Research Article

Applications of Generalized Mersenne Polynomials to a General Subclass of Bi-Bazilevič-Type Functions

N. Magesh[1] , D. S. Raju[2] and N. S. Tejas*[2]

[1] *Post-Graduate and Research Department of Mathematics, Government Arts College for Men (affiliated to Periyar University), Krishnagiri 635001, Tamilnadu, India*

[2] *Department of Mathematics, The National Institute of Engineering (affiliated to Visvesvaraya Technological University), Mysore 570008, Karnataka, India*

*Corresponding author: nstejas@gmail.com

Abstract. This paper introduces a new subclass of generalized bi-Bazilevič-type functions which encompasses several eminent and widely studied subclasses, such as bi-starlike and bi-convex functions associated with the generating function of generalized Mersenne polynomials. The use of generalized Mersenne polynomials highlights the applicability of special polynomials in geometric function theory. By applying the generating function of the generalized Mersenne polynomials, we derive coefficient bounds for the initial Taylor-Maclaurin coefficients and provide estimates for the Fekete-Szegö inequality. Furthermore, several corollaries are discussed by specifying particular parameter choices.

Keywords. Bi-univalent functions, Bi-Bazilevič functions, Bi-starlike and Bi-convex functions, Fekete-Szegö inequality, Generalized Mersenne polynomials

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1. Introduction

Consider the open unit disk in the complex plane defined by

$$\mathcal{U} = \{\mathfrak{z} \in \mathbb{C} : |\mathfrak{z}| < 1\}.$$

We begin by recalling a well-known class of analytic functions:

$$\mathcal{A} = \{\mathfrak{f} : \mathfrak{f} \text{ is analytic in } \mathcal{U}, \text{ normalized such that } \mathfrak{f}(0) = 0 \text{ and } \mathfrak{f}'(0) = 1\},$$

and the class

$$\mathcal{S} = \{\mathfrak{f} \in \mathcal{A} : \mathfrak{f} \text{ is univalent in } \mathcal{U}\}.$$

Every function $\mathfrak{f} \in \mathcal{A}$ admits a Taylor-Maclaurin series expansion of the form

$$\mathfrak{f}(\mathfrak{z}) = \mathfrak{z} + \sum_{j=2}^{\infty} b_j \mathfrak{z}^j, \quad \mathfrak{z} \in \mathcal{U}. \tag{1.1}$$

Among the many important subclasses of $\mathcal{S}$, particular attention has been given to the class of starlike functions $\mathcal{S}^*$ and the class of convex functions $\mathcal{C}$, due to their geometric properties. These are defined respectively as follows:

$$\mathcal{S}^* = \left\{\mathfrak{f} \in \mathcal{A} : \operatorname{Re}\left(\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}(\mathfrak{z})}\right) > 0 \text{ and } \mathfrak{z} \in \mathcal{U}\right\}$$

and

$$\mathcal{C} = \left\{\mathfrak{f} \in \mathcal{A} : \operatorname{Re}\left(1 + \frac{\mathfrak{z}\mathfrak{f}''(\mathfrak{z})}{\mathfrak{f}'(\mathfrak{z})}\right) > 0 \text{ and } \mathfrak{z} \in \mathcal{U}\right\}.$$

Another significant subclass of $\mathcal{S}$, which provides a connection between the classes $\mathcal{S}^*$ and $\mathcal{C}$, was introduced by Miller *et al.* [20], and is defined as follows:

$$\mathcal{M}(\beta) = \left\{\mathfrak{f} \in \mathcal{A} : \operatorname{Re}\left((1-\beta)\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}(\mathfrak{z})} + \beta\left(1 + \frac{\mathfrak{z}\mathfrak{f}''(\mathfrak{z})}{\mathfrak{f}'(\mathfrak{z})}\right)\right) > 0, \beta \leq 0, \mathfrak{z} \in \mathcal{U}\right\}.$$

In addition to the above-mentioned subclasses, the class of Bazilevič functions forms another important class connecting and extending various subclasses of $\mathcal{S}$, including the class of starlike and convex functions.

A function $\mathfrak{f} \in \mathcal{A}$ is called a Bazilevič function [7] of type (γ, β), where $\gamma > 0$ and $\beta \in \mathbb{R}$, if it can be represented as

$$\mathfrak{f}(\mathfrak{z}) = \left((\gamma + i\beta)\int_0^{\mathfrak{z}} [\mathfrak{f}_1(t)]^{\gamma} p(t) t^{i\beta - 1} \, dt\right)^{1/(\gamma + i\beta)}, \quad \mathfrak{z} \in \mathcal{U}, \tag{1.2}$$

where $\mathfrak{f}_1 \in \mathcal{S}^*$, $p \in \mathcal{A}$ with $p(0) = 1$, and $\operatorname{Re}(p(\mathfrak{z})) > 0$ for all $\mathfrak{z} \in \mathcal{U}$. The class of Bazilevič functions of type (γ, β) is denoted by $\mathcal{B}(\gamma, \beta)$. Bazilevič [7] proved that $\mathcal{B}(\gamma, \beta) \subset \mathcal{S}$.

The definition in (1.2) is equivalently expressed as:

$$\mathcal{B}(\gamma, \beta) = \left\{\mathfrak{f} \in \mathcal{A} : \operatorname{Re}\left(\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}(\mathfrak{z})}\left(\frac{\mathfrak{f}(\mathfrak{z})}{\mathfrak{f}_1(\mathfrak{z})}\right)^{\gamma}\left(\frac{\mathfrak{f}(\mathfrak{z})}{\mathfrak{z}}\right)^{i\beta}\right) > 0, \mathfrak{z} \in \mathcal{U}\right\},$$

where $\mathfrak{f}_1 \in \mathcal{S}^*$. By selecting appropriate parameter values, the following notable subclasses of $\mathcal{S}$ can be obtained:

(1) When $\mathfrak{f}_1(\mathfrak{z}) = \mathfrak{z}$, the class $\mathcal{B}(\gamma, 0) \equiv \mathcal{B}(\gamma)$, introduced and studied by Singh [29], is known as the class of Bazilevič functions of type γ, and is defined by

$$\mathcal{B}(\gamma) = \left\{\mathfrak{f} \in \mathcal{A} : \operatorname{Re}\left(\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}^{1-\gamma}(\mathfrak{z})\mathfrak{z}^{\gamma}}\right) > 0, \mathfrak{z} \in \mathcal{U}\right\}.$$

(2) For $\mathfrak{f}_1 \in \mathcal{S}^*$, the class $\mathcal{B}(1, 0) \equiv \mathcal{K}$ corresponds to the class of close-to-convex functions, defined as

$$\mathcal{K} = \left\{\mathfrak{f} \in \mathcal{A} : \operatorname{Re}\left(\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}_1(\mathfrak{z})}\right) > 0, \mathfrak{z} \in \mathcal{U}\right\}.$$

(3) Moreover, when $\mathfrak{f}_1(\mathfrak{z}) = \mathfrak{z}$, $\gamma = 1$ and $\beta = 0$, then $\mathcal{B}(1,0) \equiv \mathcal{R}$, where $\mathcal{R}$ denotes the class of analytic functions whose derivative has positive real part in $\mathcal{U}$, i.e.,

$$\mathcal{R} = \{\mathfrak{f} \in \mathcal{A} : \operatorname{Re}(\mathfrak{f}'(\mathfrak{z})) > 0, \mathfrak{z} \in \mathcal{U}\}.$$

The Bazilevič function class $\mathcal{B}(\gamma, \beta)$ includes several prominent subclasses of $\mathcal{S}$, making it a central object of study in geometric function theory. Considerable attention has been given to the study of Bazilevič functions. For further details, the reader may refer to Fitri and Muslikh [12], Fitri and Thomas [13], Keough and Miller [14], Noor [22], Noor and Al-Bany [23], Noor *et al.* [24], Owa and Obradović [25], Srivastava *et al.* [33], and Thomas [35]. The following inclusion relationships are well established (see Duren [11], Miller [19], and Sharma and Bulboacă [28]):

$$\mathcal{C} \subset \mathcal{S}^* \subset \mathcal{K} \subset \mathcal{B}(\gamma) \subset \mathcal{B}(\gamma, \beta) \subset \mathcal{S}.$$

Fitri and Muslikh [12] defined a subclass of $\mathcal{S}$, denoted by $\mathcal{B}_1^\gamma(\beta)$, as an analogue of $\mathcal{M}(\beta)$ for Bazilevič functions. A function $\mathfrak{f} \in \mathcal{A}$ belongs to the class $\mathcal{B}_1^\gamma(\beta)$ if it satisfies

$$\operatorname{Re}\left(\beta\left[\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}^{1-\gamma}(\mathfrak{z})\mathfrak{z}^\gamma} + \frac{\mathfrak{z}\mathfrak{f}''(\mathfrak{z})}{\mathfrak{f}'(\mathfrak{z})} + (\gamma-1)\left(\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}(\mathfrak{z})} - 1\right)\right] + (1-\beta)\left[\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}^{1-\gamma}(\mathfrak{z})\mathfrak{z}^\gamma}\right]\right) > 0,$$

where $\beta \geq 0$, $0 \leq \gamma \leq 1$ and $\mathfrak{z} \in \mathcal{U}$. They also derived sharp bounds and other related coefficient estimates for functions in this class.

Now let us recall the well-established relation between two analytic functions. For two functions $\mathfrak{f}_1$ and $\mathfrak{f}_2$ in $\mathcal{A}$, we say that $\mathfrak{f}_1$ is subordinate to $\mathfrak{f}_2$ in $\mathcal{U}$ if there exists an analytic function ϕ with $\phi(0) = 0$ and $|\phi(\mathfrak{z})| < 1$, such that

$$\mathfrak{f}_1(\mathfrak{z}) = \mathfrak{f}_2(\phi(\mathfrak{z})).$$

This subordination is denoted by $\mathfrak{f}_1 \prec \mathfrak{f}_2$. If $\mathfrak{f}_2$ is univalent in $\mathcal{U}$, then

$$\mathfrak{f}_1 \prec \mathfrak{f}_2, \ \mathfrak{z} \in \mathcal{U} \quad \Longleftrightarrow \quad \mathfrak{f}_1(0) = \mathfrak{f}_2(0) \text{ and } \mathfrak{f}_1(\mathcal{U}) \subset \mathfrak{f}_2(\mathcal{U}).$$

The Koebe one-quarter theorem [11] asserts that every function $\mathfrak{f} \in \mathcal{S}$ possesses an inverse $\mathfrak{f}^{-1} = \mathfrak{g}$, defined by

$$\mathfrak{g}(\mathfrak{f}(\mathfrak{z})) = \mathfrak{z}, \quad \mathfrak{z} \in \mathcal{U},$$

and

$$\mathfrak{f}(\mathfrak{g}(\mathfrak{w})) = \mathfrak{w}, \ |\mathfrak{w}| < r_0(\mathfrak{f}); \quad r_0(\mathfrak{f}) \geq \frac{1}{4},$$

where the inverse $\mathfrak{g}$ admits the expansion

$$\mathfrak{g}(\mathfrak{w}) = \mathfrak{w} + \sum_{j=2}^{\infty} d_j \mathfrak{w}^j = \mathfrak{w} - b_2\mathfrak{w}^2 + (2b_2^2 - b_3)\mathfrak{w}^3 - (5b_2^3 - 5b_2b_3 + b_4)\mathfrak{w}^4 + \cdots. \tag{1.3}$$

A function $\mathfrak{f} \in \mathcal{A}$ is said to be bi-univalent in $\mathcal{U}$ if both $\mathfrak{f}$ and its inverse $\mathfrak{f}^{-1}$ are univalent in $\mathcal{U}$. Let Σ denote the class of bi-univalent functions in $\mathcal{U}$, represented by the series expansion in (1.1). The concept of bi-univalent analytic functions was introduced by Lewin [17], who established the bound $|b_2| < 1.51$ for $\mathfrak{f} \in \Sigma$. Subsequently, Brannan and Clunie [8] conjectured that $|b_2| \leq \sqrt{2}$, while Netanyahu [21] showed that $\max |b_2| = \frac{4}{3}$ for functions in Σ. Brannan and Taha [9] introduced subclasses of Σ inspired by the familiar starlike and convex functions. The study of bi-univalent functions gained renewed interest following the work of Srivastava *et al.* [32], who provided a foundational reference for analyzing and constructing examples of functions in the class Σ. Further developments include the work of Sivasubramanian *et al.* [30], who verified

a long-standing open problem posed by Brannan and Clunie [8] for certain subclasses of Σ. It is also worth mentioning that Altınkaya and Yalçın [5] were the first to introduce a subclass of bi-univalent functions involving Lucas polynomials, thereby establishing a connection between geometric function theory and the theory of special functions. Subsequently, Abhirami *et al.* [2] defined a subclass of Σ based on Horadam polynomials, which generalize several well-known polynomials.

In recent mathematical literature, significant progress has been made in estimating the initial coefficients $|b_2|$ and $|b_3|$ for functions in the classes $\mathcal{S}$ and Σ, building on the foundational work of Srivastava *et al.* [32]. However, establishing general bounds for $|b_j|$, where $j \geq 4$, remains an open and active area of research that continues to attract interest in the study of subclasses of Σ. Recent studies have also examined bi-univalent functions associated with special families of polynomials, including Horadam, Hermite, Lucas, Laguerre, and Chebyshev polynomials (see Abirami *et al.* [2], AlRawashdeh [3], Al-Shbeil *et al.* [4], Altınkaya and Yalçın [5], [6], Altınkaya and Yalçın [10], Magesh and Bulut [18], Panigrahi and Sokol [27], Srivastava *et al.* [31, 34], Wanas and Alina [36], and Wanas and Lupas [37]).

Generating numerical sequences through structured polynomial functions has emerged as a significant direction in mathematical analysis, linking classical theory with modern applications. In this direction, the Generalized Mersenne Polynomials extend the classical Mersenne numbers $2^j - 1$, where j is a positive integer, into generating functions. Beyond their theoretical significance, these polynomials have recently found practical applications in areas such as image processing (Aarthy and Keerthi [1]). Similar studies have also been conducted using generalized bivariate Fibonacci polynomials derived from the classical Fibonacci sequence (see, Ozturk and Aktas [26]).

Kumari *et al.* [15] provided an in-depth study of the generalized Mersenne polynomials $\mathcal{W}_j(y)$, which are defined by the recurrence relation

$$\mathcal{W}_j(y) = 3y\mathcal{W}_{j-1}(y) - 2\mathcal{W}_{j-2}(y), \quad n \geq 2 \text{ and } n \in \mathbb{N}, \tag{1.4}$$

with the initial terms

$$\mathcal{W}_0(y) = a \quad \text{and} \quad \mathcal{W}_1(y) = by + c, \tag{1.5}$$

where a, b, and c are complex or real numbers not all being zero. For $a = 0$, $b = 0$ and $c = 1$, (1.4) reduces to standard Mersenne polynomials [16] and for $a = 2$, $b = 3$ and $c = 0$, it gives Mersenne Lucas polynomials. Furthermore, the generating function of the generalized Mersenne polynomials $\mathcal{W}_j(y)$, as derived in [15], is given by

$$\mathcal{G}(y, \mathfrak{z}) = \sum_{j=0}^{\infty} \mathcal{W}_j(y)\mathfrak{z}^j = \frac{a + \mathfrak{z}[(b - 3a)y + c]}{1 - 3y\mathfrak{z} + 2\mathfrak{z}^2}. \tag{1.6}$$

Inspired by the above literature on subclasses of analytic and bi-univalent functions, particularly those involving special polynomials and subordination principle, we now introduce a new subclass of Σ consisting of Bazilevič-type functions and governed by the generalized Mersenne polynomials as follows:

Definition 1.1. For $\beta \geq 0$ and $0 \leq \gamma \leq 1$, a function $\mathfrak{f} \in \Sigma$ of the form (1.1) is said to belong to the class $\mathcal{B}_{\Sigma}^{\gamma, \beta}(y)$ if

$$\beta \left[\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}^{1-\gamma}(\mathfrak{z})\mathfrak{z}^\gamma} + \frac{\mathfrak{z}\mathfrak{f}''(\mathfrak{z})}{\mathfrak{f}'(\mathfrak{z})} + (\gamma - 1)\left(\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}(\mathfrak{z})} - 1 \right) \right] + (1 - \beta)\left[\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}^{1-\gamma}(\mathfrak{z})\mathfrak{z}^\gamma} \right] < \mathcal{G}(y, \mathfrak{z}) + 1 - a,$$

and for $\mathfrak{g}(\mathfrak{w}) = \mathfrak{f}^{-1}(\mathfrak{w})$, defined by (1.3),

$$\beta\left[\frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}^{1-\gamma}(\mathfrak{w})\mathfrak{w}^\gamma} + \frac{\mathfrak{w}\mathfrak{g}''(\mathfrak{w})}{\mathfrak{g}'(\mathfrak{w})} + (\gamma-1)\left(\frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}(\mathfrak{w})} - 1\right)\right] + (1-\beta)\left[\frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}^{1-\gamma}(\mathfrak{w})\mathfrak{w}^\gamma}\right] \prec \mathcal{G}(y,\mathfrak{w}) + 1 - a,$$

where $\mathfrak{z}, \mathfrak{w} \in \mathcal{U}$, and $\mathcal{G}(y,\mathfrak{z})$ is given by (1.6).

Remark 1.2. Notable subclasses of bi-univalent functions, such as the bi-starlike, bi-convex functions and bi-Bazilevič, emerge as special cases of Definition 1.1 for particular choices of the parameters γ and β.

(1) The class $\mathcal{B}_\Sigma^{0,0}(y) \equiv \mathcal{S}_\Sigma^*(y)$ consists of functions $\mathfrak{f} \in \Sigma$ if

$$\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}(\mathfrak{z})} \prec \mathcal{G}(y,\mathfrak{z}) + 1 - a \quad \text{and} \quad \frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}(\mathfrak{w})} \prec \mathcal{G}(y,\mathfrak{w}) + 1 - a.$$

(2) The class $\mathcal{B}_\Sigma^{0,1}(y) \equiv \mathcal{C}_\Sigma(y)$ consists of functions $\mathfrak{f} \in \Sigma$ satisfying

$$1 + \frac{\mathfrak{z}\mathfrak{f}''(\mathfrak{z})}{\mathfrak{f}'(\mathfrak{z})} \prec \mathcal{G}(y,\mathfrak{z}) + 1 - a \quad \text{and} \quad 1 + \frac{\mathfrak{w}\mathfrak{g}''(\mathfrak{w})}{\mathfrak{g}'(\mathfrak{w})} \prec \mathcal{G}(y,\mathfrak{w}) + 1 - a.$$

(3) The class $\mathcal{B}_\Sigma^{0,\beta}(y) \equiv \mathcal{M}_\Sigma(\beta,y)$ consists of functions $\mathfrak{f} \in \Sigma$ satisfying

$$(1-\beta)\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}(\mathfrak{z})} + \beta\left(1 + \frac{\mathfrak{z}\mathfrak{f}''(\mathfrak{z})}{\mathfrak{f}'(\mathfrak{z})}\right) \prec \mathcal{G}(y,\mathfrak{z}) + 1 - a$$

and

$$(1-\beta)\frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}(\mathfrak{w})} + \beta\left(1 + \frac{\mathfrak{w}\mathfrak{g}''(\mathfrak{w})}{\mathfrak{g}'(\mathfrak{w})}\right) \prec \mathcal{G}(y,\mathfrak{w}) + 1 - a.$$

(4) The class $\mathcal{B}_\Sigma^{\gamma,0}(y) \equiv \mathcal{B}_\Sigma(\gamma,y)$ consists of functions $\mathfrak{f} \in \Sigma$ satisfying

$$\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}^{1-\gamma}(\mathfrak{z})\mathfrak{z}^\gamma} \prec \mathcal{G}(y,\mathfrak{z}) + 1 - a \quad \text{and} \quad \frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}^{1-\gamma}(\mathfrak{w})\mathfrak{w}^\gamma} \prec \mathcal{G}(y,\mathfrak{w}) + 1 - a.$$

(5) The class $\mathcal{B}_\Sigma^{1,0}(y) \equiv \mathcal{R}_\Sigma(y)$ consists of functions $\mathfrak{f} \in \Sigma$ satisfying

$$\mathfrak{f}'(\mathfrak{z}) \prec \mathcal{G}(y,\mathfrak{z}) + 1 - a \quad \text{and} \quad \mathfrak{g}'(\mathfrak{w}) \prec \mathcal{G}(y,\mathfrak{w}) + 1 - a.$$

For all the above subclasses ($\mathfrak{z}, \mathfrak{w} \in \mathcal{U}$), the function $\mathfrak{f}$ is given by (1.1), its inverse $\mathfrak{g} = \mathfrak{f}^{-1}$ is defined by (1.3), and $\mathcal{G}(y,\mathfrak{z})$ is defined in (1.6).

Next, we present results regarding the upper bound estimates for $|b_2|$ and $|b_3|$ for functions belonging to the class $\mathcal{B}_\Sigma^{\gamma,\beta}(y)$.

2. Coefficient Estimates

Theorem 2.1. *If $\beta \geq 0$, $0 \leq \gamma \leq 1$ and $\mathfrak{f} \in \mathcal{B}_\Sigma^{\gamma,\beta}(y)$ is of the form* (1.1)*, then*

$$|b_2| \leq \frac{\sqrt{2}|by+c|\sqrt{by+c}}{\sqrt{|(1+\gamma)(\gamma+2\beta+2)(by+c)^2 - 2(1+\gamma)^2(1+\beta)^2[3y(by+c)-2a]|}}$$

and

$$|b_3| \leq \frac{|by+c|}{(2+\gamma)(1+2\beta)} + \frac{(by+c)^2}{(1+\gamma)^2(1+\beta)^2}.$$

Proof. Let $\mathfrak{f} \in \mathcal{B}_{\Sigma}^{\gamma,\beta}(y)$ be of the form (1.1). By Definition 1.1, we have

$$\beta\left[\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}^{1-\gamma}(\mathfrak{z})\mathfrak{z}^{\gamma}} + \frac{\mathfrak{z}\mathfrak{f}''(\mathfrak{z})}{\mathfrak{f}'(\mathfrak{z})} + (\gamma-1)\left(\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}(\mathfrak{z})}-1\right)\right] + (1-\beta)\left[\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}^{1-\gamma}(\mathfrak{z})\mathfrak{z}^{\gamma}}\right] = \mathcal{G}(y, v(\mathfrak{z})) + 1 - a$$

and

$$\beta\left[\frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}^{1-\gamma}(\mathfrak{w})\mathfrak{w}^{\gamma}} + \frac{\mathfrak{w}\mathfrak{g}''(\mathfrak{w})}{\mathfrak{g}'(\mathfrak{w})} + (\gamma-1)\left(\frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}(\mathfrak{w})}-1\right)\right] + (1-\beta)\left[\frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}^{1-\gamma}(\mathfrak{w})\mathfrak{w}^{\gamma}}\right] = \mathcal{G}(y, u(\mathfrak{w})) + 1 - a,$$

where $u(\mathfrak{z}) = \sum_{j=1}^{\infty} c_j \mathfrak{z}^j$ and $v(\mathfrak{w}) = \sum_{j=1}^{\infty} d_j \mathfrak{w}^j$ are analytic functions, such that

$$u(0) = 0, \; v(0) = 0, \; |u(\mathfrak{z})| < 1 \; \text{ and } \; |v(\mathfrak{w})| < 1, \quad \mathfrak{z}, \mathfrak{w} \in \mathcal{U}.$$

Furthermore, it is well established that

$$|c_j| \le 1 \; \text{ and } \; |d_j| \le 1. \tag{2.1}$$

Let

$$F(\mathfrak{z}) := \beta\left[\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}^{1-\gamma}(\mathfrak{z})\mathfrak{z}^{\gamma}} + \frac{\mathfrak{z}\mathfrak{f}''(\mathfrak{z})}{\mathfrak{f}'(\mathfrak{z})} + (\gamma-1)\left(\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}(\mathfrak{z})}-1\right)\right] + (1-\beta)\left[\frac{\mathfrak{z}\mathfrak{f}'(\mathfrak{z})}{\mathfrak{f}^{1-\gamma}(\mathfrak{z})\mathfrak{z}^{\gamma}}\right]$$

and

$$G(\mathfrak{w}) := F^{-1}(\mathfrak{w}) = \beta\left[\frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}^{1-\gamma}(\mathfrak{w})\mathfrak{w}^{\gamma}} + \frac{\mathfrak{w}\mathfrak{g}''(\mathfrak{w})}{\mathfrak{g}'(\mathfrak{w})} + (\gamma-1)\left(\frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}(\mathfrak{w})}-1\right)\right] + (1-\beta)\left[\frac{\mathfrak{w}\mathfrak{g}'(\mathfrak{w})}{\mathfrak{g}^{1-\gamma}(\mathfrak{w})\mathfrak{w}^{\gamma}}\right].$$

In view of (1.1), it can be computed that

$$F(\mathfrak{z}) = 1 + (1+\gamma)(1+\beta)b_2\mathfrak{z} + \left(\left[\frac{(\gamma^2+\gamma-2)-2(3+\gamma)\beta}{2}\right]a_2^2 + (2+\gamma)(1+2\beta)b_3\right)\mathfrak{z}^2 + \cdots. \tag{2.2}$$

Similarly,

$$G(\mathfrak{w}) = 1 - (1+\gamma)(1+\beta)b_2\mathfrak{w} + \left(\left[\frac{(\gamma^2+5\gamma+6)+2(5+3\gamma)\beta}{2}\right]b_2^2 - (2+\gamma)(1+2\beta)b_3\right)\mathfrak{w}^2 + \cdots. \tag{2.3}$$

On the other hand,

$$\mathcal{G}(y, v(\mathfrak{z})) + 1 - a = 1 - a + \mathcal{W}_0(y) + \mathcal{W}_1(y)u(\mathfrak{z}) + \mathcal{W}_2(y)[u(\mathfrak{z})]^2 + \cdots$$

and

$$\mathcal{G}(y, u(\mathfrak{w})) + 1 - a = 1 - a + \mathcal{W}_0(y) + \mathcal{W}_1(y)v(\mathfrak{w}) + \mathcal{W}_2(y)[v(\mathfrak{w})]^2 + \cdots.$$

Equivalently,

$$\mathcal{G}(y, v(\mathfrak{z})) + 1 - a = 1 + \mathcal{W}_1(y)c_1\mathfrak{z} + [\mathcal{W}_1(y)c_2 + \mathcal{W}_2(y)c_1^2]\mathfrak{z}^2 + \cdots \tag{2.4}$$

and

$$\mathcal{G}(y, u(\mathfrak{w})) + 1 - a = 1 + \mathcal{W}_1(y)d_1\mathfrak{w} + [\mathcal{W}_1(y)d_2 + \mathcal{W}_2(y)d_1^2]\mathfrak{w}^2 + \cdots. \tag{2.5}$$

Comparing the corresponding coefficients in (2.2) and (2.4), as well as in (2.3) and (2.5), we obtain

$$(1+\gamma)(1+\beta)b_2 = \mathcal{W}_1(y)c_1, \tag{2.6}$$

$$\left(\frac{(\gamma^2+\gamma-2)-2(3+\gamma)\beta}{2}\right)b_2^2 + (2+\gamma)(1+2\beta)b_3 = \mathcal{W}_1(y)c_2 + \mathcal{W}_2(y)c_1^2, \tag{2.7}$$

$$-(1+\gamma)(1+\beta)b_2 = \mathcal{W}_1(y)d_1, \tag{2.8}$$

$$\left(\frac{(\gamma^2+5\gamma+6)+2(5+3\gamma)\beta}{2}\right)b_2^2 - (2+\gamma)(1+2\beta)b_3 = \mathcal{W}_1(y)d_2 + \mathcal{W}_2(y)d_1^2. \tag{2.9}$$

From (2.6) and (2.8), we get

$$c_1 = -d_1, \tag{2.10}$$

$$2(1+\gamma)^2(1+\beta)^2 a_2^2 = [\mathcal{W}_1(y)]^2(c_1^2 + d_1^2). \tag{2.11}$$

Adding (2.7) and (2.9), we obtain

$$(1+\gamma)(2+\gamma+2\beta)b_2^2 = \mathcal{W}_1(y)(c_2+d_2) + \mathcal{W}_2(y)(c_1^2+d_1^2). \tag{2.12}$$

Substituting (2.11) into (2.12), we get

$$b_2^2 = \frac{[\mathcal{W}_1(y)]^3(c_2+d_2)}{(1+\gamma)(2+\gamma+2\beta)[\mathcal{W}_1(y)]^2 - 2\mathcal{W}_2(y)(1+\gamma)^2(1+\beta)^2}. \tag{2.13}$$

In view of (1.5), (2.1), and (2.13), we have

$$|b_2| \le \frac{\sqrt{2}|by+c|\sqrt{by+c}}{\sqrt{|(1+\gamma)(\gamma+2\beta+2)(by+c)^2 - 2(1+\gamma)^2(1+\beta)^2[3y(by+c)-2a]|}}.$$

Subtracting (2.9) from (2.7), we get

$$b_3 = \frac{\mathcal{W}_1(y)(c_2-d_2)}{2(2+\gamma)(1+2\beta)} + b_2^2. \tag{2.14}$$

From (2.11) and (2.14), we have

$$b_3 = \frac{\mathcal{W}_1(y)(c_2-d_2)}{2(2+\gamma)(1+2\beta)} + \frac{[\mathcal{W}_1(y)]^2(c_1^2+d_1^2)}{2(1+\gamma)^2(1+\beta)^2}. \tag{2.15}$$

In view of (1.5), (2.1), and (2.15), we get

$$|a_3| \le \frac{|by+c|}{(2+\gamma)(1+2\beta)} + \frac{(by+c)^2}{(1+\gamma)^2(1+\beta)^2}.$$

This completes the proof of the theorem. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

Theorem 2.1 leads to the following corollaries, as a consequence of Remark 1.2.

Corollary 2.2. *If* $\mathfrak{f} \in \Sigma$, *of the form* (1.1)*, belongs to the class* $\mathcal{S}_\Sigma^*(y)$*, then*

$$|b_2| \le \frac{|by+c|\sqrt{by+c}}{\sqrt{|(by+c)^2 - 3y(by+c) + 2a|}} \quad and \quad |b_3| \le \frac{|by+c|}{2} + (by+c)^2.$$

Remark 2.3. It is interesting to note that for the particular choice $a = 3$ and $b = c = y = 1$, we obtain the classical Brannan and Clunie conjecture [8], which states that $|b_2| \le \sqrt{2}$.

Corollary 2.4. *Let* $\mathfrak{f} \in \Sigma$ *be of the form* (1.1)*. If* $\mathfrak{f}$ *belongs to the class* $\mathcal{C}_\Sigma(y)$*, then*

$$|b_2| \le \frac{|by+c|\sqrt{by+c}}{\sqrt{|2(by+c)^2 - 12y(by+c) + 8a|}} \quad and \quad |b_3| \le \frac{|by+c|}{6} + \frac{(by+c)^2}{4}.$$

Corollary 2.5. *Suppose* $\mathfrak{f} \in \Sigma$ *is given by* (1.1)*. If* $\mathfrak{f} \in \mathcal{M}_\Sigma(\beta,y)$*, then*

$$|b_2| \le \frac{|by+c|\sqrt{by+c}}{\sqrt{|(1+\beta)(by+c)^2 - (1+\beta)^2[3y(by+c)-2a]|}}$$

and

$$|b_3| \le \frac{|by+c|}{2(1+2\beta)} + \frac{(by+c)^2}{(1+\beta)^2}.$$

Corollary 2.6. *If* $\mathfrak{f} \in \Sigma$*, of the form* (1.1)*, belongs to the class* $\mathcal{B}_\Sigma(\gamma, y)$*, then*

$$|b_2| \le \frac{\sqrt{2}|by+c|\sqrt{by+c}}{\sqrt{|(\gamma^2+3\gamma+2)(by+c)^2-2(1+\gamma)^2[3y(by+c)-2a]|}}$$

and

$$|b_3| \le \frac{|by+c|}{(2+\gamma)} + \frac{(by+c)^2}{(1+\gamma)^2}.$$

Corollary 2.7. *Let* $\mathfrak{f} \in \Sigma$ *be of the form* (1.1)*. If* $\mathfrak{f}$ *belongs to the class* $\mathcal{R}_\Sigma(y)$*, then*

$$|b_2| \le \frac{|by+c|\sqrt{by+c}}{\sqrt{|3(by+c)^2-12y(by+c)+8a|}} \quad and \quad |b_3| \le \frac{|by+c|}{3} + \frac{(by+c)^2}{4}.$$

3. Fekete-Szegö Estimate

The following theorem derives the famous Fekete-Szegö estimate for the class $\mathcal{B}_\Sigma^{\gamma,\beta}(y)$.

Theorem 3.1. *Consider a function* $\mathfrak{f}$ *of the form* (1.1) *belonging to the class* $\mathcal{B}_\Sigma^{\gamma,\beta}(y)$*. Then, for* $\beta \ge 0$*,* $0 \le \gamma \le 1$*, and* $\mu \in \mathbb{R}$*, the following result holds*

$$|b_3 - \mu b_2^2| \le \begin{cases} \dfrac{|by+c|}{(2+\gamma)(1+2\beta)}, \\ \quad if \ |1-\mu| \le \dfrac{|(1+\gamma)(2+\gamma+2\beta)(by+c)^2-2(1+\gamma)^2(1+\beta)^2[3y(by+c)-2a]|}{2(2+\gamma)(1+2\beta)(by+c)^2}, \\[2ex] \dfrac{2|by+c|^3|1-\mu|}{|(1+\gamma)(2+\gamma+2\beta)(by+c)^2-2(1+\gamma)^2(1+\beta)^2[3y(by+c)-2a]|}, \\ \quad if \ |1-\mu| \ge \dfrac{|(1+\gamma)(2+\gamma+2\beta)(by+c)^2-2(1+\gamma)^2(1+\beta)^2[3y(by+c)-2a]|}{2(2+\gamma)(1+2\beta)(by+c)^2}. \end{cases}$$

Proof. From (2.14), for any real number $\mu \in \mathbb{R}$, we have

$$b_3 - \mu b_2^2 = \frac{\mathcal{W}_1(y)(c_2-d_2)}{2(\gamma+2)(1+2\beta)} + (1-\mu)b_2^2. \tag{3.1}$$

Substituting (2.13) into (3.1), we obtain

$$b_3 - \mu b_2^2 = \frac{\mathcal{W}_1(y)(c_2-d_2)}{2(\gamma+2)(1+2\beta)} + \frac{(1-\mu)[\mathcal{W}_1(y)]^3(c_2+d_2)}{(1+\gamma)(2+\gamma+2\beta)[\mathcal{W}_1(y)]^2-2(1+\gamma)^2(1+\beta)^2\mathcal{W}_2(y)}$$

$$= \mathcal{W}_1(y)\left[\left(\Omega + \frac{1}{2(2+\gamma)(1+2\beta)}\right)c_2 + \left(\Omega - \frac{1}{2(2+\gamma)(1+2\beta)}\right)d_2\right], \tag{3.2}$$

where

$$\Omega = \frac{(1-\mu)[\mathcal{W}_1(y)]^2}{(1+\gamma)(2+\gamma+2\beta)[\mathcal{W}_1(y)]^2-2(1+\gamma)^2(1+\beta)^2\mathcal{W}_2(y)}.$$

Based on the result in [38, Lemma 7, p. 2], for any $k,l \in \mathbb{R}$ and $z_1, z_2 \in \mathbb{C}$ with $|z_1| < R$ and $|z_2| < R$, the following inequality holds

$$|(k+l)z_1 + (k-l)z_2| \le \begin{cases} 2|l|R, & if \ |k| \le |l|, \\ 2|k|R, & if \ |k| \ge |l|. \end{cases} \tag{3.3}$$

Applying (3.3) to (3.2), we get

$$|b_3 - \mu b_2^2| \leq \begin{cases} \dfrac{|\mathcal{W}_1(y)|}{(2+\gamma)(1+2\beta)}, & \text{if } 0 \leq |\Omega| \leq \dfrac{1}{2(2+\gamma)(1+2\beta)}, \\[3mm] 2|\mathcal{W}_1(y)||\Omega|, & \text{if } |\Omega| \geq \dfrac{1}{2(2+\gamma)(1+2\beta)}. \end{cases}$$

In view of (1.5), the proof of Theorem 3.1 is complete. $\qquad\square$

In view of Remark 1.2, the following corollaries are stated:

Corollary 3.2. *If $\mathfrak{f} \in \Sigma$, of the form (1.1), belongs to the class $\mathcal{S}_\Sigma^*(y)$, then for $\mu \in \mathbb{R}$, we have*

$$|b_3 - \mu b_2^2| \leq \begin{cases} \dfrac{|by+c|}{2}, & \text{if } |1-\mu| \leq \dfrac{|(by+c)^2 - 3y(by+c)+2a|}{2(by+c)^2}, \\[3mm] \dfrac{|by+c|^3|1-\mu|}{|(by+c)^2 - 3y(by+c)+2a|}, & \text{if } |1-\mu| \geq \dfrac{|(by+c)^2 - 3y(by+c)+2a|}{2(by+c)^2}. \end{cases}$$

Corollary 3.3. *Let $\mathfrak{f} \in \Sigma$ be of the form (1.1) and $\mu \in \mathbb{R}$. If $\mathfrak{f}$ belongs to the class $\mathcal{C}_\Sigma(y)$, then*

$$|b_3 - \mu b_2^2| \leq \begin{cases} \dfrac{|by+c|}{6}, & \text{if } |1-\mu| \leq \dfrac{|(by+c)^2 - 6y(by+c)+4a|}{3(by+c)^2}, \\[3mm] \dfrac{|by+c|^3|1-\mu|}{|2(by+c)^2 - 12y(by+c)+8a|}, & \text{if } |1-\mu| \geq \dfrac{|(by+c)^2 - 6y(by+c)+4a|}{3(by+c)^2}. \end{cases}$$

Corollary 3.4. *Suppose $\mathfrak{f} \in \Sigma$ is given by (1.1). If $\mathfrak{f} \in \mathcal{M}_\Sigma(\beta,y)$ and $\mu \in \mathbb{R}$, then*

$$|b_3 - \mu b_2^2| \leq \begin{cases} \dfrac{|by+c|}{2(1+2\beta)}, & \text{if } |1-\mu| \leq \dfrac{|(1+\beta)(by+c)^2 - (1+\beta)^2[3y(by+c)-2a]|}{2(1+2\beta)(by+c)^2}, \\[3mm] \dfrac{|by+c|^3|1-\mu|}{|(1+\beta)(by+c)^2 - (1+\beta)^2[3y(by+c)-2a]|}, & \\[2mm] & \text{if } |1-\mu| \geq \dfrac{|(1+\beta)(by+c)^2 - (1+\beta)^2[3y(by+c)-2a]|}{2(1+2\beta)(by+c)^2}. \end{cases}$$

Corollary 3.5. *If $\mathfrak{f} \in \Sigma$, of the form (1.1), belongs to the class $\mathcal{B}_\Sigma(\gamma,y)$ and $\mu \in \mathbb{R}$, then*

$$|b_3 - \mu b_2^2| \leq \begin{cases} \dfrac{|by+c|}{2+\gamma}, & \text{if } |1-\mu| \leq \dfrac{|(\gamma^2+3\gamma+2)(by+c)^2 - 2(1+\gamma)^2[3y(by+c)-2a]|}{2(2+\gamma)(by+c)^2}, \\[3mm] \dfrac{2|by+c|^3|1-\mu|}{|(1+\gamma)(2+\gamma+2\beta)(by+c)^2 - 2(1+\gamma)^2(1+\beta)^2[3y(by+c)-2a]|}, & \\[2mm] & \text{if } |1-\mu| \geq \dfrac{|(\gamma^2+3\gamma+2)(by+c)^2 - 2(1+\gamma)^2[3y(by+c)-2a]|}{2(2+\gamma)(by+c)^2}. \end{cases}$$

Corollary 3.6. *Let $\mathfrak{f} \in \Sigma$ be of the form (1.1) and $\mu \in \mathbb{R}$. If $\mathfrak{f}$ belongs to the class $\mathcal{R}_\Sigma(y)$, then*

$$|b_3 - \mu b_2^2| \leq \begin{cases} \dfrac{|by+c|}{3}, & \text{if } |1-\mu| \leq \dfrac{|3(by+c)^2 - 12y(by+c)+8a|}{3(by+c)^2}, \\[3mm] \dfrac{|by+c|^3|1-\mu|}{|3(by+c)^2 - 12y(by+c)+8a|}, & \text{if } |1-\mu| \geq \dfrac{|3(by+c)^2 - 12y(by+c)+8a|}{3(by+c)^2}. \end{cases}$$

4. Conclusion

We introduced a new subclass of generalized bi-Bazilevič-type functions associated with the generating function of generalized Mersenne polynomials in this paper. This class includes several well-known subclasses of bi-univalent functions. By applying the generating function of the generalized Mersenne polynomials, we obtained coefficient bounds for the initial Taylor-Maclaurin coefficients and provided estimates for the Fekete-Szegö inequality. Several corollaries were also obtained by specifying particular choices of parameters. Although the results presented are not sharp, future research may focus on determining sharp bounds and examining higher-order coefficients, which remains an open problem of interest.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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