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Research Article

On Upper and Lower *fsgb*-Continuous Multifunctions in Fuzzy Topological Spaces

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Abstract. In this study, the weaker forms of fuzzy multifunctions such as upper and lower *fsgb*-continuous multifunctions are introduced and their properties are examined. Further, the characteristics of upper and lower *fsgb*-irresolute multifunctions have also been studied.

Keywords. *fsgb-CS*, *fsgb-OS*, $u.$*fsgb-c*, $l.$*fsgb-c* multifunctions, $l.$*fsgb-i*, $u.$*fsgb-i*, *fsgb*-irresolute

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1. Introduction

The theory of multifunctions has developed greatly during the past three decades, and there are many applications of it, particularly in fixed point theory and functional analysis. Many writers have recently explored fuzzy multifunctions and characterised some properties of fuzzy multifunctions defined on a fuzzy topological space, such as Albrycht and Matłoka [1] and Beg [3, 4]. For fuzzy multifunctions, various authors have examined various forms of fuzzy continuity. Recently, applications to investigate graphs —which are employed in physics and smart cities— have made use of multifunctions in fuzzy topological spaces.

This work aims to provide a new weaker form of continuous functions known as upper *fsgb*-continuous multifunctions and lower *fsgb*-continuous multifunctions. Furthermore, features of an upper *fsgb*-continuous (or lower *fsgb*-continuous) multifunction are introduced and explored, along with their preservation theorems. In addition, the concept of upper *fsgb*-irresolute (resp. lower *fsgb*-irresolute) multifunctions are examined, along with an examination of their characteristics.

Throughout this study (L,τ), (M,σ), and (N,γ) (or simply L, M, and N) are fuzzy topological spaces (in short, *fts*). The interior, closure, and compliment of a fuzzy subset P of (L,τ) are denoted by $Int(P)$, $Cl(P)$, and P^c, respectively. Unless otherwise specified, no separation axioms are expected. If L is a *fts* and P is a subset of L, *fsgb*-closed set (*fsgb-CS*) (Kulkarni and Karnel [7]) if $bCl(P) \subseteq Q$, whenever $P \subseteq Q$ and Q is fuzzy generalized open set (*fg-OS*) in L.

For a fuzzy multifunction $g : L \to M$, we will indicate the lower and upper inverse of a set Q of M by $g^-(Q)$ and $g^+(Q)$ respectively, that is $g^-(Q) = \{l \in L : g(l) \cap Q \neq \varphi\}$ and $g^+(Q) = \{l \in L : g(l) \subset Q\}$.

2. Upper and Lower *fsgb*-Continuous Multifunctions

Definition 2.1. A fuzzy multifunction $g : L \to M$ is called upper *fsgb*-continuous (briefly, u.*fsgb-c*) at a point $l \in L$ if for every open subset Q in M with $g(l) \subseteq Q$, there exist an *fsgb-OS* P in L containing l such that $g(P) \subseteq Q$.

Definition 2.2. A fuzzy multifunction $g : L \to M$ is called lower *fsgb*-continuous (briefly, l.*fsgb-c*) at a point $l \in L$, if for each open subset Q in M with $g(l) \cap Q \neq \varphi$, there exist an *fsgb-OS* P containing l such that $g(r) \cap Q \neq \varphi$, for every $r \in P$.

Theorem 2.3. *If L and M be any two fuzzy topological spaces. Then, for a multifunction $g : L \to M$ the following properties are equivalent:*

(i) *g is u.fsgb-c at a point $l \in L$.*

(ii) *For every fuzzy closed set P with $g(l) \subseteq P$, there exists an fsgb-OS $Q(x)$ such that if $M \in Q$, then $g(y) \subseteq P$.*

(iii) *For every OS $Q \in O(M)$, then $g^+(Q)$ is fsgb-OS.*

(iv) *For every OS $R \in C(M)$, then $g^-(Q)$ is fsgb-CS.*

Proof. (i)$\Longleftrightarrow$(ii) is obvious.

(i)$\Longrightarrow$(iii): Suppose P be any f-*CS* in M and there exist a point $l \in g^+(P)$. Then by (i), there exist an *fsgb-OS* $P(l)$ such that $P(l) \subset g^+(P)$. Hence, $l \in Int(g^+(P))$ and thus $g^+(P)$ is a *fsgb-OS* in L.

(iii)$\Longrightarrow$(iv): Let Q be a f-*OS* in M, then $M \backslash Q$ is a *fsgb-CS* in M. By (iii), $g^+(M\backslash Q)$ is *fsgb*-open. As $g^+(M\backslash Q) = M \backslash g^-(Q)$. Hence $g^-(Q)$ is *fsgb-CS*.

(iv)$\Longrightarrow$(iii): It is similar to that of (iii)$\Longrightarrow$(iv).

(iii)$\Longrightarrow$(i): Suppose Q be any f-*CS* in M and $\in g^+(Q)$. Then by (iii), $g^+(Q)$ is *fsgb-OS* in L. Let $P = g^+(Q)$ then $P \subset g^+(Q)$. Hence, g is u.*fsgb-c*. □

Theorem 2.4. *The following properties are equivalent for a multifunction* $g : L \to M$:

(i) g *is* l*.fsgb-c at a point* $l \in L$.

(ii) $g^{-}(P)$ *is open for every fsgb-CS* P *in* M.

(iii) $g^{+}(Q)$ *is closed for every fsgb-OS* Q *in* M.

Theorem 2.5. *If* $g : L \to M$ *is* u*.fsgb-c iff for every OS* Q *in* M, $g^{-}(Q)$ *is open in* L.

Proof. Suppose $Q \in O(M)$ and Then by definition of u*.fsgb-c*, there exist $R \in$ *fsgb-O*(L) with $g(R) \subseteq Q$, where $g^{+}(Q) \in O(L)$. Let $g^{+}(Q)$ is open and $l \in g^{-}(Q)$, then $g^{+}(Q) = \{l \in Q : g(L) \subseteq Q\}$. Thus, g is u*.fsgb-c*. □

Theorem 2.6. *A fuzzy multifunction* $g : L \to M$ *is* l*.fsgb-c iff for every OS* Q *in* M, $g^{-}(Q)$ *is open in* L.

Proof. Suppose $Q \in O(M)$ and $l \in g^{+}(Q)$. Then by l*.fsgb-c*, there exists $R \in$ *fsgb-O*(L) with $g(R) \cap Q \neq \varphi$. As $r \in R$ and thus $g^{-}(Q) \in O(L)$. If $g^{-}(Q)$ is open and $l \in g^{-}(Q)$. Then, $g^{-}(Q) = \{ l \in L : g(L)) \cap Q \neq \varphi\}$. Therefore, g is l*.fsgb-c*. □

Theorem 2.7. *For a fuzzy multifunction* $g : (L,\tau) \to (M,\sigma)$, *the following statements are equivalent:*

(i) g *is* u*.fsgb-c*.

(ii) g*(fsgb-Cl*$(Q)) \subseteq cl(g(Q))$, *for every* $Q \subset L$.

(iii) *fsgb-Cl*$(g^{+}(P)) \subseteq (g^{+}Cl(P))$, *for every* $P \subset M$.

(iv) $g^{-}(Int(P)) \subseteq$ *fsgb-Int*$(g^{-}(P))$, *for every* $P \subset M$.

(v) $Int(g(Q)) \subseteq g$*(fsgb-Int*$(Q))$, *for every* $Q \subset L$.

Proof. (i)⟹(ii): Consider $Q \subseteq L$, we have $g(Q) \subset cl(g(Q))$, where $Cl(g(Q))$ is closed in M. Also, g is u*.fsgb-c*, $Q \subset (g^{+}Clg(Q))$. From Theorem 2.3, $g^{+}Cl(g(Q))$ is *fsgb*-closed in L. Therefore, *fsgb-Cl*$(Q) \subseteq g^{+}Cl(g(Q))$ and hence g*(fsgb-Cl*$(Q)) \subseteq Cl(g(Q))$.

(ii)⟹(iii): Consider $P \subset M$, so $(g^{+}(P) \subset L$. From (ii), g*(fsgb-Cl*$(g^{+}(P)) \subseteq Cl(g(g^{+}(P))) + Cl(P)$, hence *fsgb-Cl*$(g^{+}(P)) \subseteq g^{+}(Cl(P))$.

(iii)⟹(iv): Consider $P \subset M$, apply (iii) to $M \backslash P$ then *fsgb-Cl*$(g^{+}(M\backslash P)) \subseteq g^{+}Cl(M\backslash P)) \Longleftrightarrow$ *fsgb-Cl*$(L\backslash g^{-}(P)) \subseteq g^{+}(M\backslash Int(A)) \Leftrightarrow L\backslash$*fsgb-Int*$(g^{-}(P)) \subseteq L\backslash g^{-}(Int(P)) \Longleftrightarrow g^{-}(Int(P)) \subseteq$ *fsgb-Int*$(g^{-}(P))$.

(iv)⟹(v): Consider $Q \subset L$, so $g(Q) \subset M$. From (iv), $g^{-}(Int(g(P)) \subseteq$ *fsgb-Int*$(g^{-}(g(P))) =$ *fsgb-Int*(P). Hence, $Int(g(P)) \subseteq g(fsgb - Int(P))$.

(v)⟹(i): Let $l \in L$ and $P \in O(M, g(l))$. So, $l \in g^{+}(P)$, where $g^{+}(P) \subset L$. From (v), we have $Int(g(g^{+}(P)) \subseteq g$*(fsgb-Int*$(g^{+}(P)))$. Then, $Int(P) \subseteq g$*(fsgb-Int*$(g^{+}(A)))$. As P is open, $P \subseteq g$*(fsgb-Int*$(g^{+}(P)))$ that is $g^{+}(P) \subseteq$ *fsgb-Int*$(g^{+}(P))$. Hence $g^{+}(P) \in$ *fsgb-O*(L,l) and $g(g^{+}(P)) \subseteq P$. Therefore, g is u*.fsgb-c*. □

Theorem 2.8. *Suppose a fuzzy multifunction* $g : L \to M$ *is fuzzy upper-continuous and* M *is submaximal then* g *is* u*.fsgb-c*.

Proof. Consider $P \in l\text{-}O(M)$. As M is submaximal, then $P \in O(M)$ (Balasubramanian [2]). As, g is fuzzy upper-continuous $g^+(P)$ is open in L and therefore $g^+(P)$ is *fsgb*-open in L. Hence, g is $u.fsgb\text{-}c$. □

Theorem 2.9. *Consider $g: L \to M$ be $u.fsgb\text{-}c$. If M is fuzzy closed subset of N, then $g: L \to N$ is $u.fsgb\text{-}c$.*

Proof. Suppose $P \subset C(N)$ and so $P \cap N$ is *fsgb*-closed. By $u.fsgb\text{-}c$, $g^+(P \cap N) \in fsgb\text{-}C(L)$ with $g(L) \in M$, for all $l \in L$. Hence, $g^+(P) = g^+(P \cap N)$ is *fsgb*-closed in L. From Theorem 2.3, g is $u.fsgb\text{-}c$. □

Theorem 2.10. *Suppose $g:L \to M$ be $u.fsgb\text{-}c$ and $P \in fsgb\text{-}C(L)$. Then, $g \backslash P : P \to M$ is $u.fsgb\text{-}c$.*

Proof. Consider $Q \in C(M)$. As g is $u.fsgb\text{-}c$, then $g^+(Q) \in fsgb\text{-}C(L)$. As intersection of two *fsgb*-closed sets is fuzzy closed, then $g^+(Q) \cap P = P_1$, where $P_1 \in fsgb\text{-}C(L)$, and thus $(g \backslash P)^+(Q) = P_1$ is *fsgb-CS* in M. Hence $g \backslash P$ is $u.fsgb\text{-}c$. □

Theorem 2.11. *Suppose $g_1 : L \to M$, $g_2 : M \to N$ be two fuzzy multifunctions. Then, $g_1 \circ g_2 : L \to N$ is $u.fsgb\text{-}c$ if g_2 is fuzzy-irresolute and g_1 is $u.fsgb\text{-}c$.*

Proof. Consider $P \in bcl(N)$ and so $g_2^+(P) \in bcl(M)$ as g_2 is fuzzy-irresolute. Thus $g_2(P) \in fsgb\text{-}C(M)$, as g_1 is $u.fsgb\text{-}c$. Then, $g_1^+(g_2(P)) \in fsgb\text{-}C(L)$ and thus $g_1 \circ g_2$ is $u.fsgb\text{-}c$. □

3. Upper (Lower) *fsgb*-Irresolute Fuzzy Multifunctions

Definition 3.1. A fuzzy multifunction $g : L \to M$ is known as upper *fsgb*-irresolute (briefly, $u.fsgb\text{-}i$) if for every $l \in L$ and every $P \in fsgb\text{-}O(M, g(l))$, there exists $Q \in fsgb\text{-}O(L, l)$ such that $g(Q) \subseteq P$.

Definition 3.2. A fuzzy multifunction $g : L \to M$ is known as lower *fsgb*-irresolute (briefly, $l.fsgb\text{-}i$) if for every $l \in L$ and every *fsgb-OS* P with $g(l) \cap P \neq \emptyset$, there exists $Q \in fsgb\text{-}O(L, l)$ such that $Q \subseteq g^-(P)$.

Theorem 3.3. *Consider $g : L \to M$ be a fuzzy multifunction. Then $(fsgb\text{-}Cl(g))^-(P) = g^-(P)$, for each $P \in fsgb\text{-}O(M)$.*

Proof. Suppose $P \in fsgb\text{-}O(M)$ with $l \in (fsgb\text{-}Cl(g))\text{-}C(P)$, thus that $P \cap (fsgb\text{-}Cl(g))(l) \neq \varphi$, as $P \in fsgb\text{-}O(M)$, and thus $P \cap g(l) \neq \varphi$. Hence $l \in g^-(P)$.
Conversely, let $l \in g^-(P)$. Then $P \cap g(l) \subseteq (fsgb\text{-}Cl(g))(l) \cap P \neq \varphi$, and thus $l \in (fsgb\text{-}Cl(g))^-(P)$. Hence $(fsgb\text{-}Cl(g))^-(P) = g^-(P)$. □

Theorem 3.4. *For a fuzzy multifunction $g : L \to M$, the following statements are equivalent:*

(i) *g is $u.fsgb\text{-}i$*

(ii) *for each $l \in L$, for every fsgb-nbd P of $g(l)$, $g^+(P)$ is fsgb-nbd of l.*

(iii) *for each $l \in L$, for every fsgb-nbd P of $g(l)$, there exists fsgb-nbd Q of l with $g(Q) \subseteq P$.*

(iv) *$g^+(P) \in fsgb\text{-}O(L)$, for every $P \in fsgb\text{-}O(M)$.*

(v) *$g^-(P) \in fsgb\text{-}C(L)$, for every $P \in fsgb\text{-}C(M)$.*

(vi) $fsgb\text{-}Cl(g^-(R)) \subset g^-(fsgb\text{-}Cl(g^-(R))$, *for every* $R \subset M$.

Proof. (i)$\Longrightarrow$(ii): Consider $l \in L$ and V be *fsgb*-nbd of $g(l)$. Then, there exists $P \in fsgb\text{-}O(M)$ with $g(l) \subset P \subset V$. Since g is $u.fsgb\text{-}i$, then there exists $Q \in fsgb\text{-}O(L,l)$ such that $g(Q) \subseteq P$. Hence $l \in Q \subset g^+(P) \subset g^+(V)$ and so $g^+(V)$ is a *fsgb*-nbd of l.

(ii)$\Longrightarrow$(iii): Consider $l \in L$ and P be a *fsgb*-nbd of $g(l)$. Let $Q = g^+(P)$. Then by (ii), Q is *fsgb*-nbd of l with $g(Q) \subseteq P$.

(iii)$\Longrightarrow$(iv): Let $P \in fsgb\text{-}O(M)$ and $l \in g^+(P)$. Then, there exists *fsgb*-nbd W of l with $g(W) \subseteq P$. Hence for some $Q \in fsgb\text{-}O(L,l)$ with $Q \subseteq W$ and $g(Q) \subseteq P$. So, $l \in P \subset g^+(P)$, and therefore $g^+(P) \in fsgb\text{-}O(M)$.

(iv)$\Longrightarrow$(v): Consider $U \in fsgb\text{-}C(M)$ then $L \backslash g^-(U) = g^+(M \backslash K) \in fsgb\text{-}O(L)$. Hence $g^-(U) \in fsgb\text{-}C(M)$.

(v)$\Longrightarrow$(vi): Consider $R \subset M$. As $fsgb\text{-}Cl(R)$ is *fsgb*-closed in M. So $g^-(fsgb\text{-}Cl(R))$ is *fsgb*-closed in L with $g^-(R) \subset g^-(fsgb\text{-}Cl(R)$. Hence $fsgb\text{-}Cl(g^-(R)) \subset g^-(fsgb\text{-}Cl(R))$.

(vi)$\Longrightarrow$(i): Consider $l \in L$ and $P \in fsgb\text{-}O(M)$ with $g(l) \subset P$. So $g(l) \cap (M \backslash P) = \varphi$. Thus, $l \in g^-(M \backslash P)$. From (vi) $l \in fsgb\text{-}Cl(g^-(M \backslash P))$ and so there exists $Q \in fsgb\text{-}O(L,l)$ such that $Q \cap g^-(M \backslash P) = \varphi$. Therefore, $g(Q) \subseteq P$ and so g is $u.fsgb\text{-}i$. □

Theorem 3.5. *For a fuzzy multifunction* $g : L \to M$, *the following statements are equivalent:*

(i) g *is* $l.fsgb\text{-}i$.

(ii) *for every* $P \in fsgb\text{-}O(M)$ *and every* $l \in g^-(P)$, *there exists* $Q \in fsgb\text{-}O(L,l)$ *such that* $Q \subset g^-(P)$.

(iii) $g^-(P) \in fsgb\text{-}O(L)$, *for every* $P \in fsgb\text{-}O(M)$.

(iv) $g^+(K) \in fsgb\text{-}C(L)$, *for every* $K \in fsgb\text{-}C(M)$.

(v) *for every* $U \in L$, $g(fsgb\text{-}Cl(U)) \subset fsgb\text{-}Cl(g(U))$.

(vi) $fsgb\text{-}Cl(g^+(V)) \subset g^+(fsgb\text{-}Cl(V))$, *for every* $V \subset M$.

Proof. (i)$\Rightarrow$(ii): Follows from Definition 3.1.

(ii)$\Longrightarrow$(iii): Let $P \in fsgb\text{-}O(M)$ with $l \in g^-(P)$. From (ii) there exists $Q \in fsgb\text{-}O(L,l)$ such that $Q \subset g^-(P)$. Therefore, $l \in Q \subset Cl(Int(Q) \cup Int(Cl(Q)) \subset Cl(Int(g^-(Q))) \cup Int(Cl(g^-(Q)))$. Thus, $g^-(P) \in fsgb\text{-}O(L)$.

(iii)$\Longrightarrow$(iv): Consider $K \in fsgb\text{-}C(M)$, then $L \backslash g^+(K) = g^-(M \backslash K) \in fsgb\text{-}O(L)$ and hence $g^+(K) \in fsgb\text{-}C(L)$.

(iv)$\Longrightarrow$(v): Follows from the definition.

(v)$\Longrightarrow$(vi): Follows from the definition.

(vi)$\Longrightarrow$(i): Let $l \in L$ and $P \in fsgb\text{-}O(M)$ with $g(l) \cap P \neq \varphi$. So $g(l) \cap (M \backslash P) = \varphi$. Then $g(l) \nsubseteq M \backslash P$ and $l \notin g^+(M \backslash P)$. Since $M \backslash P \in fsgb\text{-}C(M)$, and from (vi), $l \notin fsgb\text{-}Cl(g^+(M \backslash P))$. So that there exists $Q \in fsgb\text{-}O(L,l)$ with $Q \cap g^-(M \backslash P) = Q \cap (L \backslash g^-(P)) = \varphi$. Hence $Q \subset L \backslash g^-(P)) = g^-(P)$, that is $Q \subset g^-(P)$. Therefore g is $l.fsgb\text{-}i$. □

Theorem 3.6. *Let* $P, Q \subseteq L$. *Then*

(i) *if* $P \in fsgb\text{-}O(L)$ *and* $Q \in L$ *then* $P \cap Q \in fsgb\text{-}O(Q)$.

(ii) *if* $P \in fsgb\text{-}O(Q)$ *and* $Q \in fsgb\text{-}O(L)$ *then* $P \in fsgb\text{-}O(L)$.

Theorem 3.7. *Consider* $g : L \to M$ *be a fuzzy multifunction and* $P \in O(L)$*. If* g *is* $u.fsgb\text{-}i$ *(resp.,* $l.fsgb\text{-}i$*) then* $g_{l_p} : P \to M$ *is an* $u.fsgb\text{-}i$ *(resp.,* $l.fsgb\text{-}i$*).*

Proof. Consider $Q \in fsgb\text{-}O(M)$, let $l \in P$ and $l \in g^{-}_{l_p}(Q)$. Since g is $l.fsgb\text{-}i$, there exists $U \in fsgb\text{-}O(L,l)$ with $U \subseteq g^{-}(Q)$ and so $l \in U \cap P \in fsgb\text{-}O(P)$ and $U \cap P \subseteq g_{l_p}(Q)$. Therefore, g_{l_p} is $l.fsgb\text{-}i$. Similarly, we can prove for $u.fsgb\text{-}i$. □

4. Conclusion

We have examined new weaker forms of two new categories of continuous functions in this work: upper *fsgb*-continuous multifunctions and lower *fsgb*-continuous multifunctions. The characteristics and preservation theorems of an upper *fsgb*-continuous (or lower *fsgb*-continuous) multifunction have been established. These recently established ideas have scope for further research and development in fuzzy topological spaces.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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