are given by:

$$f_X(x)=\begin{cases}\Theta^{-1}q_X(x), & \text{if } x\in(-\infty,\alpha_1)\cup(\beta_1,\alpha_2)\cup\ldots\cup(\beta_{K-1},\alpha_K)\cup(\beta_K,\infty),\\ 0, & \text{otherwise,}\end{cases} \tag{3}$$

$$F_X(x)=\begin{cases}\Theta^{-1}Q_X(x), & a<x<\alpha_1,\\ \Theta^{-1}[Q_Y(\alpha_1)+Q_X(x)-Q_Y(\beta_1)], & \beta_1<x<\alpha_2,\\ \quad\vdots & \\ \Theta^{-1}[Q_Y(\alpha_1)+Q_Y(\alpha_2)-Q_Y(\beta_1)+\ldots & \\ \quad +Q_Y(\alpha_{K-1})-Q_Y(\beta_{K-2})+Q_X(x)-Q_Y(\beta_{K-1})], & \beta_{K-1}<x<\alpha_K,\\ \Theta^{-1}[Q_Y(\alpha_1)+Q_Y(\alpha_2)-Q_Y(\beta_1)+\ldots & \\ \quad +Q_Y(\alpha_K)-Q_Y(\beta_{K-1})+Q_X(x)-Q_Y(\beta_K)], & \beta_K<x<b,\end{cases} \tag{4}$$

respectively, where $\Theta^{-1} = 1 - Q_Y(\beta_K) + \sum_{i=1}^{K}[Q_Y(\alpha_i) - Q_Y(\beta_{i-1})]$, $\beta_0 = -\infty$ and $a < \alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \ldots < \alpha_K < \beta_K < b$ are the points at which the truncation was made. As a result, the probability that the concealed target in $[\beta_{i-1}, \alpha_i]$ will become $p_i = F_X(\alpha_i) - F_X(\beta_{i-1})$ (that is, we identify the sub-interval that has to be searched where p_i depends on the boundaries of the sub-intervals based on the data provided).

2.1 The Searcher's Mechanism

Every sub-interval $[\beta_{i-1}, \alpha_i]$ in our model has a single searcher searching for the target. As seen in Figure 1, the searcher is free to move freely into the searched sub-intervals (that is, to move from one sub-interval to another on $[a, b]$). Every sub-interval's search takes one unit of time and is carried out separately from earlier searches. After finishing the search, the user can immediately move to a different sub-interval or carry on looking for the same sub-interval. Every sub-interval is distinct from the others in its character. As a result, the detection probability in each subinterval is solely dependent on the searcher's overall effort, not on how that effort is applied. Furthermore, we consider that no effort is lost in the search process. In order to reduce the expected value of searching cost, the searcher should split his effort z_{ij} (the effort which used in $[\beta_{i-1}, \alpha_i]$, it may be energy, money, time and etc.) among the sub-intervals after the jth search.

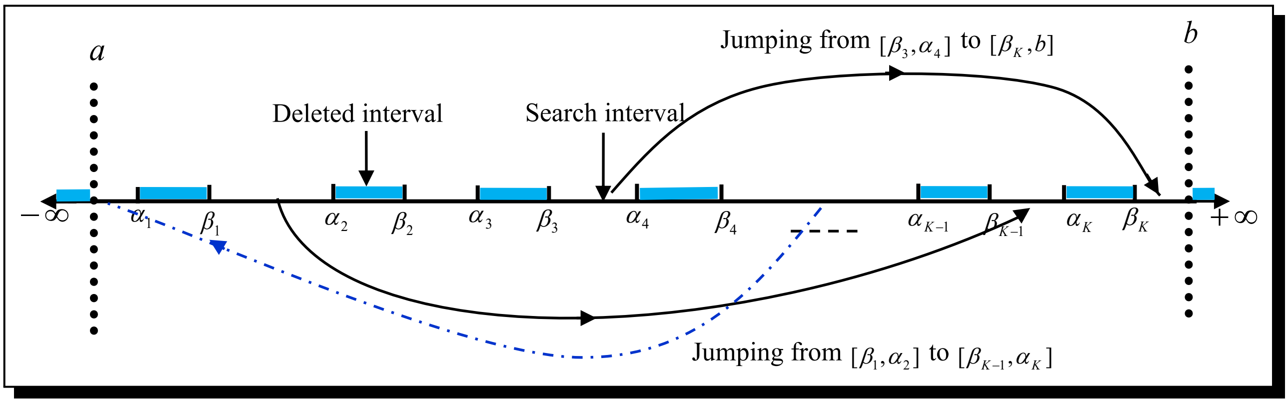

Figure 1. K sub-intervals within a bounded search space, with unrestricted jumping between them

This model's flexibility allows the searcher to move freely inside the sub-intervals where details about each sub-interval, including its boundaries and the searcher's speed, are known. As a result, this model may simultaneously show the probability of detection and the shortest estimated time of detection, two crucial metrics in search theory. Additionally, this model is more adaptable in situations when wireless signals are inefficient, such looking for people under debris after earthquakes or disasters, and where it cannot employ moving sensors.

3. Detection Probability Under the Influence of a Discount Effort Reward

When a natural catastrophe strikes, it is normal for certain areas of the search area to be given priority in the search based on the information that is now available, allowing for the rescue of any victims who may still be alive beneath the debris. Since the distribution of effort fluctuates between sub-intervals, we propose that a normal random variable serve as its boundary. Furthermore, the variable's values are established based on the type of search procedure. Thus, if we consider the search effort Z to be a variable whose value is bound by a normal random variable W, then the total search effort function to detect the target at the jth search is given by,

$$\Phi(Z)=\sum_{i=1}^{K}\Phi_i(Z)=\sum_{i=1}^{K}\sum_{j=1}^{N}Z_{ij}, \tag{5}$$

where $0\le\Phi_i(Z)\le W_i$, $i=1,2,\dots,K$.

3.1 Formalization of the Detection Probability Under the Impact of the Discount Effort Reward Parameter

In our search model, the target's location is randomly distributed and the searcher jumps in K sub-intervals to address decision-making under uncertainty. These processes can be thought of as mathematical models that concentrate on the optimal path for a decision-maker who has to make several choices with different outcomes throughout time. The inclusion of uncertainty in the search process would increase the reliability of the outcomes of a thorough analysis because many future occurrences regarding the target's location are unpredictable. As a result, for the searcher on the bounded interval over time, we can regulate the leaps between sub-intervals. This enables us to optimize the probability of detecting the target and minimize the amount of work expended by the searcher by utilizing the discount factor. As a result, we may incorporate the potential for effort modeling, similar to what Lanillos *et al*. [14] did. Blum *et al*. [4] demonstrated how the minimum search effort can be translated into a maximum discounted effort reward depending on the detection probability. In this case, the potential rewards at time interval $i=1,2,\dots,K$ will be decreased by using the exponential function $w_j(i)=\lambda_j^i$, $0<\lambda_j<1$. The tuning parameter λ_j allows us to indirectly control the search time and speed, or alternatively, the significance of the future actions the searcher will take. Consequently, the parameter λ_j has a direct affect on the searching time T_{ij} at the jth search in the inspecting sub-interval number i ($[\beta_{i-1},\alpha_i]$). As a result, we should have a discounting time $\lambda_j^i T_{ij}=t_j$ which gives a direct impact on the detection probability. If we consider H_{ij} be a detection event in $[\beta_{i-1},\alpha_i]$ after the jth search, then the probability of detecting the target after the 1st search, is given by

$$\begin{aligned}\Gamma_{i1}&=P[H_{i1}]\\&=\sum_{t=1}^{N}P(H_{i1}\mid\lambda_1^iT_{i1}=t)\cdot P(\lambda_1^iT_{i1}=t)\\&=\sum_{t=1}^{N}P(L_i,S_i(t),D_i(1,t))\cdot P(\lambda_1^iT_{i1}=t),\end{aligned}$$

where L_i and $S_i(t)$ are the existence and the detection events of the target at time t when the searching process done in $[\beta_{i-1},\alpha_i]$, respectively. If we consider $t_\ell<t_\wp$ which means the set of times $\{t_\ell,t_\ell+1,\dots,t_\wp-2,t_\wp-1\}$, then we have

$$\Gamma_{i1}=\sum_{t=1}^{N}P(D_i(1,t))\cdot P(L_i\mid D_i(1,t))\cdot P(S_i(t)\mid L_i\cap D_i(1,t))\cdot P(\lambda_1^iT_{i1}=t),$$

where $D_i(1,t)$ is the no detection event during $[1,t)$. This leads to,

$$\Gamma_{i1}=\sum_{t=1}^{N}P(D_i(1,t))\cdot\frac{P(L_i\cap D_i(1,t))}{P(D_i(1,t))}\cdot P(S_i(t)\mid L_i\cap D_i(1,t))\cdot P(\lambda_1^iT_{i1}=t)$$

$$= \sum_{t=1}^{N} P(L_i \cap D_i(1,t)) \cdot P(S_i(t) \mid L_i \cap D_i(1,t)) \cdot P(\lambda_1^i T_{i1} = t).$$

From the independence principle of the searching process which done here, we have $P(S_i(t) \mid L_i \cap D_i(1,t)) = \xi_i$ where ξ_i is the probability of detecting the target in $[\beta_{i-1}, \alpha_i]$ and $P(L_i \cap D_i(1,t)) = P(L_i) = p_i = F_X(\alpha_i) - F_X(\beta_{i-1})$. Then,

$$\Gamma_{i1} = \xi_i (F_X(\alpha_i) - F_X(\beta_{i-1})) \sum_{t=1}^{N} P(\lambda_1^i T_{i1} = t).$$

In case of complete discovery of the target, we have $\sum_{t=1}^{N} P(\lambda_1^i T_{i1} = t) = 1$. As a result of the target distribution in $[\beta_{i-1}, \alpha_i]$, we get $\Gamma_{i1} = \xi_i (F_X(\alpha_i) - F_X(\beta_{i-1}))$.

Since the target cannot be found in the earlier search steps $j-1, j-2, \dots, 2, 1$ if the detection is carried out at search step j, $j = 2, 3, \dots, N$, then we have

$$\begin{aligned}
\Gamma_{ij} &= P[H_{ij}] \\
&= \sum_{t_j = t_{j-1}+1}^{N} \sum_{t_{j-1} = t_{j-2}+1}^{N-1} \cdots \sum_{t_1=1}^{N-j+1} P(H_{ij}, H_{i(j-1)}^c, \dots, H_{i1}^c \mid \lambda_j^i T_{i1} = t_1, \dots, \lambda_{j-1}^i T_{i(j-1)} = t_{j-1}, \lambda_j^i T_{ij} = t_j) \\
&\qquad \times P(\lambda_1^i T_{i1} = t_1, \dots, \lambda_{j-1}^i T_{i(j-1)} = t_{j-1}, \lambda_j^i T_{ij} = t_j) \\
&= \sum_{t_j = t_{j-1}+1}^{N} \sum_{t_{j-1} = t_{j-2}+1}^{N-1} \cdots \sum_{t_1=1}^{N-j+1} P(L_i, S_i(t_j), S_i^c(t_{j-1}), \dots, S_i^c(t_1), D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times P(\lambda_1^i T_{i1} = t_1, \dots, \lambda_{j-1}^i T_{i(j-1)} = t_{j-1}, \lambda_j^i T_{ij} = t_j) \\
&= \sum_{t_j = t_{j-1}+1}^{N} \sum_{t_{j-1} = t_{j-2}+1}^{N-1} \cdots \sum_{t_1=1}^{N-j+1} P(D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times P(L_i \mid D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times P(S_i(t_j) \mid L_i, S_i^c(t_{j-1}), \dots, S_i^c(t_1), D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times P(S_i^c(t_{j-1}) \mid L_i, S_i^c(t_{j-2}), \dots, S_i^c(t_1), D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times \dots \times P(S_i^c(t_1) \mid L_i, D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times P(\lambda_1^i T_{i1} = t_1, \dots, \lambda_{j-1}^i T_{i(j-1)} = t_{j-1}, \lambda_j^i T_{ij} = t_j) \\
&= \sum_{t_j = t_{j-1}+1}^{N} \sum_{t_{j-1} = t_{j-2}+1}^{N-1} \cdots \sum_{t_1=1}^{N-j+1} P(L_i, D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times P(S_i(t_j) \mid L_i, S_i^c(t_{j-1}), \dots, S_i^c(t_1), D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times P(S_i^c(t_{j-1}) \mid L_i, S_i^c(t_{j-2}), \dots, S_i^c(t_1), D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times \dots \times P(S_i^c(t_1) \mid L_i, D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times P(L_i, D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) \\
&\qquad \times P(\lambda_1^i T_{i1} = t_1, \dots, \lambda_{j-1}^i T_{i(j-1)} = t_{j-1}, \lambda_j^i T_{ij} = t_j),
\end{aligned}$$

where H_{ij}^c and $S_i^c(t)$ are the complement events of H_{ij} and $S_i(t)$, respectively.

Also in each sub-interval, we have

$$P(S_i(t_j) \mid L_i, S_i^c(t_{j-1}), \dots, S_i^c(t_1), D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) = \xi_i,$$

(independence principle of the searching process). This leads to,

$$P(S_i^c(t_{j-1}) \mid L_i, S_i^c(t_{j-2}), \dots, S_i^c(t_1), D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j))$$

$$= P(S_i^c(t_{j-2}) \mid L_i, S_i^c(t_{j-1}), \dots, S_i^c(t_1), D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j))$$
$$\vdots$$
$$= P(S_i^c(t_1) \mid L_i, D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) = 1 - \xi_i.$$

Also, we get $P(L_i, D_i(1,t_1), \dots, D_i(t_{j-2}, t_{j-1}), D_i(t_{j-1}, t_j)) = P(L_i) = p_i = F_X(\alpha_i) - F_X(\beta_{i-1})$ when the detection done in $[\beta_{i-1}, \alpha_i]$. Consequently,

$$\Gamma_{ij} = \xi_i (F_X(\alpha_i) - F_X(\beta_{i-1}))(1-\xi_i)^{j-1} \cdot \sum_{t_j = t_{j-1}+1}^{N} \sum_{t_{j-1} = t_{j-2}+1}^{N-1} \dots \sum_{t_1=1}^{N-j+1} P(\lambda_1^i T_{i1} = t_1, \dots, \lambda_{j-1}^i T_{i(j-1)} = t_{j-1}, \lambda_j^i T_{ij} = t_j).$$

If the target will be detected at jth search, then we have

$$\sum_{t_j = t_{j-1}+1}^{N} \sum_{t_{j-1} = t_{j-2}+1}^{N-1} \dots \sum_{t_1=1}^{N-j+1} P(\lambda_1^i T_{i1} = t_1, \dots, \lambda_{j-1}^i T_{i(j-1)} = t_{j-1}, \lambda_j^i T_{ij} = t_j).$$

Then the probability of detecting the target after the jth search, is given by

$$\Gamma_{ij} = \xi_i (F_X(\alpha_i) - F_X(\beta_{i-1}))(1-\xi_i)^{j-1}. \tag{6}$$

3.2 Maximizing the Detection Probability

We can utilize the conditional probability of identifying the target at search step j, given that it is hidden in $[\beta_{i-1}, \alpha_i]$ as a detection function $1 - \Omega(i, j, Z_{ij}) = 1 - e^{-\frac{Z_{ij}}{\phi_j}}$ (see, Hong *et al.* [12, 13]), where the nature of the search area (sub-interval) at the jth search determines a factor called ϕ_j, as the searcher is free to jump on these sub-intervals to search them with the effort Z_{ij}. We obtain the probability of finding the target by

$$\Psi(Z) = \sum_{i=1}^{K} \sum_{j=1}^{N} \Gamma_{ij} (1 - e^{-\frac{Z_{ij}}{\phi_j}}) = \sum_{i=1}^{K} \sum_{j=1}^{N} \xi_i (F_X(\alpha_i) - F_X(\beta_{i-1}))(1-\xi_i)^{j-1} (1 - e^{-\frac{Z_{ij}}{\phi_j}}), \tag{7}$$

disregarding the manner in which the search effort is applied.

By combining λ_j with (7), we need to solve the following optimization problem (P1), to maximize detection probability at the jth search in the sub-interval $[\beta_{i-1}, \alpha_i]$,

$$\text{P1}: \quad \max_{\xi_i} \Psi(Z) = \sum_{i=1}^{K} \sum_{j=1}^{N} \lambda_j^i \xi_i (F_X(\alpha_i) - F_X(\beta_{i-1}))(1-\xi_i)^{j-1} (1 - e^{-\frac{Z_{ij}}{\phi_j}})$$
$$\text{subject to } \Phi(Z) = \sum_{i=1}^{K} \sum_{j=1}^{N} \lambda_j^i Z_{ij} = \left\{ Z_{ij} \in R^{NK} \,\middle|\, \Phi(Z) = \sum_{i=1}^{K} \Phi_i(Z) = \sum_{i=1}^{K} \sum_{j=1}^{N} \lambda_j^i Z_{ij} \le W \right\},$$
$$\sum_{j=1}^{N} \Gamma_{ij} = 1, \quad Z_{ij} \ge 0,\ 0 < \lambda_j < 1,\ i = 1, 2, \dots, K,\ j = 1, 2, \dots, N,$$

where the feasible set of limited choices is denoted by R^{NK}. The convexity of $\Psi(Z)$ and $Z(w)$, which allows us to apply the Kuhn-Tucker conditions covered in Mangasarian [17] to get the greatest value of ξ_i, ensures the unique solution. However, doing so will increase Z_{ij}'s effort in every jth search sub-interval. By simultaneously decreasing the likelihood of non-detection (which is equal to maximizing the chance of detection) and Z_{ij}, the decision variable, we may avoid this contradiction.

We take into consideration $P(\Phi_i(Z) \le w_i) \le \beta$, where $w_i \in W$, $i = 1,2,\dots,N$ and β is the probability of the exerted effort, since the distribution of the effort to search for the target varies from one sub-interval to the next. We have $P\left(\frac{\Phi_i(Z)-\mu}{\sigma} \le \frac{w_i-\mu}{\sigma}\right) \le \beta_i$, if W has a normal distribution with mean μ and variance σ^2. As a result, $\frac{w_i-\mu}{\sigma}$, $i = 1,2,\dots,N$ is a standard Normal random variable. We obtain the constraint $\psi\left(\frac{\Phi_i(Z)-\mu}{\sigma}\right) \le \psi(A_i^{(p)})$ if A_p is the value of $\frac{W-\mu}{\sigma}$ at which $\psi(A_i^{(p)}) = \beta_i$. As a result, we have $\Phi_i(Z) - \mu \le \sigma A_i^{(p)}$. Additionally, we need to minimize the probability of going no detection (which equivalent to maximization of detection probability); to do this, we employ the $w_j(i)$, complement function, or $1-\lambda_j^i$. No, we may formulate an equivalent nonlinear discrete optimization problem P2 based on these constraints:

$$\text{P2}: \quad \min_{Z_{ij}} \Psi(Z) = \prod_{j=1}^{N}\sum_{i=1}^{K}(1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}e^{-\frac{Z_{ij}}{\phi_j}}$$

$$\text{subject to } B(w_i) = \{Z_{ij} \in R^{NK} \mid g_i(Z_{ij}) = \Phi_i((1-\lambda_j^i)Z_{ij}) - \mu - \sigma A_i^{(p)} \le 0\},$$

$$\sum_{j=1}^{N}[\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}] = 1,$$

$$Z_{ij} \ge 0,\ \lambda_j - 1 < 0,\ i = 1,2,\dots,K,\ j = 1,2,\dots,N.$$

We need to demonstrate the convexity of the set of feasible constraints $B(w_i)$, where $i = 1,2,\dots,K$, and it is evident that the detection function in P2 is a convex function where $e^{-\frac{Z_{ij}}{\phi_j}}$ is convex. We may now apply the Kuhn-Tucker conditions as in Mangasarian [17] because the functions $\Psi(Z)$ and $g_i(Z_{ij})$ are convex. Thus, for $U_s \ge 0$ (the Lagrange multipliers), we get

$$-\frac{\xi_q(1-\lambda_d^q)(F_X(\alpha_q)-F_X(\beta_{q-1}))(1-\xi_q)^{d-1}}{\phi_d} \times e^{-\frac{Z_{qd}}{\phi_d}}\prod_{\substack{j=1\\ q\ne j}}^{N}\sum_{i=1}^{K}(1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}e^{-\frac{Z_{ij}}{\phi_j}} + U_q = 0, \tag{8}$$

$$\Phi_s(Z) - \mu - \sigma A_s^{(p)} \le 0, \tag{9}$$

$$U_s(\Phi_s(Z) - \mu - \sigma A_s^{(p)}) = 0, \tag{10}$$

where $\Phi_s(Z) = \sum_{j=1}^{N}(1-\lambda_j^s)Z_{sj}$. We should examine a large number of cases for U_s in order to determine the minimal values of the search effort and the chance of not detection. First of all, $\sum_{j=1}^{N}(1-\lambda_j^s)Z_{sj} \le \mu + \sigma A_s^{(p)}$ and $\sum_{i=1}^{K}(1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}e^{-\sum_j \frac{Z_{ij}}{\phi_j}} = 0$ result if $U_s = 0$, $s = 1,2,\dots,K$. We have $\Gamma_{ij} > 0$ since the target hiding in the sub-interval $[\beta_{i-1}, \alpha_i]$ will be detected at the jth search. This results in an impossible case where $\sum_{i=1}^{K}(1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}e^{-\sum_j \frac{Z_{ij}}{\phi_j}} = 0$. We still get the same impossible case with $U_s = 0$, $s = 1,2,\dots,t$ and $U_s > 0$, $s = t+1, t+2,\dots,n$. As a result, $U_s > 0$, $s = t+1,t+2,\dots,n$ yields the optimal solution scenario that can be considered. Therefore, using equation (10), one can obtain $\sum_{j=1}^{N}(1-\lambda_j^s)Z_{sj} \le \mu + \sigma A_s^{(p)}$ and hence

$$Z_{sj} = \ln\left[\frac{(1-\lambda_j^s)\xi_s(F_X(\alpha_s)-F_X(\beta_{s-1}))(1-\xi_s)^{j-1}}{\left[\left(\prod_{j=1}^{N}((1-\lambda_j^s)\xi_s(F_X(\alpha_s)-F_X(\beta_{s-1}))(1-\xi_s)^{j-1})^{\phi_j}\right)\left(\prod_{j=1}^{N}(\phi_j)^{\phi_j}\right)^{-1}e^{-l_s}\right]^{\frac{1}{C}}\phi_j}\right]^{\phi_j}, \tag{11}$$

where $C = \sum_{j=1}^{N}\phi_j$ and $l_s = \mu + \sigma A_s^{(p)} = \sum_{j=1}^{N}(1-\lambda_j^s)Z_{sj}$, $s = 1,2,\dots,K$.

The minimum amount of effort needed to find the target at the jth search, where $j = 1,2,\dots,N$, if it is hidden in the sub-interval $[\beta_{i-1},\alpha_i]$, is determined from (11) by

$$\begin{aligned} Z_{ij}^* = \phi_j\Bigg\{&\ln((1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}) \\ &-\frac{1}{C}\sum_{j=1}^{N}\phi_j\ln\left(\frac{(1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}}{T_j}\right)+\frac{l_i}{C}-\ln\phi_j\Bigg\}. \end{aligned} \tag{12}$$

As from (12), the minimal probability of the non-detection function will now become

$$\begin{aligned} \Psi^*(Z) = \prod_{j=1}^{N}\sum_{i=1}^{K}&\frac{((1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1})^2}{\phi_j} \\ &\cdot\exp\left[\frac{l_i-\sum_{j=1}^{N}\phi_j\ln\left(\frac{(1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}}{\phi_j}\right)}{C}\right]. \end{aligned} \tag{13}$$

Since the target is hidden in $[\beta_{i-1},\alpha_i]$, then by the complement of $\Psi^*(Z)$ (which is represented by $\widetilde{\Psi}(Z) = 1-\Psi^*(Z)$) one may therefore determine the maximum probability of detection at search step j. We have $\phi_j = 1$, $j = 1,2,\dots,N$ and

$$Z_{ij}^{**} = \ln((1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}) - \ln\prod_{j=1}^{N}G_i + \frac{l_i}{N}, \tag{14}$$

$$\Psi^{**}(Z) = \prod_{j=1}^{N}\sum_{i=1}^{K}\frac{((1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1})^2}{G_i}e^{\frac{l_i}{N}}, \tag{15}$$

when the searching sub-intervals are the same, where

$$G_i = ((1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1})^{\frac{1}{N}}$$

is the Geometric mean.

3.3 Influencing of Discount Effort Reward Parameter on the Minimum Search Effort Stability

Since the search conditions for the hidden target vary by sub-interval, the effort distribution will be unstable. Therefore, studying the stability of the solution based on the discount effort parameter is one of great importance to ensure the optimal distribution of minimum search effort.

The stability set of the minimum search effort is defined as follows: $S(Z^*_{ij}) = \{w_i \in B(w_i) \mid Z^*_{ij}$ is an optimal solution of P2, where $Z^*_{ij} \geq 0$ corresponds to $w_i \in B(w_i)$. Furthermore, this ideal solution yields the optimal value of $\Psi^*(Z)$ such that $0 \leq \Psi^*(Z) \leq 1$. As a result, we may obtain the first stability condition that yields the minimum amount of effort by

$$l_i \leq C \ln((1-\lambda^i_j)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}) \\ - \sum_{j=1}^{N} \phi_j \ln\left(\frac{(1-\lambda^i_j)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}}{\phi_j}\right) + C \ln T_j, \tag{16}$$

if $Z^*_{ij} \geq 0$. Furthermore, since $0 \leq \Psi^*(Z) \leq 1$ then $\ln\{\Psi(Z_{ij})\} \leq 0$, this allows to obtain the second condition, which provides the lowest probability of not detection as in the following:

$$\ln \prod_{j=1}^{N} \sum_{i=1}^{K} \frac{((1-\lambda^i_j)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1})^2}{\phi_j} \\ \cdot \exp\left[\frac{l_i - \sum_{j=1}^{N} \phi_j \ln\left(\frac{(1-\lambda^i_j)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}}{\phi_j}\right)}{C}\right] \leq 0.$$

Then,

$$\sum_{i=1}^{K} \sum_{j=1}^{N} \ln\left[\frac{((1-\lambda^i_j)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1})^2}{\phi_j} \\ \cdot \exp\left[\frac{l_i - \sum_{j=1}^{N} \phi_j \ln\left(\frac{(1-\lambda^i_j)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}}{\phi_j}\right)}{C}\right]\right] \leq 0.$$

Hence,

$$\sum_{i=1}^{K} \sum_{j=1}^{N} \ln\left[\frac{((1-\lambda^i_j)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1})^2}{\phi_j}\right] \\ \leq NC^{-1} \sum_{i=1}^{K} \sum_{j=1}^{N} \phi_j \ln\left(\frac{(1-\lambda^i_j)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}}{\phi_j}\right) - NC^{-1} \sum_{i=1}^{K} l_i.$$

This gives,

$$\sum_{i=1}^{K} l_i \leq \sum_{i=1}^{K} \sum_{j=1}^{N} \left[\phi_j \ln\left(\frac{(1-\lambda^i_j)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}}{\phi_j}\right) \\ - \frac{C}{N} \ln\left[\frac{((1-\lambda^i_j)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1})^2}{\phi_j}\right]\right], \tag{17}$$

which directly affects the stability for the maximum possible probability of detection. Therefore, the stability set comes from equations (16) and (17),

$$S(Z^*_{ij}) = \left\{\forall\ i = 1,2,\ldots,K,\ w_i \in B(w_i) \,\middle|\, \sum_{j=1}^{N} (1-\lambda^i_j) Z^*_{ij} = \mu + \sigma A^{(p)}_i,\right.$$

$$
\begin{aligned}
l_i &\leq C\ln\Bigg((1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}\Bigg) \\
&\quad -\sum_{j=1}^{N}\phi_j\ln\left(\frac{(1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}}{\phi_j}\right)+C\ln\phi_j, \\
\sum_{i=1}^{K} l_i &\leq \sum_{i=1}^{K}\sum_{j=1}^{N}\left[\phi_j\ln\left(\frac{(1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}}{\phi_j}\right)\right. \\
&\quad \left.\left. -\frac{C}{N}\ln\left[\frac{((1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1})^2}{\phi_j}\right]\right]\right\}.
\end{aligned} \tag{18}
$$

When sub-intervals are identical, the minimum search effort's stability set under the influence of the discount parameter becomes

$$
\begin{aligned}
\widetilde{S}(Z_{ij}^{**}) = \Bigg\{\forall\ i, w_i \in B(w_i) \,\Bigg|\, &\sum_{j=1}^{N}(1-\lambda_j^i)Z_{ij}^{**} = \mu+\sigma A_i^{(p)}, \\
&l_i \leq N[\ln((1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1})-\ln G_i], \\
&\sum_{i=1}^{K} l_i \leq \sum_{i=1}^{K}\sum_{j=1}^{N}\ln\left(\frac{1}{(1-\lambda_j^i)\xi_i(F_X(\alpha_i)-F_X(\beta_{i-1}))(1-\xi_i)^{j-1}}\right)\Bigg\}.
\end{aligned} \tag{19}
$$

4. Application

In light of the recent earthquake that struck Turkey and Syria (February 2023), reports from the Turkish Disaster and Emergency Management Presidency indicate that most of the victims pulled out from under the rubble died because the stairs collapsed on them while they were trying to escape, while those who survived stayed where they were. Assuming that victims (target) were reported trapped in a specific area $[a,b]=[-10,10]$ on a street. This poses a major challenge to our probabilistic search model and its statistical processing, as the target is randomly placed in the search area. Additionally, we consider a target that is randomly located on the line in accordance with a normal distribution (see [1]) with mean $\tilde{\mu}=2$ and variance $\tilde{\sigma}^2=10$. The target probability distribution of the new random variable Y in the interval $[-10,10]$ with the following probability density and cumulative distribution functions,

$$
q_Y(y)=\begin{cases}\frac{1}{7.881}\exp\left[-\frac{(y-2)^2}{10}\right], & \text{if } -10\leq y\leq 10,\\ 0, & \text{otherwise,}\end{cases} \tag{20}
$$

$$
Q_Y(y)=\begin{cases}\frac{1}{0.994}\left(\frac{1}{2}\left[1+\operatorname{erf}\left(\frac{y-2}{\sqrt{20}}\right)\right]-0.000074\right), & \text{if } -10\leq y\leq 10,\\ 0, & \text{otherwise,}\end{cases} \tag{21}
$$

which are obtained from (1) and (2) respectively by double truncating the original random variable of the target distribution on the entire street. Given the assumption that the searcher would obtain positional data about the target, we can partition the search region (interval $[-10,10]$) into a specified number of sub-intervals in order to maximize the detection probability.

[1] *Normal Distribution*, Wikipedia, URL: https://en.wikipedia.org/wiki/Normal_distribution.

Next, the target should be redistributed to the areas with the highest probability, excluding a number of sub-intervals where the target's probability is low. The computational target probability in the remaining three sub-intervals will therefore depend on the distribution of a new random variable X (using (4)), whose cumulative distribution function is provided by,

$$F_X(x)=\begin{cases}\frac{1}{0.397}\left(\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{x-2}{\sqrt{20}}\right)\right]-0.000074\right), & -10<x<-5,\\ \frac{1}{0.3995}\left(\frac{1}{0.994}\left(\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{x-2}{\sqrt{20}}\right)\right]-0.000074\right)-0.365\right), & 1<x<4,\\ \frac{1}{0.3995}\left(\frac{1}{0.994}\left(\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{x-2}{\sqrt{20}}\right)\right]-0.000074\right)-0.6005\right), & 8<x<10,\end{cases} \tag{22}$$

if two sub-intervals $[\alpha_1,\beta_1]=[-5,1]$ and $[\alpha_2,\beta_2]=[4,8]$ are removed from the search area. Furthermore, we set $\xi_1=0.2$, $\xi_2=0.6$, and $\xi_3=0.4$ as the probabilities of finding the target in the searching sub-intervals $[-10,-5]$, $[1,4]$, and $[8,10]$, with the discount parameters $\lambda_j^1=0.7$, $\lambda_j^2=0.8$, and $\lambda_j^3=0.9$, respectively. The probability of finding the target in $[-10,-5]$, $[1,4]$, and $[8,10]$ are $\Gamma_{1j}=0.00201\times(0.8)^{j-1}$, $\Gamma_{2j}=0.1962\times(0.4)^{j-1}$, $\Gamma_{3j}=0.07448\times(0.6)^{j-1}$, respectively, if it is found at the jth search, $j=1,2,3$. We additionally allow $\phi_j=1$, $j=1,2,3$ and $A_i^{(p)}$, $i=1,2,3$ to take the values $\{3,4,5\}$, respectively, and suppose that the random variable of the search effort W has a Normal distribution with a mean $\mu=0.82$ and variance $\sigma^2=0.04$. Table 1 shows the optimal values of Z_{ij}^*, $i=1,2,3$, $j=1,2,3$ that are derived from equation (12) and, consequently, the minimum detection function $\Psi^*(Z)$ from equation (13). Even though the target is present in any of these sub-intervals, the searching process might not yield any results in any of them. As a result, repeat searches over the same sub-intervals are feasible. Additionally, when a searcher enters a sub-interval for the first time, it is typical for the search to be intensive. Consequently, we see that, as the number of searching times grows, the searching effort lessens upon entering, as shown in Table 1.

Table 1. $\Psi^*(Z)$ corresponding to Z_{ij}^*, $i=1,2,3$, $j=1,2,3$

i	j	Z_{ij}^*	$\Psi^*(Z)$
1	1	3.184	
	2	3.077	
	3	2.970	
2	1	1.405	
	2	0.819	0.087
	3	0.233	
3	1	0.520	
	2	0.340	
	3	0.159	

Because of the minimal effort required and the fact that a single sub-interval remains the same no matter how many times it is searched, Z_{ij}^* values are uniform. For instance, we see that the effort Z_{ij}^* has a maximum value that is somewhat larger than its value in other sub-intervals when Z_{ij}^*, $i=3$, $j=1,2,3$ is used. The probability that the target will be present leads to this. Given that the target is randomly located and that $0.520\le Z_{ij}^*\le 0.159$, Z_{ij}^*'s value lowers slightly, the probability of any event occurring in the presence of this homogeneity is approximately equal. Consequently, following jth search, we can ascertain each sub-interval's maximum probability of detection, from which we can choose whether or not to carry out another search for that particular sub-interval.

Naturally, at the beginning of the search process, searchers suggest the sub-interval with the highest probability of the target's existence. Hence, we note that the effort expended to search for the target gradually decreases with the increase in time as in Table 1. This also illustrates the usefulness of the discount parameter as it increases the probability of the target being present in subsequent search sub-intervals with less effort. Therefore, the value of Z^*_{ij} in sub-intervals represents the maximum probability of again entering. Furthermore, we found that $1-\Psi^*(Z) = 0.913$ is the greatest value of the detection probability function. That is a noteworthy probability that helps predict whether or not the searcher will go back to the sub-intervals.

5. Conclusion and Future Work

To find a hidden target on a line with a bounded interval, a novel technique has been presented. A collection of sub-intervals was created from this interval. To increase the probability of finding the target within the search sub-intervals, this division is made. The distribution of the target position and the sub-interval boundaries affect this probability. By applying El-Hadidy's [5] definition of the truncated distribution with N intervals eliminated, we may determine the probability of the target on each of the sub-intervals that were searched. The target is hidden within one of a series of these sub-intervals in the discrete search problem that was created from the original problem. Furthermore, by applying the discounted effort reward approach on the probability detection function, we save a significant amount of effort and raise the probability of finding the hidden target. A normal distributed random variable serves as the boundary for the search effort. We found a minimum search effort and a maximum detection probability by solving an intriguing discrete nonlinear optimization problem using the Kuhn-Tucker criteria. Depending on the discount parameter, we study the stability of the solution based on the discount effort parameter to get the great importance that ensure the optimal distribution of minimum search effort. To demonstrate the efficacy and practicality of our concept, an application with a normal distribution target position has been provided.

In future research, this model can be expanded to use multiple searchers on M bounded intervals of M real lines, each interval being subdivided into a distinct set of subintervals, in order to detect a set of hidden targets.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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