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Research Article

A New More General Integral Transform: Sharad Transform and Its Applications

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Abstract. In this paper, we are presenting the new broad essential transform called the Sharad transform. Additionally, we demonstrated a few theorems and properties, similar to the linearity property and the convolution theorem. Additionally, we fostered a connection between the new transform and another old useful transform, similar to the Laplace transform, the Sumudu transform, the natural transform, the Elzaki transform, the Abhoodh transform, the Formable transform, and so forth. Here we presume that these transforms are the extraordinary instances of the new broad basic transform, the Sharad transform. This transform is applied to ordinary differential equations and partial differential equations. This transform will be utilised in the future to tackle a lot more the ordinary differential equations, the partial differential equations, integro-differential equations, fractional differential equations, difference equations, differential-difference equations, and so forth.

Keywords. Sharad transform, Laplace transform, Partial Differential Equation, Ordinary Differential Equation, Integral transform

Mathematics Subject Classification (2020). 35A22, 44A35, 44A05, 44A10, 42B10

1. Introduction

As of late, addressing differential equations like partial and ordinary differential equations, integro-differential equations, fractional differential equations, and so forth has become vital as so many everyday life problems include them. Subsequently, numerous techniques were developed to address these. The strategy for utilising the integral transform has demonstrated its presentation and relevance in addressing the ordinary and the partial differential equations, integro-differential equations, fractional differential equations, and so on. These transforms are used to convert differential equations into algebraic equations, solve this problem in this domain, and then apply inverse transforms to it for the original solution of our problem. The main aim of the technique is to solve differential equations using an easier method. This technique plays a vital role in obtaining solutions to the differential equations.

In the last thirty years, many transforms have been introduced and proven efficient for solving differential equations. Initially, in 1780, French mathematician and physician P. S. Laplace (cf. Debnath and Bhatta [2]), introduced the Laplace transform. Also in 1822, J. Fourier (cf. Debnath and Bhatta [2]) introduced the Fourier transform. These two transforms have powerful applications in applied mathematics as well as other sciences like physics, engineering, and astronomy. Then many transforms were introduced, like the Sumudu transform (Watugala [19]), the Natural transform (Khan and Khan [6]), the Elzaki transform (Elzaki [3], and Kuffi *et al.* [7]), the formable integral transform (Saadeh and Ghazal [14]), NE transform (Xhaferraj [20]) etc., to prove their efficacy in solving the differential equations. Some more transforms are discussed in relation between Sharad transform and other transform relation.

In this paper, we introduced an integral transform called the Sharad transform in Section 2, proved some theorems and properties like the convulsion theorem and the linearity property in Section 3, and found the Sharad transform of some specific functions in Section 3. Find the relationship between the Sharad transform and certain ancient transforms in the table in Section 4. The Sharad transform was then applied to an ordinary differential equation in its subsequent part.

2. Sharad Transform

In this section, we will define a new integral transform called the Sharad transform. Also, its inverse transform.

We know that "A function $g(t)$ is called the exponential order of α if there exists a constant P and T such that $g(t) \le Pe^{\alpha t}$, for all $t \ge T$."

Definition 1 (Sharad Transform)**.** A new integral transform, the Sharad transform of the function $g(t)$ is denoted by the symbol $\mathcal{T}[g(t)]$, and it is defined as

$$\mathcal{T}[g(t)] = G(s,u) = p(s)\int_0^{\infty} e^{-q(s)t} g(ut)dt \tag{2.1}$$

provided the integral exists for $t \ge 0$, $u > 0$, $t,u \in \mathbb{R}$ and $p(s) \ne 0$ and $q(s)$ are complex functions of variable $s \in \mathbb{C}$.

Here s and u are transform variables. For its existence, the integral of the function $g(t)$ must be of exponential order as well as piece-wise continuous on the interval $[0,\infty)$. The equation (2.1) is equivalent to

$$\mathcal{T}[g(t)] = G(s,u) = \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t} g(t)dt.$$

Definition 2. The inverse Sharad transform of the function $G(s,u)$ is the original function $g(t)$, which is defined as

$$\mathcal{T}^{-1}[G(s,u)] = g(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{q'(s)}{p(s)} e^{\frac{q(s)t}{u}} G(s,u)ds\,.$$

Here $p(s) \neq 0$ and $q'(s)$ is a derivative of $q(s)$ with respect to s assumed to exist. The expression in the definition can be proved by using the Fourier transform and its inverse transform.

3. Properties and Theorems

In this section, the author verified some properties and theorems of the Sharad transform.

Theorem 1 (Linearity Property). *Let $g(t)$ and $f(t)$ be functions whose Sharad transform exists, and they are denoted by $G(t)$ and $F(t)$, respectively. Then, the Sharad transform of $\gamma f(t) + \lambda g(t)$ exists and is as below:*

$$\mathcal{T}[\gamma f(t) + \lambda g(t)] = \gamma F(t) + \lambda G(t), \quad \text{where } \gamma \text{ and } \lambda \text{ are constant.}$$

Proof.

$$\begin{aligned}\mathcal{T}[\gamma f(t) + \lambda g(t)] &= \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t}[\gamma f(t) + \lambda g(t)]dt \\ &= \gamma\left[\frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t} f(t)dt\right] + \lambda\left[\frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t} g(t)dt\right] \\ &= \gamma F(t) + \lambda G(t).\end{aligned}$$

□

Theorem 2 (The Differentiation of Sharad Transform). *If $g(t)$ has a Sharad transform $G(s,u)$, the Sharad transform of a derivative of different order is as follows:*

(1) $\mathcal{T}[g'(t)] = \frac{q(s)}{u}\mathcal{T}[g(t)] - \frac{p(s)}{u}g(0)$,

(2) $\mathcal{T}[g''(t)] = \frac{[q(s)]^2}{u^2}\mathcal{T}[g(t)] - \frac{p(s)q(s)}{u^2}g(0) - \frac{p(s)}{u}g'(0)$,

(3) $\mathcal{T}[g'''(t)] = \frac{[q(s)]^3}{u^3}\mathcal{T}[g(t)] - \frac{[q(s)]^2}{u^2}\frac{p(s)}{u}g(0) - \frac{q(s)}{u}\frac{p(s)}{u}g'(0) - \frac{p(s)}{u}g''(0)$,

(4) $\mathcal{T}[g^{(n)}(t)] = \left(\frac{q(s)}{u}\right)^n \mathcal{T}[g(t)] - \frac{p(s)}{u}\sum_{k=0}^{n-1}\left(\frac{q(s)}{u}\right)^{n-1-k} g^k(0)$.

Proof.

(1)
$$\begin{aligned}\mathcal{T}[g'(t)] = G'(s,u) &= \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t} g'(t)dt \\ &= \frac{p(s)}{u}\lim_{x\to\infty}\left(\int_0^x e^{-\frac{q(s)}{u}t} g'(t)dt\right) \qquad \text{(integrating by parts)} \\ &= \frac{p(s)}{u}\left[\left(e^{-\frac{q(s)}{u}t} g(t)\right)_0^x - \int_0^\infty \left(-\frac{q(s)}{u}\right) e^{-\frac{q(s)}{u}t} g(t)dt\right]\end{aligned}$$

$$= -\frac{p(s)}{u}g(0) + \frac{q(s)}{u}\left(\frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t}g(t)dt\right)$$
$$= \frac{q(s)}{u}\mathcal{T}[g(t)] - \frac{p(s)}{u}g(0),$$

(2) $$\mathcal{T}[g''(t)] = G'(s,u) = \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t}g''(t)dt$$
$$= \frac{p(s)}{u}\lim_{x\to\infty}\left(\int_0^x e^{-\frac{q(s)}{u}t}g''(t)dt\right) \qquad \text{(integrating by parts)}$$
$$= \frac{p(s)}{u}\left[\left(e^{-\frac{q(s)}{u}t}g'(t)\right)_0^x - \int_0^\infty\left(-\frac{q(s)}{u}\right)e^{-\frac{q(s)}{u}t}g'(t)dt\right]$$
$$= -\frac{p(s)}{u}g'(0) + \frac{q(s)}{u}\left(\frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t}g'(t)\,dt\right)$$
$$= -\frac{p(s)}{u}g'(0) + \frac{q(s)}{u}\left(\frac{q(s)}{u}\mathcal{T}[g(t)] - \frac{p(s)}{u}g(0)\right) \qquad \text{(by proof of (2.1))}$$
$$= \frac{[q(s)]^2}{u^2}\mathcal{T}[g(t)] - \frac{p(s)q(s)}{u^2}g(0) - \frac{p(s)}{u}g'(0),$$

(3) $$\mathcal{T}[g'''(t)] = G'''(s,u) = \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t}g'''(t)dt$$
$$= \frac{p(s)}{u}\lim_{x\to\infty}\left(\int_0^x e^{-\frac{q(s)}{u}t}g'''(t)dt\right) \qquad \text{(integrating by parts)}$$
$$= \frac{p(s)}{u}\left[\left(e^{-\frac{q(s)}{u}t}g''(t)\right)_0^x - \int_0^\infty\left(-\frac{q(s)}{u}\right)e^{-\frac{q(s)}{u}t}g''(t)dt\right]$$
$$= -\frac{p(s)}{u}g''(0) + \frac{q(s)}{u}\left(\frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t}g''(t)\,dt\right)$$
$$= -\frac{p(s)}{u}g''(0) + \frac{q(s)}{u}\left(\frac{[q(s)]^2}{u^2}\mathcal{T}[g(t)] - \frac{p(s)q(s)}{u^2}g(0) - \frac{p(s)}{u}g'(0)\right)$$
(by proof of (6.1))
$$= \frac{[q(s)]^3}{u^3}\mathcal{T}[g(t)] - \frac{[q(s)]^2}{u^2}.\frac{p(s)}{u}g(0) - \frac{q(s)}{u}\frac{p(s)}{u}g'(0) - \frac{p(s)}{u}g''(0),$$

(4) Proof can be obtained by using mathematical induction on n. □

Theorem 3 (First Translation Property or Shifting Property). *Let $g(t)$ be the continues function and $t \geq 0$, then*
$$\mathcal{T}[e^{\alpha t}g(t)] = \frac{q(s)}{q(s)-\alpha u}G\left(s, \frac{u}{q(s)-\alpha u}\right).$$

Proof.
$$\mathcal{T}[e^{\alpha t}g(t)] = G(s,u) = p(s)\int_0^\infty e^{-q(s)t}e^{\alpha ut}g(ut)dt$$
$$= p(s)\lim_{x\to\infty}\left(\int_0^x e^{-q(s)t}e^{\alpha ut}g(ut)dt\right)$$
$$= p(s)\lim_{x\to\infty}\left(\int_0^x e^{-(q(s)-\alpha u)t}g(ut)dt\right)$$
$$= \frac{p(s)q(s)}{q(s)-\alpha u}\int_0^\infty e^{-q(s)t}g\left(\frac{uq(s)t}{q(s)-\alpha u}\right)dt \qquad \text{(by change of variable)}$$

$$= \frac{q(s)}{q(s)-\alpha u} G\left(s, \frac{u}{q(s)-\alpha u}\right). \qquad \square$$

Theorem 4 (Change of Scale Property). *Let $g(t)$ be the continues function and $t \geq 0$, then*

$$\mathcal{T}[g(\alpha t)] = G(s, au)$$

and

$$\mathcal{T}\left[g\left(\frac{t}{\alpha}\right)\right] = G\left(s, \frac{u}{\alpha}\right), \quad \alpha \neq 0.$$

Proof.

$$\mathcal{T}[g(\alpha t)] = p(s)\int_0^\infty e^{-q(s)t} g(\alpha ut)dt$$

$$= \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t} g(\alpha t)dt$$

put $w = \alpha t$, then, we get

$$= \frac{p(s)}{\alpha u}\int_0^\infty e^{-\frac{q(s)}{\alpha u}t} g(w)dw$$

$$= G(s, au).$$

For second proof in above proof just replace α by $\frac{1}{\alpha}$. $\square$

Theorem 5 (Convolution). *Let $g_1(t)$ and $g_2(t)$ be the functions with the Sharad transform $G_1(s,u)$ and $G_2(s,u)$, respectively. Then, the convolution of the functions g_1 and g_2 is as follows:*

$$\mathcal{T}[g_1 * g_2] = \frac{u}{p(s)} G_1(s,u)G_2(s,u), \quad where\ g_1 * g_2 = \int_0^t g_1(\tau)g_2(t-\tau)d\tau.$$

Proof. We can write by using the definition of the Sharad transform,

$$\mathcal{T}[g_1 * g_2] = p(s)\int_0^\infty e^{-q(s)t}(g_1 * g_1)(ut)dt$$

$$= p(s)\int_0^\infty e^{-q(s)t}\left(\int_0^{ut} g_1(\tau)g_2(ut-\tau)d\tau\right)dt \quad (\text{put } \tau = ux \text{ and } d\tau = udx \text{ then } x \text{ is in } [0,t])$$

$$= up(s)\int_0^\infty e^{-q(s)t}\left(\int_0^t g_1(ux)g_2(u(t-x))dx\right)dt \qquad (\text{put } y = t-x \text{ then } dy = dt)$$

$$= up(s)\int_0^\infty e^{-q(s)(y+x)} g_1(ux)g_2(uy)dxdy$$

$$= \frac{u}{p(s)} G_1(s,u)G_2(s,u). \qquad \square$$

4. Sharad Transform of Standard Functions

In this section, we are able to find the Sharad transform of some special functions.

(1) Let $g(t) = k$, where k is constant. Applying Sharad transform to both sides, we get

$$\mathcal{T}[g(t)] = G(s,u) = \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t} k dt$$

$$= \frac{kp(s)}{u}\lim_{x\to\infty}\left(\int_0^x e^{-\frac{q(s)}{u}t} dt\right)$$

$$= \frac{kp(s)}{u}\left[\left(-\frac{u}{q(s)}e^{-\frac{q(s)}{u}t}\right)_0^x\right]$$
$$= \frac{kp(s)}{q(s)}.$$

(2) Let $g(t) = t$, for all $t > 0$. Applying Sharad transform to both sides, we get

$$\mathcal{T}[g(t)] = G(s,u) = \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t}\, t\,dt$$
$$= \frac{p(s)}{u}\lim_{x\to\infty}\left(\int_0^x e^{-\frac{q(s)}{u}t}\, t\,dt\right) \qquad \text{(integrating by parts)}$$
$$= \frac{p(s)}{u}\left[\left(-\frac{ut}{q(s)}e^{-\frac{q(s)}{u}t}\right)_0^x - \left(1\left(\frac{u^2}{[q(s)]^2}\right)e^{-\frac{q(s)}{u}t}\right)_0^x\right]$$
$$= \frac{up(s)}{[q(s)]^2}.$$

(3) Let $g(t) = t^2$, for all $t > 0$. Applying Sharad transform to both sides, we get

$$\mathcal{T}[g(t)] = G(s,u) = \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t}\, t^2 dt$$
$$= \frac{p(s)}{u}\lim_{x\to\infty}\left(\int_0^x e^{-\frac{q(s)}{u}t}\, t^2 dt\right) \qquad \text{(integrating by parts)}$$
$$= \frac{p(s)}{u}\left[\left(-\frac{ut^2}{q(s)}e^{-\frac{q(s)}{u}t}\right)_0^x - \lim_{x\to\infty}\left(\int_0^x -\frac{u}{q(s)}2t\,e^{-\frac{q(s)}{u}t}\,dt\right)\right]$$
$$= \frac{2p(s)}{q(s)}\lim_{x\to\infty}\left(\int_0^x t\,e^{-\frac{q(s)}{u}t}\,dt\right)$$
$$= \frac{2u^2p(s)}{[q(s)]^3}.$$

(4) Let $g(t) = t^n$, for all $t > 0$ and $n \in N$. Then

$$\mathcal{T}[g(t)] = \frac{\Gamma(n+1)u^n p(s)}{[q(s)]^{n+1}}.$$

We can prove it by using mathematical induction on $n \in N$.

(5) Let $g(t) = e^{at}$, for all $t, a > 0$. Applying Sharad transform to both sides, we get

$$\mathcal{T}[g(t)] = G(s,u) = \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t}\, e^{at} dt$$
$$= \frac{p(s)}{u}\lim_{x\to\infty}\left(\int_0^x e^{-\frac{q(s)}{u}t}\, e^{at} dt\right)$$
$$= \frac{p(s)}{u}\lim_{x\to\infty}\left(\int_0^x e^{\left(\frac{au-q(s)}{u}\right)t} dt\right)$$
$$= \frac{p(s)}{u}\left[\left(\frac{u}{au-q(s)}e^{\left(\frac{au-q(s)}{u}\right)t}\right)_0^x\right]$$
$$= \frac{p(s)}{q(s)-au}, \qquad \text{where } q(s) > au.$$

(6) Let $g(t)=e^{iat}$, for all $t,a>0$. Applying Sharad transform to both sides, we get

$$\begin{aligned}\mathcal{T}[g(t)]=G(s,u) &= \frac{p(s)}{u}\int_0^\infty e^{-\frac{q(s)}{u}t}e^{iat}dt \\ &= \frac{p(s)}{u}\lim_{x\to\infty}\left(\int_0^x e^{-\frac{q(s)}{u}t}e^{iat}dt\right) \\ &= \frac{p(s)}{u}\lim_{x\to\infty}\left(\int_0^x e^{\left(\frac{iau-q(s)}{u}\right)t}dt\right) \\ &= \frac{p(s)}{u}\left[\left(\frac{u}{au-q(s)}e^{\left(\frac{iau-q(s)}{u}\right)t}\right)_0^x\right] \\ &= \frac{p(s)}{q(s)-iau} \qquad \text{(because } q(s)>iau) \\ &= \frac{p(s)[q(s)+iau]}{[q(s)]^2+a^2u^2} \qquad \text{(by rationalisation)} \\ &= \frac{p(s)q(s)}{[q(s)]^2+a^2u^2}+i\frac{au\,p(s)}{[q(s)]^2+a^2u^2}.\end{aligned}$$

We know that, $e^{(iat)}=\cos(at)+i\sin(at))$, so we get

$$\mathcal{T}[\cos(at)]=\frac{p(s)q(s)}{[q(s)]^2+a^2u^2} \quad \text{and} \quad \mathcal{T}[\sin(at)]=\frac{au\,p(s)}{[q(s)]^2+a^2u^2}.$$

(7) Similarly, by using the sine and cosine formulas and the Sharad transform of e^t, we can find the Sharad transform of hyperbolic sine and hyperbolic cosine,

$$\mathcal{T}[\cosh(at)]=\frac{p(s)q(s)}{[q(s)]^2-a^2u^2} \quad \text{and} \quad \mathcal{T}[\sinh(at)]=\frac{au\,p(s)}{[q(s)]^2-a^2u^2}.$$

The Sharad transform for some standard functions is collected in Table 2.

Table 1. Sharad transform of some standard function

Sr. No.	Standard function	Sharad transform $\mathcal{T}[g(t)]$
1	k, constant	$\frac{kp(s)}{q(s)}$
2	t	$\frac{up(s)}{[q(s)]^2}$
3	t^2	$\frac{2u^2p(s)}{[q(s)]^3}$
4	t^n	$\frac{\Gamma(n+1)u^np(s)}{[q(s)]^{n+1}}$
4	e^{at}	$\frac{p(s)}{q(s)-au}$
5	$\cos(at)$	$\frac{p(s)q(s)}{[q(s)]^2+a^2u^2}$
6	$\sin(at)$	$\frac{au\,p(s)}{[q(s)]^2+a^2u^2}$
7	$\cosh(at)$	$\frac{p(s)q(s)}{[q(s)]^2-a^2u^2}$
8	$\sinh(at)$	$\frac{au\,p(s)}{[q(s)]^2-a^2u^2}$

5. Relation Between the Sharad Transform and Other Transforms

In this section, we will give a relation between the Sharad transform and some other transform. If we put the particular values of u, $p(s)$, and $q(s)$ in definition of the Sharad transform, we get the other transform. We formatted Table 2 with some substitution of u, $p(s)$, and $q(s)$ in the Sharad transform.

In a similar way, by substituting values, we get many other transforms. Thus, we can conclude that this is the more general form of the integral transform.

Table 2. Relation between Sharad transform and other useful transform

Sr. No.	u	$p(s)$	$q(s)$	$\mathcal{T}[g(t)]$	Sharad transform converted into
1	1	1	s	$\int_0^\infty e^{-st}g(t)dt$	Laplace transform (Debnath and Bhatta [2])
2	–	$\frac{1}{s}$	s	$\frac{1}{s}\int_0^\infty e^{-st}g(ut)dt$	NE transform (Xhaferraj [20])
3	–	s	s	$s\int_0^\infty e^{-st}g(ut)dt$	Formable transform (Saadeh and Ghazal [14])
4	1	1	1	$\int_0^\infty e^{-t}g(ut)dt$	Sumudu transform (Watugala [19])
5	–	1	s	$\int_0^\infty e^{-st}g(ut)dt$	Natural transform (Khan and Khan [6])
6	1	s	$\frac{1}{s}$	$s\int_0^\infty e^{-\frac{t}{s}}g(t)dt$	Elzaki transform (Elzaki [3])
7	1	$\frac{1}{s}$	s	$\frac{1}{s}\int_0^\infty e^{-st}g(t)dt$	Abhoodh transform (Aboodh [1])
8	–	s	s	$s\int_0^\infty e^{-st}g(ut)dt$	ZZ transform (Zafar *et al.* [21])
9	1	s	s	$s\int_0^\infty e^{-st}g(t)dt$	Mahgoub transform (Mahgoub and Alshikh [10])
10	1	s^2	s	$s^2\int_0^\infty e^{-st}g(t)dt$	Mohan transform (Mahgoub [9])
11	1	1	1	$\int_0^\infty e^{-t}g(t)dt$	Kamal transform (Sedeeg [15])
12	1	s^3	s	$s^3\int_0^\infty e^{-st}g(t)dt$	Rohit transform (Gupta [4])
13	1	$\frac{1}{s^n}$	s	$\frac{1}{s^n}\int_0^\infty e^{-st}g(t)dt$	SEE transform (Mansour *et al.* [12])
14	1	$\frac{1}{s}$	s	$\frac{1}{s}\int_0^\infty e^{-s^\alpha t}g(t)dt,\ \alpha\neq 0$	Soham transform (Khakale and Patil [5])
15	1	s	s^α	$s\int_0^\infty e^{-s^\alpha t}g(t)dt,\ \alpha\neq 0$	Kushare transform (Kushare *et al.* [8])
16	1	–	–	$p(s)\int_0^\infty e^{-q(s)t}g(t)dt$	A new general transform (Mansour *et al.* [13])
17	1	$\frac{1}{s^2}$	s	$\frac{1}{s^2}\int_0^\infty e^{-st}g(t)dt$	Emad-Sara transform (Maktoof *et al.* [11])
18	1	s^β	s^α	$\frac{1}{s^\beta}\int_0^\infty e^{-s^\alpha t}g(t)dt$	Sadiq transform (Shaikh [16])
19	1	s^5	s	$s^5\int_0^\infty e^{-st}g(t)dt$	Dinesh Verma transform (Verma [18])
20	–	1	s	$\int_0^\infty e^{-st}g(ut)dt$	Ramadan Group transform (Soliman *et al.* [17])

6. Application of Sharad Transform

Here, we apply the Sharad transform to solve some ordinary differential equations.

Example 1. Solve the *Initial Value Problem* (IVP) $y'(t)+5y(t)=0$ with the initial condition $y(0)=2$.

Solution. We have to solve this example by applying the Sharad transform,

$$y'(t)+5y(t)=0, \tag{6.1}$$

$$y(0)=2. \tag{6.2}$$

Applying the Sharad transform to equation (6.1),

$$\mathfrak{T}[y'(t)]+\mathfrak{T}[5y(t)]=0,$$

$$\frac{q(s)}{u}\mathfrak{T}[y(t)]-\frac{p(s)}{u}y(0)+5\mathfrak{T}[y(t)]=0. \qquad \text{(using Theorem 2)}$$

using the initial condition (6.2) and after simplification, we get

$$\mathfrak{T}[y(t)]=\frac{2p(s)}{q(s)+5u}.$$

By taking inverse Sharad transform, we get the required solution,

$$y(t)=2e^{-5t}.$$

Example 2. Solve the IVP $y''(t)-3y'(t)+2y(t)=4e^{3t}$ with the initial condition $y(0)=-3$ and $y'(0)=5$.

Solution. We have to solve this example by applying the Sharad transform,

$$y''(t)-3y'(t)+2y(t)=4e^{3t}, \tag{6.3}$$

$$y(0)=-3 \text{ and } y'(0)=5. \tag{6.4}$$

Applying the Sharad transform to equation (6.3) and using Theorem 2, we get

$$\frac{[q(s)]^2}{u^2}\mathfrak{T}[y(t)]-\frac{p(s)q(s)}{u^2}y(0)-\frac{p(s)}{u}y'(0)-3\frac{q(s)}{u}\mathfrak{T}[y(t)]+3\frac{p(s)}{u}y(0)+2\mathfrak{T}[y(t)]=\frac{4p(s)}{q(s)-3u}.$$

Using the initial condition (6.4) and after simplification, we get

$$\left(\frac{[q(s)]^2}{u^2}-3\frac{q(s)}{u}+2\right)\mathfrak{T}[y(t)]=\frac{p(s)(-38u^2+23q(s)u-3[q(s)]^2)}{u^2(q(s)-3u)},$$

$$\mathfrak{T}[y(t)]=\frac{2p(s)}{q(s)-3u}+\frac{4p(s)}{q(s)-2u}-\frac{9p(s)}{q(s)-u}.$$

By taking inverse Sharad transform, we get the required solution,

$$y(t)=2e^{3t}+4e^{2t}-9e^{t}.$$

Example 3. Solve the IVP

$$y'''(t)+2y''(t)+2y'(t)+3y(t)=\sin(t)+\cos(t) \tag{6.5}$$

with the initial condition

$$y(0)=y''(0)=0 \text{ and } y'(0)=1. \tag{6.6}$$

Solution. We have to solve this example by applying the Sharad transform. Applying the Sharad transform to equation (6.5) and using Theorem 2, we get

$$\frac{[q(s)]^3}{u^3}\mathfrak{T}[y(t)]-\frac{[q(s)]^2}{u^2}\frac{p(s)}{u}y(0)-\frac{q(s)}{u}\frac{p(s)}{u}y'(0)-\frac{p(s)}{u}y''(0)+2\frac{[q(s)]^2}{u^2}\mathfrak{T}[y(t)]$$
$$-2\frac{p(s)q(s)}{u^2}y(0)-2\frac{p(s)}{u}y'(0)+2\frac{q(s)}{u}\mathfrak{T}[y(t)]-2\frac{p(s)}{u}y(0)+3\mathfrak{T}[y(t)]$$

$$= \frac{up(s)}{[q(s)]^2+u^2} + \frac{p(s)q(s)}{[q(s)]^2+u^2}.$$

Using the initial condition (6.6) and after simplification, we get

$$\left(\frac{[q(s)]^3}{u^3} + 2\frac{[q(s)]^2}{u^2} + 2\frac{q(s)}{u} + 3\right)\mathcal{T}[y(t)] = \frac{p(s)q(s)}{u^2} + 2\frac{p(s)}{u} + \frac{up(s)}{[q(s)]^2+u^2} + \frac{p(s)q(s)}{[q(s)]^2+u^2},$$

$$\left(\frac{[q(s)]^3 + 2u[q(s)]^2 + 2u^2q(s) + 3u^3}{u^3}\right)\mathcal{T}[y(t)] = \frac{ps(s)([q(s)]^3 + 2u[q(s)]^2 + 2u^2q(s) + 3u^3)}{u^2([q(s)]^2+u^2)},$$

$$\mathcal{T}[y(t)] = \frac{up(s)}{[q(s)]^2+u^2}.$$

By taking inverse Sharad transform, we get the required solution

$$y(t) = \sin(t).$$

7. Conclusion

We define the more general new integral transform, the Sharad transform, and prove some properties and theorems about it. Also, we apply this to solving some ordinary differential equations. Here we conclude that this transform is a more generalisation of the integral transform. It covers many of the integral transforms, like the Laplace transform, the Sumudu transform, the Formable transform, and many more. Hence, it will become very useful in solving ordinary and partial differential equations, integro-differential equations, and many more in modern life.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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