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Research Article

Study of Hybrid Nano Particles Flow in an Inclined Circular Artery

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Abstract. In the present article, steady fluid flow through an inclined tube of nonuniform cross section with stenosis and dilation has been investigated under the influence of hybrid nanofluid particles. The homotopy perturbation method is applied to solve the flow equations assuming mild stenosis to determine the axial velocity, wall shear stress, pressure gradient, and streamlines. Studies have examined how parameters effects on pressure gradient, impedance resistance λ and wall shear stress S_{rz}.

Keywords. Homotopy Perturbation Method, Hybrid Nanoparticles, Stenosis, Wall Shear Stress, Dilatation

Mathematics Subject Classification (2020). 76Z05,76A05

1. Introduction

Stenosis is a *Common Cardiovascular Disease* (CVD) where blood vessels become narrowed or constricted, hindering the normal flow of blood. This condition typically develops due to the gradual build-up of fatty deposits, cholesterol, and other matter on the inner walls of arteries, a process known as atherosclerosis. As stenosis progresses, it can have serious consequences. If the stenosis becomes severe, it may result in less blood flow to the heart, causing chest pain or potentially leading to a heart attack. In cases of stenosis in the carotid arteries (arteries in the neck that supply blood to the brain), it can result in a cerebral stroke.

There have been several theoretical and experimental studies conducted on blood flow under different situations in the stenotic region (Young [19], Padmanabhan [14], Misra *et al*. [13], and Roy *et al*. [16]). Most of these investigations have modelled blood as a Newtonian fluid, and it is accepted that blood, as a cell suspension, behaves as a non-Newtonian fluid at low shear rates in small-diameter channels.

Post-stenotic dilatation, defined as arterial expansion downstream to a stenosis, is influenced by hemodynamic factors such as high flow rates and wall shear stress observing the constriction (Tandon *et al*. [18]). Some people have a weak nervous system (particularly the elderly). When blood clots in a specific position, the arterial wall bulges out (due to increased pressure). If it continues to rise, it may cause damage to the arterial walls, which also leads to death.

Hybrid Nanofluids are advanced fluids made by suspending two or more types of nanosized particles, such as metals, metal oxides, or their mixtures, in a base fluid. They provide superior thermal conductivity and heat transfer efficiency compared to single-particle nanofluids. They are widely used in heat pipes, heat exchangers, and micro-channels.

Many researchers have extended the use of hybrid nanoparticles, especially in studying blood flow through narrowed arteries. Ardahaei *et al*. [3] investigate their properties, while Das *et al*. [7] examined how they behave in blood flow with porous materials under magnetic fields and heat. Other researchers, like Abo-Elkhair *et al*. [2], Sreedevi *et al*. [17], Dolui *et al*. [9], looked at medical uses, including drug delivery and treating heart and lung conditions. Further investigation by Abdelsalam *et al*. [1], Basha *et al*. [4], and Zaman *et al*. [20] focused on how these nanofluids behave in arteries.

The flow of hybrid nanoparticles (Cu-CuO/blood) through constricted arteries under strong electromagnetic fields has not been investigated in previous research. We model electro-magneto-hydrodynamic flow in a porous tube while taking Hall current effects into consideration in order to close this gap.

Assuming a low Reynolds number and mild stenosis, the governing equations for the blood flow model are simplified. The homotopy perturbation method is then used to analytically solve the normalized momentum equation. This study explores how key parameters affect physiological flow properties, comparing the behavior of hybrid nanoblood (Cu-CuO/blood) with conventional nanoblood (Cu-blood). The results are visualized and analyzed through detailed graphs, offering key insights into flow dynamics under these specific conditions.

2. Mathematical Description of the Problem

Suppose a steady, two-dimensional, incompressible fluid flow in an inclined axisymmetric artery with stenosis and dilatation over a finite length L and it is concentrated with Hybrid Nanoparticles (Cu-CuO NPs). Cylindrical polar coordinates $(\widetilde{r}_d, \widetilde{\theta}, \widetilde{z})$ are chosen so that the z-axis coincides with the tube centreline and, variation $\widetilde{\theta}$ is neglected due to axisymmetric flow. Let 'α' be an inclined angle of the tube to the horizontal axis as shown in Figure 1. A constant magnetic field B_0 is applied in the radial direction. A uniform temperature (T_0) is maintained for the stenosis artery. The nanoparticles of Copper (Cu) and copper oxide (CuO) were mixed with blood to form the hybrid nanofluid. Heat transfer is examined with the assumption that the arterial wall temperature remains constant.

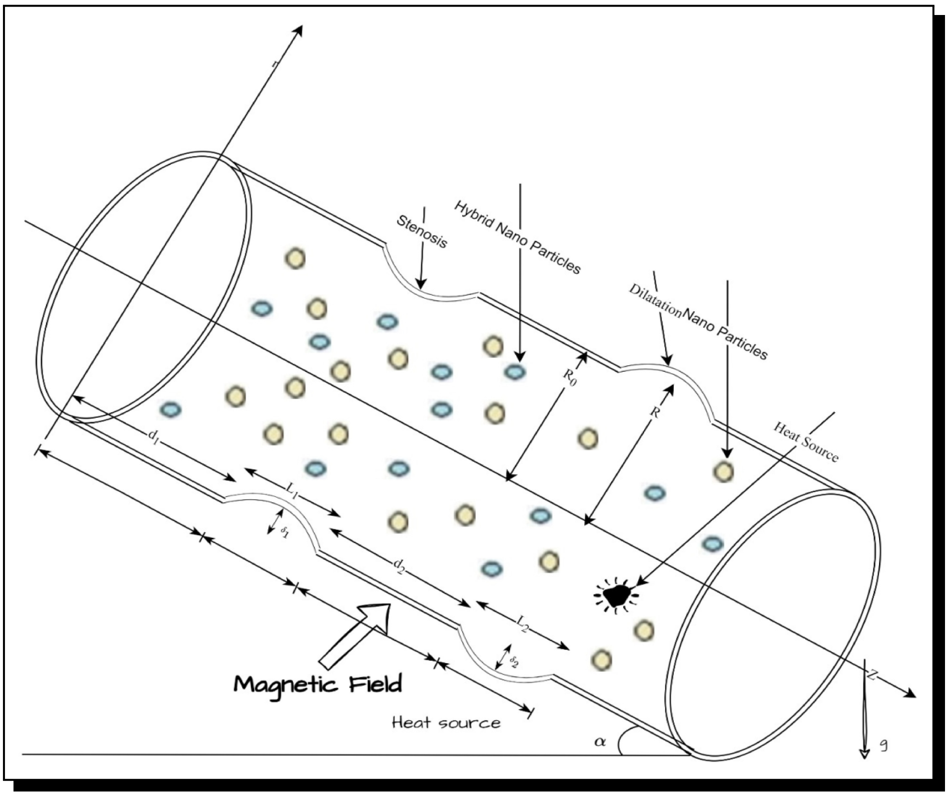


Figure 1. Geometry of the discussed problem

The equation pertaining to the geometry of the wall is given by (Prasad *et al.* [15]) as

$$h(\tilde{z}) = \frac{R(\tilde{z})}{R_0} = \begin{cases} 1 - \frac{\bar{\delta}_i}{2R_0}\left[1 + \cos\frac{2\pi}{L_i}\left(\tilde{z} - \alpha_i - \frac{L_i}{2}\right)\right], & \alpha_i \le \tilde{z} \le \beta_i, \\ 1, & \text{otherwise}, \end{cases} \tag{1}$$

where $\bar{\delta}_i$ denotes the maximum distance from the ith unusual sections that extends into the artery lumen. It is negative for aneurysms and positive for stenosis. The artery's radius and the radius of the normal artery are represented by R_0 and R, respectively. The length of the ith abnormal segment by L_i, and the distance from the starting point to the start of the ith unusual sections, which is represented as α_i is given as:

$$\alpha_i = \sum_{j=1}^{i}(d_j + L_j) - L_i\,. \tag{2}$$

The distance from the starting point to the the ith unusual sections is denoted by β_i and it is defined as

$$\beta_i = \sum_{j=1}^{i}(d_j + L_j), \tag{3}$$

where the distance from the origin to the end of the ith unusual portion from the end of the $(i-1)$th section is represented by d_i. Das *et al.* [7], and Cowling [6] gives the formula for Ohm's law applied to partly ionized fluid, which is represented as

$$\mathrm{K} + \frac{\upsilon_e\theta_e}{\mathrm{H}_0}(\mathrm{K}\times\mathrm{H})\sigma_{HNf}(\varepsilon + q\times\mathrm{H}) \tag{4}$$

In this formulation, $q, \mathrm{H}, \varepsilon, \mathrm{K}, \sigma_{HNf}, v_e, \theta_e$ represent the velocity, magnetic field, electric field, current density vectors, effective electrical conductivity of the hybrid nanofluid, the electron collision frequency, and time, respectively.

Electromagnetic influences such as ion slip, thermoelectric effects, and electron pressure gradients are assumed negligible. The left-hand side of the equation includes two components: electron-ion drag and Hall currents, the latter resulting from the Lorentz force. Under weak magnetic fields, Hall effects are minimal, allowing their exclusion. However, in strong fields, Hall currents significantly influence electromagnetic forces by altering both the direction and magnitude of current density, making them critical to analyze in blood flow studies.

Given the assumptions stated above, equation (4) for the generalized Ohm's law becomes

$$\mathrm{K}_{\widetilde{r}_d} + m\,\mathrm{K}_{\widetilde{z}} = \sigma_{HNf}(\varepsilon_{\widetilde{r}_d} + \widetilde{v}\mathrm{H}_0), \tag{5}$$

$$\mathrm{K}_{\widetilde{r}_d} - m\mathrm{K}_{\widetilde{z}} = \sigma_{HNf}(\varepsilon_{\widetilde{z}} - \widetilde{\chi}\mathrm{H}_0), \tag{6}$$

Here $(\widetilde{v}, \widetilde{\chi})$, $(\mathrm{K}_{\widetilde{r}_d}, \mathrm{K}_{\widetilde{z}})$, $(\varepsilon_{\widetilde{r}_d}, \varepsilon_{\widetilde{z}})$ represents velocity, current density, and electric field in radial and axial components, respectively. The Hall parameter, defined as $m = ve\theta_e$, is the ratio of the electron collision frequency to the electron-atom collision frequency. The condition $F = 0$ indicates the absence of applied and polarization voltages.

Solving equation (5) and equation (6) for $\mathrm{K}_{\widetilde{r}_d}$ and $\mathrm{K}_{\widetilde{z}}$ yields

$$\mathrm{K}_{\widetilde{r}_d} = \frac{\sigma_{HNf}\mathrm{H}_0}{1+m^2}(\widetilde{v} + m\widetilde{\chi}), \tag{7}$$

$$\mathrm{K}_{\widetilde{z}} = \frac{\sigma_{HNf}\mathrm{H}_0}{1+m^2}(m\widetilde{v} - \widetilde{\chi}), \tag{8}$$

Changdar and De [5], Das *et al*. [8], and Ijaz *et al*. [12] provide momentum and energy equations for blood flow with suspended hybrid nanoparticles in a symmetric artery based on the previously mentioned assumptions and employing Boussinesq's approximation as:

$$\frac{1}{\widetilde{r}_d}\frac{\partial}{\partial \widetilde{r}_d}(\widetilde{r}_d\widetilde{\chi}) + \frac{\partial}{\partial \widetilde{z}}(\widetilde{v}) = 0, \tag{9}$$

$$\rho_{HNf}\left(\widetilde{\chi}\frac{\partial \widetilde{\chi}}{\partial \widetilde{r}_d} + \widetilde{v}\frac{\partial \widetilde{\chi}}{\partial \widetilde{z}}\right) = -\frac{\partial \widetilde{p}}{\partial \widetilde{r}_d} + \mu_{HNf}\left(\frac{\partial^2 \widetilde{\chi}}{\partial \widetilde{r}_d^2} + \frac{1}{\widetilde{r}_d}\frac{\partial \widetilde{\chi}}{\partial \widetilde{r}_d} + \frac{\partial^2 \widetilde{\chi}}{\partial \widetilde{z}^2} - \frac{\widetilde{\chi}^2}{\widetilde{r}_d^2}\right) + g(\rho\beta)_{HNf}(\widetilde{T} - T_0)\cos\alpha, \tag{10}$$

$$\rho_{HNf}\left(\widetilde{\chi}\frac{\partial \widetilde{v}}{\partial \widetilde{r}_d} + \widetilde{v}\frac{\partial \widetilde{v}}{\partial \widetilde{z}}\right) = -\frac{\partial \widetilde{p}}{\partial \widetilde{z}} + \mu_{HNf}\left(\frac{\partial^2 \widetilde{v}}{\partial \widetilde{r}_d^2} + \frac{1}{\widetilde{r}_d}\frac{\partial \widetilde{v}}{\partial \widetilde{r}_d} + \frac{\partial^2 \widetilde{v}}{\partial \widetilde{z}^2} - \frac{\widetilde{v}}{k^*}\right) + g(\rho\beta)_{HNf}(\widetilde{T} - T_0)\sin\alpha - \frac{\sigma_{HNf}B_0^2}{1+m^2}(\widetilde{v} + m\widetilde{\chi}), \tag{11}$$

$$(\rho c_p)_{HNf}\left(\widetilde{\chi}\frac{\partial \widetilde{T}}{\partial \widetilde{r}_d} + \widetilde{v}\frac{\partial \widetilde{T}}{\partial \widetilde{z}}\right) = k_{HNf}\left(\frac{\partial^2 \widetilde{T}}{\partial \widetilde{r}_d^2} + \frac{1}{\widetilde{r}_d}\frac{\partial \widetilde{T}}{\partial \widetilde{r}_d} + \frac{\partial^2 \widetilde{T}}{\partial \widetilde{z}^2}\right) + Q_0. \tag{12}$$

Here, the subscripts denote the partial derivatives:

$$(\ )_r = \frac{\partial}{\partial r}, \quad (\ )_z = \frac{\partial}{\partial z}, \quad (\ )_{rr} = \frac{\partial^2}{\partial r^2}, \quad (\ )_{zz} = \frac{\partial^2}{\partial z^2},$$

where $\widetilde{\chi}$, $\widetilde{v}$ are the velocity components in the radial $\widetilde{r}_d$ and axial $\widetilde{z}$ directions. Also $\widetilde{T}, \widetilde{p}$ denote blood temperature and blood pressure. Q_0 is the constant heat source, whereas g the acceleration due to gravity. k^* is the permeability of the porous media, ρ_{HNf} the density and μ_{HNf} denotes the dynamic viscosity. σ_{HNf} denotes electrical conductivity, β_{HNf} the thermal expansion coefficient, $(\rho c_p)_{HNf}$ represents heat capacitance, and k_{HNf} indicates the thermal conductivity of Cu-CuO/blood.

According to the geometry of the problem the boundary conditions are

$$\left.\begin{array}{ll} \frac{\partial \tilde{\chi}}{\partial \tilde{r}_d}=0,\ \frac{\partial \tilde{T}}{\partial \tilde{r}_d}=0 & \text{at } \tilde{r}_d=0, \\ \tilde{\chi}=0,\ \tilde{T}=T_0 & \text{at } \tilde{r}_d=\frac{R(\tilde{z})}{R_0}. \end{array}\right\} \tag{13}$$

Introducing the non-dimensional variables

$$r=\frac{\tilde{r}_d}{R_0},\ z=\frac{\tilde{z}}{L_0},\ d=\frac{d_0}{L_0},\ \delta=\frac{\delta_0}{R_0},\ u=\frac{\tilde{\chi}_0}{\delta U_0},\ w=\frac{\tilde{v}}{U_0},\ p=\frac{\tilde{p}R_0^2}{U_0L_0\mu_f},\ \theta=\frac{\tilde{T}-T_0}{T_0}, \tag{14}$$

where U_0 is used as the reference velocity.

In microvascular systems, the flow velocity in the radial direction is assumed to be negligible compared to its axial component. Under the low Reynolds number approximation, the axial viscous term is ignored. Owing to the minimal radial velocity, the pressure gradient's radial variation $\frac{\partial \tilde{p}}{\partial \tilde{z}}$ within the artery can be ignored. Hence, the axial pressure gradient $\frac{\partial \tilde{p}}{\partial \tilde{z}}$ is treated as a function of the axial coordinate $\tilde{z}$ only.

Substituting equation (14) into equations (10)-(12) and applying the mild stenosis hypothesis ($\delta \ll 1$) alongside the flow constraint $\frac{L_0}{R_0} \sim O(1)$, where the stenotic length is significantly smaller than the artery's radius (Das *et al*. [8]).

The governing equations for blood flow reduce to the following dimensionless form as:

$$\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\left(\frac{1}{Da}+\frac{\xi_3}{\xi_1}M^2\right)w-\frac{1}{\xi_1}\left(\frac{\partial p}{\partial z}+\theta\,\xi_4 Gr\sin\alpha\right)=0, \tag{15}$$

$$\frac{\partial^2 \theta}{\partial r^2}+\frac{1}{r}\frac{\partial \theta}{\partial r}+\frac{S}{\xi_5}=0, \tag{16}$$

where

$M^2=\dfrac{\sigma_f B_0^2 R_0^2}{\mu_f}$ denotes the 2nd power of the squared Hartmann number,

$Da=\dfrac{k^*}{R_0^2}$ the Darcy number,

$Gr=\dfrac{g\beta_f R_0^2 T_0\rho_f}{U_0\mu_f}$ the Grashof number,

$Re=\dfrac{L_0U_0\rho_f}{\mu_f}$ the Reynolds number,

$S=\dfrac{Q_0R_0^2}{k_fT_0}$ the heat source parameter, and

$$\left.\begin{array}{l} \xi_1=\dfrac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}},\ \xi_2=(1-\phi_2)\left[(1-\phi_1)+\phi_1\dfrac{\rho_{s_1}}{\rho_f}\right]+\phi_2\frac{\rho_{s_2}}{\rho_f}, \\ \xi_3=\dfrac{\sigma_{HNf}}{\sigma_f},\ \xi_4=(1-\phi_2)\left[(1-\phi_1)+\phi_1\dfrac{(\rho\beta)_{s_1}}{(\rho\beta)_f}\right]+\phi_2\dfrac{(\rho\beta)_{s_2}}{(\rho\beta)_f},\ \xi_5=\dfrac{k_{HNf}}{k_f}. \end{array}\right\} \tag{17}$$

Since we employed the mathematical relationships between thermophysical properties of blood, copper Nanoparticles (Cu-NPs), and Copper Oxide nanoparticles (CuO-NPs), along with the thermophysical properties of nanofluids and hybrid nanofluids, from Ijaz *et al*. [12], Elnaqeeb *et al*. [10].

Here, the solid volume ratios of (Cu-NPs) and (CuO-NPs) are denoted by ϕ_1, ϕ_2. Further, f stands for base fluid which is blood, Nf and HNf stands for Cu-blood, and Cu-CuO/blood. Cu-NPs and CuO-NPs symbolizes s_1, s_2. Whereas the condition $\phi_1, \phi_2 = 0$ describes the base fluid, which corresponds to no Cu and CuO-NPs.

Making use of equation (14) in to equation (1) gives

$$h(z) = \frac{R(z)}{R_0} = \begin{cases} 1 - \frac{\delta_i}{2R_0}\left[1 + \cos\frac{2\pi}{L_i}\left(z) - \alpha_i - \frac{L_i}{2}\right)\right], & \alpha_i \le z \le \beta_i, \\ 1, & \text{otherwise.} \end{cases} \tag{18}$$

Dimensionless boundary conditions obtained as

$$\left.\begin{aligned} &\text{At } r = 0 &&: \frac{\partial w}{\partial r} = 0,\ \frac{\partial \theta}{\partial r} = 0 \\ &\text{At } r = h(z) &&: w = 0,\ \theta = 0 \end{aligned}\right\} \tag{19}$$

Using the boundary conditions (19), on solving equation (16) give the temperature function as:

$$\theta(r,z) = \frac{S}{4\xi_5}(h^2 - r^2). \tag{20}$$

3. Method of Solution

The momentum equation (15) takes the form

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + A(r^2 - h^2) - Bw - C = 0, \tag{21}$$

where

$$A = \frac{\xi_4 S Gr}{4\xi_1\xi_5}\sin\alpha, \quad B = \frac{1}{Da} + \frac{\xi_3 M^2}{\xi_1(1+m^2)}, \quad C = \frac{1}{\xi_1}\frac{dp}{dz}. \tag{22}$$

Following HPM, the homotopy equation is defined as:

$$\breve{H}(r,\breve{q}) = (1-\breve{q})[\breve{L}(\breve{w}) - \breve{L}(\breve{w}_o)] + \breve{q}[\breve{L}(\breve{w}) + A(r^2 - h^2) - B\breve{w} - C] = 0. \tag{23}$$

The initial guess $\breve{w}_0$ is defined by Das *et al*. [7] as

$$\breve{w}_o(r,z) = \frac{1}{r}(r^2 - h^2), \tag{24}$$

where, we have taken $\widehat{L} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$.

Define

$$\breve{w}(r,z) = \breve{w}_0 + \breve{w}_1\breve{q} + \breve{w}_2\breve{q}^2 + \dots. \tag{25}$$

By adopting the same procedure as described by Das *et al*. [7], the solution for the axial velocity $w(r,z)$ for $\breve{q} = 1$ can be expressed as

$$w(r,z) = w_0 + w_1 r^2 + w_2 r^4 + w_3 r^6, \tag{26}$$

where

$$\left.\begin{aligned} w_0 &= -\frac{C}{4}h^2 + \frac{3}{64}(BC - 4A)h^4 + \frac{19}{2304}B(B-4A)h^6, \\ w_1 &= \frac{C}{4} + \frac{1}{16}(4A - BC)h^2, \\ w_2 &= \frac{1}{256}[4(BC - 4A) - Bh^2], \\ w_3 &= \frac{1}{2304}(B^2 - 4AB). \end{aligned}\right\} \tag{27}$$

The stream function ψ describe the blood flow pattern is

$$w = \frac{1}{r}\frac{\partial \psi}{\partial r}. \tag{28}$$

Integrating equation (28) and apply the boundary condition $\psi = 0|_{r=0}$ gives the resulting stream function as

$$\psi(r,z) = \frac{1}{2}w_0 r^2 + \frac{1}{4}w_1 r^4 + \frac{1}{6}w_2 r^6 + \frac{1}{8}w_3 r^8. \tag{29}$$

The non-dimensional form wall shear stress is computed as:

$$S_{rz} = -\frac{\mu_{hnf}}{\mu_f}\left(\frac{\partial w}{\partial r}\right)_{r=h} = -2\xi_2 h(w_1 + 2w_2 h^2 + 3w_3 h^4). \tag{30}$$

As given by Ijaz and Nadeem [11], and Das *et al*. [7], the dimensionless volumetric flow rate is

$$F = \int_0^h r\,w\,dr \tag{31}$$

The axial pressure gradient can be obtained by substituting equation (26) in to equation (31), we get

$$\frac{dp}{dz} = \frac{96\xi_1}{(6 - Bh^2)h^4}\left[F + \frac{Ah}{24} - \frac{11}{6144}B(B-4A)h^8\right]. \tag{32}$$

Using equation (32), we can determine the pressure drop between the segment $\tilde{z} = 0$ to $\tilde{z} = L$through the stenosis, which is defined as

$$\Delta p = \int_0^L \left(-\frac{dp}{dz}\right) dz \tag{33}$$

The arterial segment's resistance impedence to blood flow is calculated using equation (33) as follows

$$\lambda = \frac{\Delta p}{F} = \frac{1}{F}\int_0^L \left(-\frac{dp}{dz}\right) dz \tag{34}$$

At the wall of the artery coefficient of heat transfer is given by Das *et al*. [7] as:

$$Z = h_z \theta_r|_{r=h} = -\frac{1}{2}\frac{\pi\delta S h}{\xi_5}\sin 2\pi\left(z - d - \frac{1}{2}\right). \tag{35}$$

Here suffix represents the partial derivative.

4. Result and Discussions

The axial velocity, temperature profile, pressure gradient, wall shear stress, and resistance impedance are expressed by equations (26), (20), (32), (30) and (34), respectively. The impact of different flow parameters on these variables has been studied. All graphs were plotted by taking $d_1 = 0.2$, $d_2 = 0.4$, $L_1 = 0.2$ and $L_2 = 0.2$.

The axial pressure gradient $\frac{dp}{dz}$ variation with respect to key parameters such as the magnetic parameter (M^2), Hall parameter (m) and Grashof number (Gr) is illustrated in Figures 2-4.

Figure 2 shows the M^2 effects the axial pressure gradient. It's noticed that higher values of M^2 significantly reduce the axial pressure. This happens because a stronger magnetic field increases the Lorentz force, which opposes blood flow in the artery, hence lowering the pressure gradient. The pressure gradient is highest when there is no magnetic field ($M^2 = 0$).

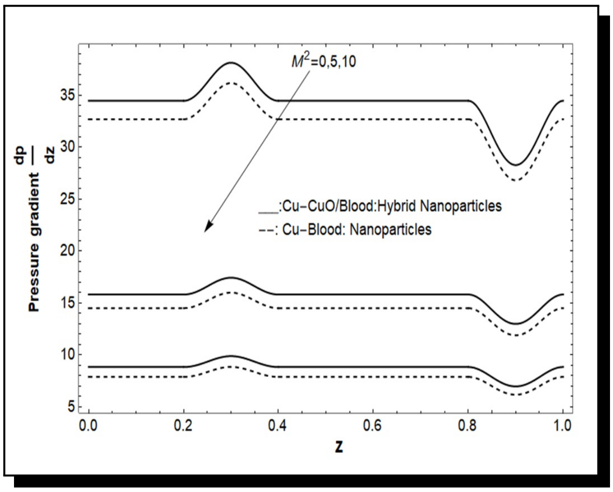


Figure 2. Effect of z on $\frac{dp}{dz}$ with M^2 varying

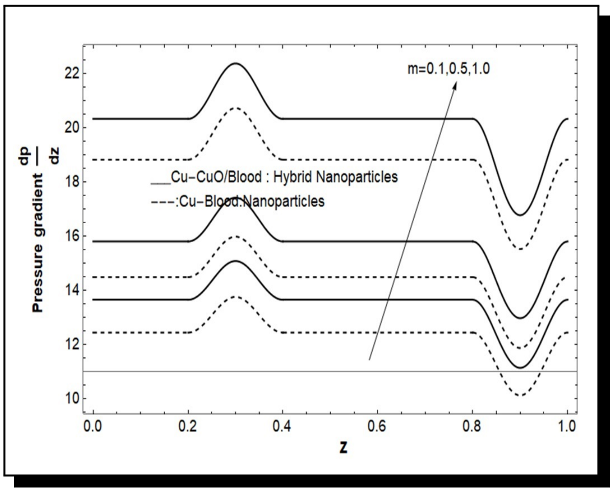


Figure 3. Effect of z on $\frac{dp}{dz}$ with m varying

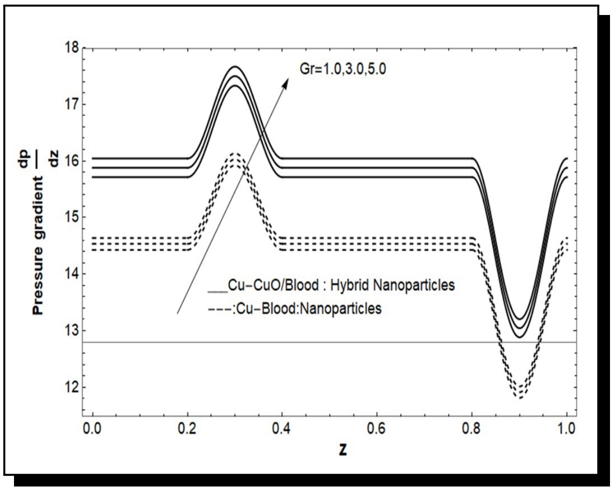


Figure 4. Effect of z on $\frac{dp}{dz}$ with Gr varying

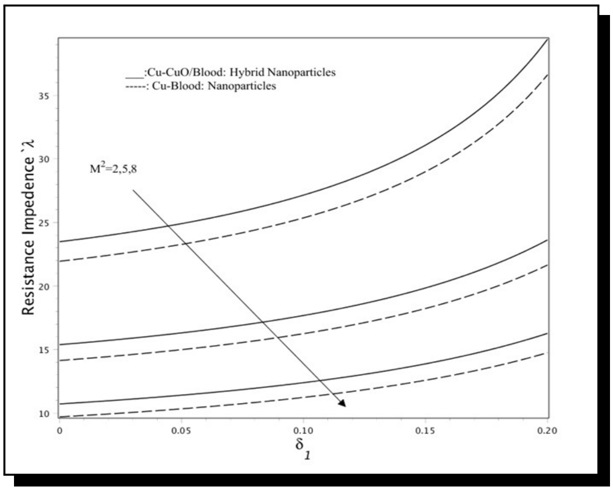


Figure 5. Effect of δ_1 on λ with M^2 varying

Figure 3 shows Hall parameter (m) effects the axial pressure gradient. As (m) increases, the pressure gradient rises because the term $(1+m^2)$ in the Lorentz force reduces its strength, allowing blood to flow faster (see equation (32)). Figure 4 illustrates the pressure gradient increases with (Gr), as higher Gr strengthens thermal buoyancy forces from blood density variations, overcoming viscous forces and creating stronger convection currents. These findings highlight the roles of Hall currents and Gr in enhancing blood flow and influencing flow dynamics.

Figures 5-9 show how impedance resistance (λ) changes with stenotic height (δ_1), (M^2), (m), Darcy number (Da), and heat source parameter (S). In Figure 5, increasing (M^2) reduces λ as Lorentz forces oppose blood flow, while Figure 6 shows that higher m increases (λ) by weakening these forces. Figure 7 reveals that (λ) rises with (Da) due to stronger blood-artery wall interactions. Finally, Figures 8-9 indicate that higher nanoparticle concentrations (ϕ_1, ϕ_2) raise (λ) by thickening the blood, making it harder to flow through the narrowed artery.

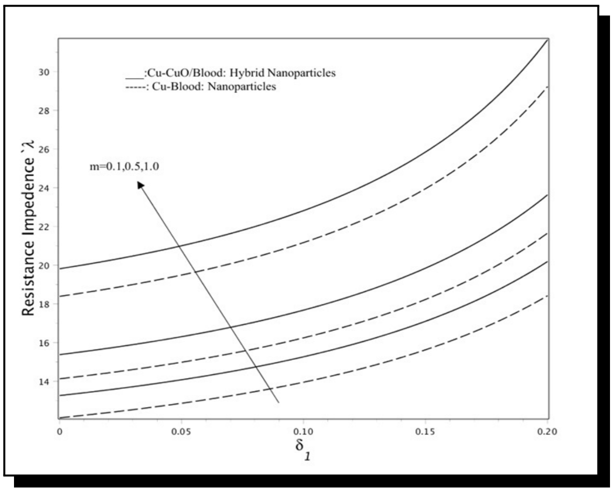


Figure 6. Effect of δ_1 on λ with m varying

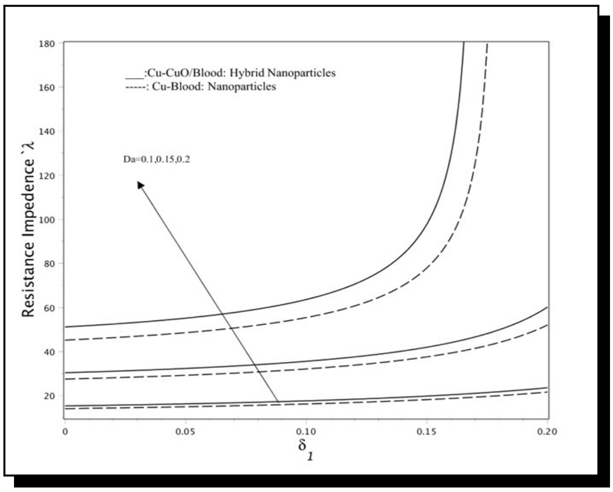


Figure 7. Effect of δ_1 on λ with Da varying

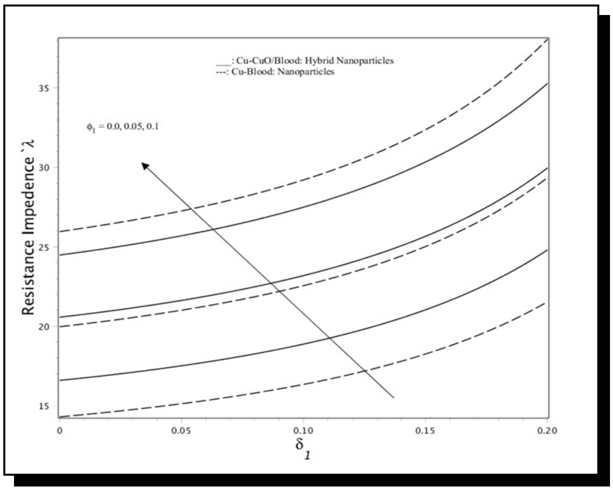


Figure 8. Effect of δ_1 on λ with ϕ_1 varying

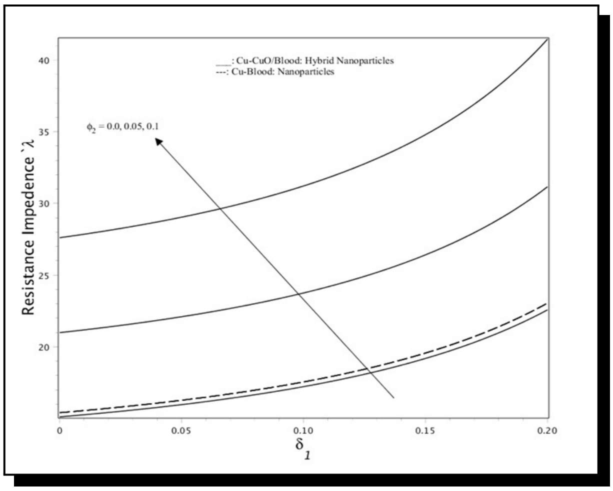


Figure 9. Effect of δ_1 on λ with ϕ_2 varying

Figures 10-14 show how the dimensionless *Wall Shear Stress* (WSS) S_{rz} changes with m, Gr, Da, (δ_1) and S, Srz decreases as m, Gr, S, Da, (δ_1) increase, but it increases with higher Da, as seen in Figures 10-14. The decrease in S_{rz} with higher m, Gr, S, and (δ_1) means these parameters reduce the frictional force at the wall. On the other hand, the increase in S_{rz} with Da (Darcy number) means that greater permeability increases the shear stress at the wall. These changes are important for understanding fluid behavior in systems like porous media or heat exchangers.

The streamline pattern for various effects of M^2, m, Da, Gr, ϕ_1, ϕ_2 in stenosis and dilatation are observed in Figures 15(a-h). It is observed from Figures 15(a-b) that the closed region is moving (occlusion) towards the center axis when the magnetic parameter increases, but the closed region is not formed in the dilation region, but the width of the center is increasing (dilation case). This analysis is more useful for doctors in diagnosis. It is noticed that the same phenomenon is observed in increasing the Da and Gr.

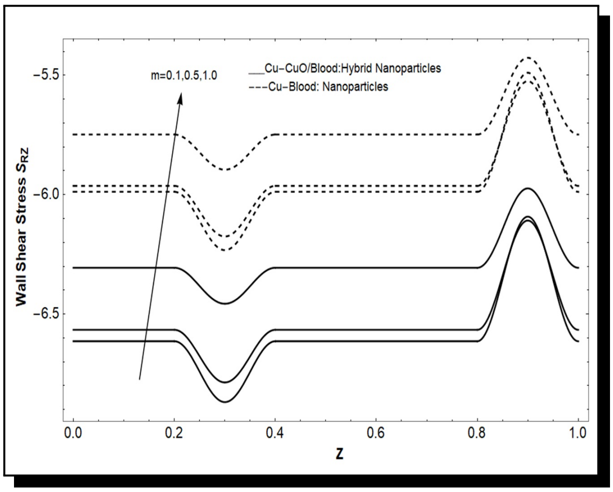


Figure 10. Effect of z on S_{RZ} with m varying

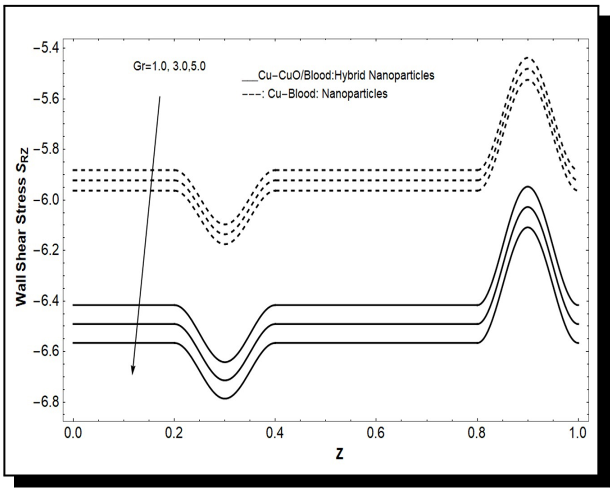


Figure 11. Effect of z on S_{RZ} with Gr varying

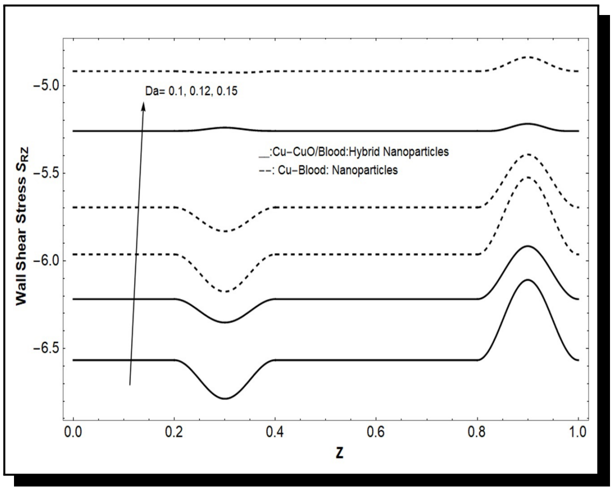


Figure 12. Effect of z on S_{RZ} with Da varying

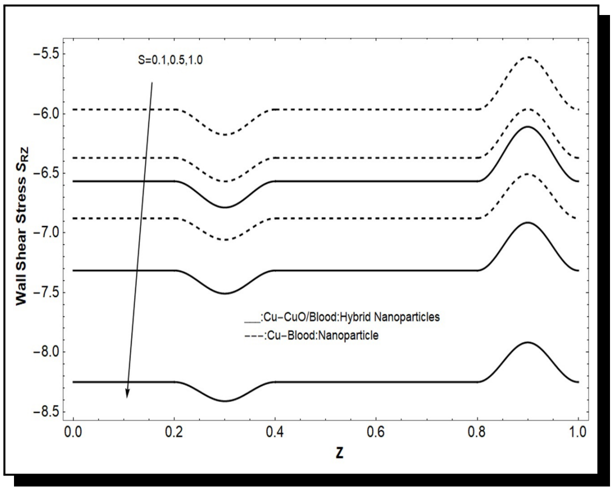


Figure 13. Effect of z on S_{RZ} with S varying

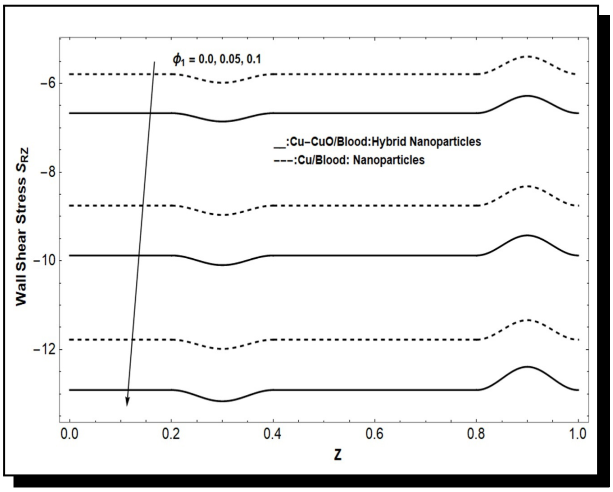
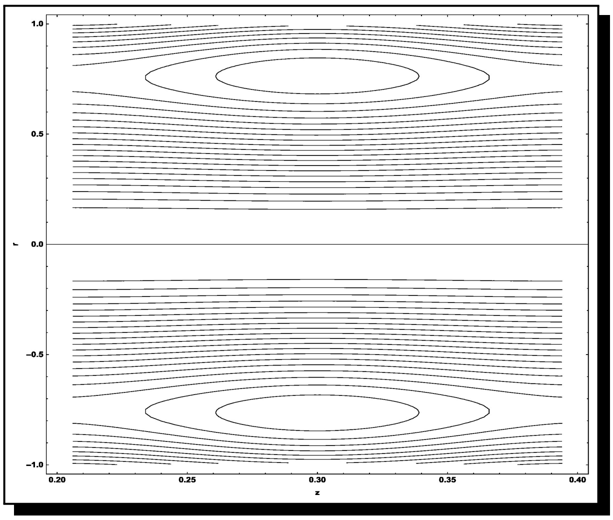
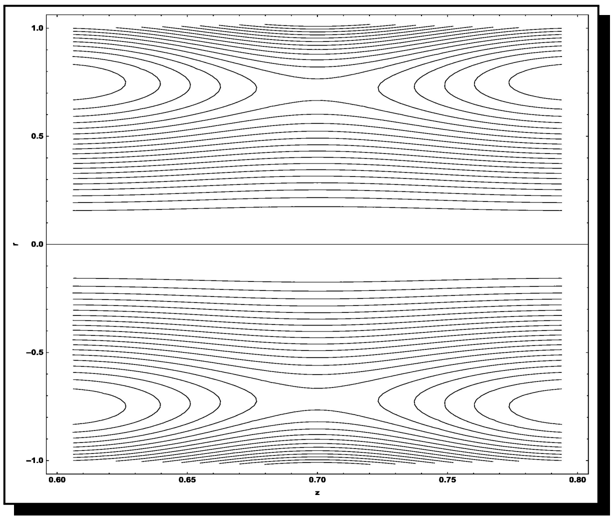


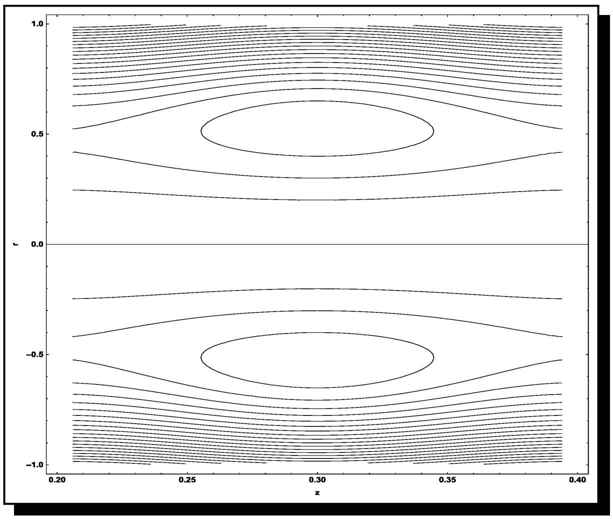
Figure 14. Effect of z on S_{RZ} with ϕ_1 varying



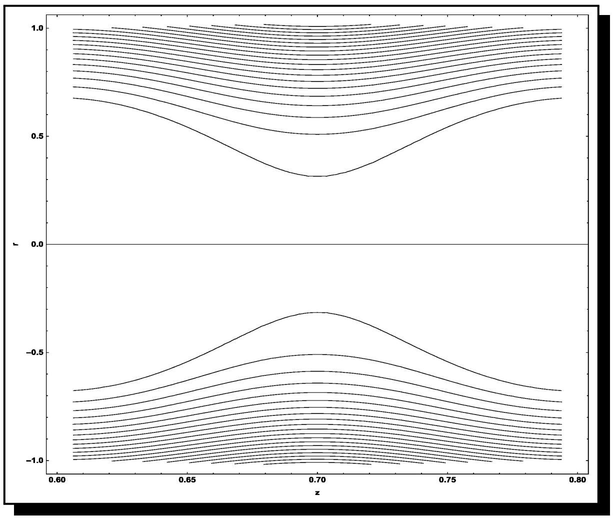
(a) Stream lines for Stenosis region when $M^2 = 5$



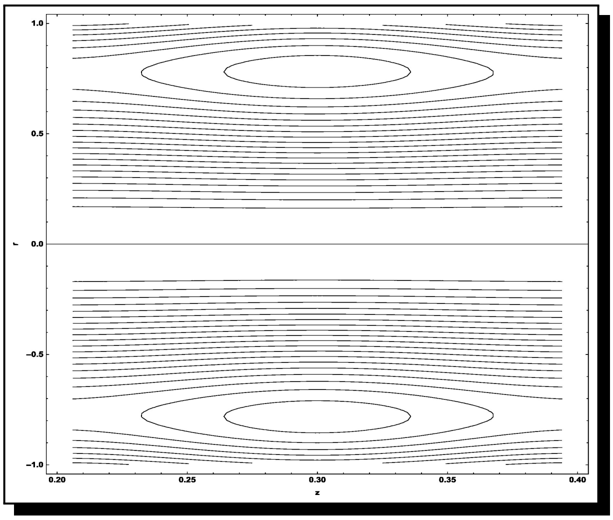
(b) Stream lines for Dilatation region when $M^2 = 5$



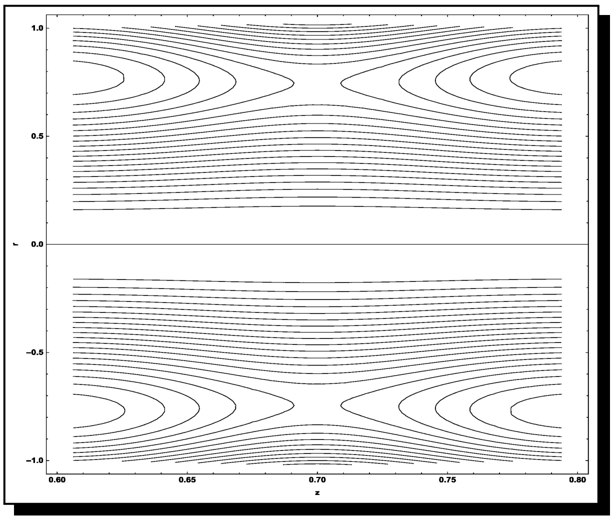
(c) Stream lines for Stenosis region when $M^2 = 10$



(d) Stream lines for Dilatation region when $M^2 = 10$

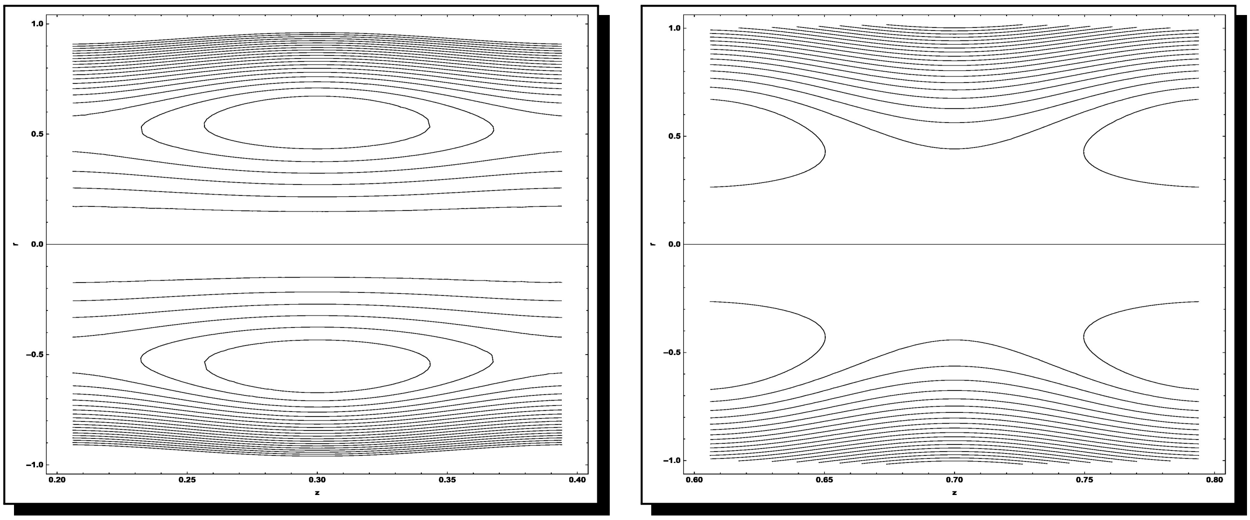


(e) Stream lines for Stenosis region when $Da = 0.2$



(f) Stream lines for Dilatation region when $M^2 = 10$

Figure 15 (*continued*)



(g) Stream lines for Stenosis region when $Gr = 5$

(h) Stream lines for Dilatation region when $Gr = 5$

Figure 15

5. Conclusions

The influence of hybrid nano paricles flow in an inclined circular artery with stenosis and dilatation has been studied. By deriving the solutions to the flow characteristics expressions by Homotopy method, it was possible to analyses the impact of various parameters on stenosis and dilatation regions on the axial velocity, temperature profile, pressure gradient, shear stress at the wall and impedance.

The observations are:

- Axial pressure gradient $\frac{dp}{dz}$ increases with m and Gr (buoyancy), but decreases with M^2.
- *Wall Shear Stress* (WSS) increases with Darcy number (Da) (permeability) and inclination angle (α), but decreases with M^2, Grashof number (Gr), heat source (S), and stenosis height (δ_1).
- Flow Resistance (λ) increases with stenosis height (δ_1), heat source (S), nanoparticles (ϕ_1, ϕ_2), and Darcy number (Da), whereas the reverse occurs with magnetic parameter M^2 (due to Lorentz force) and Hall parameter (m) (weakens Lorentz force).

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Competing Interests

The author declares that he has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

References

[1] S. I. Abdelsalam, Kh. S. Mekheimer and A. Z. Zaher, Alterations in blood stream by electroosmotic forces of hybrid nanofluid through diseased artery: Aneurysmal/stenosed segment, *Chinese Journal of Physics* **67** (2020), 314 – 329, DOI: 10.1016/j.cjph.2020.07.011.

[2] R. E. Abo-Elkhair, M. M. Bhatti and K. S. Mekheimer, Magnetic force effects on peristaltic transport of hybrid bio-nanofluid (Ausingle bondCu nanoparticles) with moderate Reynolds number: An expanding horizon, *International Communications in Heat and Mass Transfer* **123** (2021), 105228, DOI: 10.1016/j.icheatmasstransfer.2021.105228.

[3] S. S. Ardahaie, A. J. Amiri, A. Amouei, Kh. Hosseinzadeh and D. D. Ganji, Investigating the effect of adding nanoparticles to the blood flow in presence of magnetic field in a porous blood arterial, *Informatics in Medicine Unlocked* **10** (2018), 71 – 81, DOI: 10.1016/j.imu.2017.10.007.

[4] H. T. Basha, K. Rajagopal, N. A. Ahammad, S. Sathish and G. S. Rao, Finite difference computation of Au–Cu/magneto-bio-hybrid nanofluid flow in an inclined uneven stenosis artery, *Complexity* **2022**(1) (2022), 2078372, DOI: 10.1155/2022/2078372.

[5] S. Changdar and S. De, Investigation of nanoparticle as a drug carrier suspended in a blood flowing through an inclined multiple stenosed artery, *BioNanoScience* **8** (2017), 166 – 178, DOI: 10.1007/s12668-017-0446-7.

[6] T. G. Cowling, Magnetohydrodynamics, *Reports on Progress in Physics* **25**(1) (1962), 244, DOI: 10.1088/0034-4885/25/1/307.

[7] S. Das, T. K. Pal and R. N. Jana, Outlining impact of hybrid composition of nanoparticles suspended in blood flowing in an inclined stenosed artery under magnetic field orientation, *BioNanoScience* **11** (2021), 99 – 115, DOI: 10.1007/s12668-020-00809-y.

[8] S. Das, T. K. Pal, R. N. Jana and B. Giri, Significance of Hall currents on hybrid nano-blood flow through an inclined artery having mild stenosis: Homotopy perturbation approach, *Microvascular Research* **137** (2021), 104192, DOI: 10.1016/j.mvr.2021.104192.

[9] S. Dolui, B. Bhaumik, S. De and S. Changdar, Effect of a variable magnetic field on peristaltic slip flow of blood-based hybrid nanofluid through a nonuniform annular channel, *Journal of Mechanics in Medicine and Biology* **23**(01) (2023), 2250070, DOI: 10.1142/S0219519422500701.

[10] T. Elnaqeeb, N. A. Shah and K. S. Mekheimer, Hemodynamic characteristics of gold nanoparticle blood flow through a tapered stenosed vessel with variable nanofluid viscosity, *BioNanoScience* **9** (2019), 245 – 255, DOI: 10.1007/s12668-018-0593-5.

[11] S. Ijaz and S. Nadeem, A biomedical solicitation examination of nanoparticles as drug agents to minimize the hemodynamics of a stenotic channel, *The European Physical Journal Plus* **132** (2017), article number 448, DOI: 10.1140/epjp/i2017-11703-6.

[12] S. Ijaz, Z. Iqbal, E. N. Maraj and S. Nadeem, Investigation of Cu–CuO/blood mediated transportation in stenosed artery with unique features for theoretical outcomes of hemodynamics, *Journal of Molecular Liquids* **254** (2018), 421 – 432, DOI: 10.1016/j.molliq.2018.01.098.

[13] J. C. Misra, A. Sinha and G. C. Shit, Mathematical modeling of blood flow in a porous vessel having double stenoses in the presence of an external magnetic field, *International Journal of Biomathematics* **4**(2) (2011), 207 – 225, DOI: 10.1142/S1793524511001428.

[14] N. Padmanabhan, Mathematical model of arterial stenosis, *Medical and Biological Engineering and Computing* **18** (1980), 281 – 286, DOI: 10.1007/BF02443380.

[15] K. M. Prasad, T. Sudha and M. V. Phanikumari, The effects of post-stenotic dilatations on the flow of couple stress fluid through stenosed arteries, *American Journal of Computational Mathematics* **6**(4) (2016), 365 – 376, DOI: 10.4236/ajcm.2016.64036.

[16] M. Roy, B. S. Sikarwar, M. Bhandwal and P. Ranjan, Modelling of blood flow in stenosed arteries, *Procedia Computer Science* **115** (2017), 821 – 830, DOI: 10.1016/j.procs.2017.09.164.

[17] P. Sreedevi, P. S. Reddy and M. A. Sheremet, Impact of homogeneous-heterogeneous reactions on heat and mass transfer flow of Au–Eg and Ag–Eg Maxwell nanofluid past a horizontal stretched cylinder, *Journal of Thermal Analysis and Calorimetry* **141** (2020), 533 – 546, DOI: 10.1007/s10973-020-09581-3.

[18] P. N. Tandon, U. V.S. Rana, M. Kawahara and V. K. Katiyar, A model for blood flow through a stenotic tube, *International Journal of Bio-Medical Computing* **32**(1) (1993), 61 – 78, DOI: 10.1016/0020-7101(93)90007-S.

[19] D. F. Young, Effects of time-dependent stenosis on flow through a tube, *Journal of Manufacturing Science and Engineering* **90**(2) (1968), 248 – 254, DOI: 10.1115/1.3604621.

[20] A. Zaman, N. Ali and A. A. Khan, Computational biomedical simulations of hybrid nanoparticles on unsteady blood hemodynamics in a stenotic artery, *Mathematics and Computers in Simulation* **169** (2020), 117 – 132, DOI: 10.1016/j.matcom.2019.09.010.