Communications in Mathematics and Applications
Vol. 16, No. 1, pp. 315–326, 2025
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v16i1.2944

http://www.rgnpublications.com

Research Article

Impact of Variable Viscosity on Heat Transfer in a Horizontal Channel With Uniform Transverse Magnetic Field and Non-Uniform Wall Temperature

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Received: November 24, 2024 **Revised:** December 19, 2024 **Accepted:** January 6, 2025

Abstract. This study reconnoiters variable viscosity fluid flow through horizontal channel with parallel walls imperiled to non-uniform temperature and uniform transverse magnetic field. The governing equations have been solved analytically by perturbation process for velocity and temperature fields. The volume flow rate athwart channel, skin friction, Nusselt number at lower and upper plates are calculated and epitomized the impacts of significant parameters through tables. The verdicts show that velocity, temperature are enhanced by increasing viscosity variation parameter.

Keywords. Magnetic field, Variable viscosity, Channel flow, Heat transfer

Mathematics Subject Classification (2020). 83C50, 35D40, 94A40, 80A19

1. Introduction

The heat relocation and magneto hydrodynamic flows have been the main areas of difficulty for many researchers in recent yearsdue to their significant role in aerodynamic heating, magnetic generators, and magnetic pumps. Maximum fluids used in engineering and industrial systems are at danger in the interim from severe conditions such shear stress, high temperatures, and pressure. High shear stress and peripheral heating might set the stage for the fluid to reach high temperatures. This might have a significant impact on the fluid's characteristics. The biggest obstacle to high viscosity is high energy consumption. Setayesh and Sahai [19] investigated characteristics of variable transport as well as heat transfer in evolving magnetohydrodynamic Poiseuille flow. Herwig *et al*. [9] investigated varying viscosity effects across channel with solved Navier-Stokes equations using finite difference method. Pop *et al*. [15] investigated the effect of varying viscosity on heat transfer laminar flow with moving plate where fluid viscosity varies as an inverse linear function of temperature. Barakat [4] examined how varying viscosity affected fluid flow during heat transfer when a magnetic field was present. A comparable solution for viscous fluid channel flow—where viscosity is exponentially dependent on temperature, heat cohort is very large—was provided by Pearson [14]. Makinde [13] investigated temperature, velocity, and divergencies in a constant laminar flow with varying viscosity over an inclined surface, taking into account thermal criticality. Shateyi and Motsa [20] investigated the effects of hall current, changing viscosity, thermal diffusivity on unsteady MHD heat transfer viscous and incompressible fluid flow via semi-infinite stretching sheet.

In a thin film of viscous flow, Khan *et al*. [11] looked into a flow model caused by varying viscosity and heat conductivity along stretching sheets. Using Runge-Kutta method, Tshehla [21] examined effects of heat transfer flow with temperature-dependent variable viscosity in an inclined surface. The effects of variable viscosity fluid with suction, thermal radiation on heat and mass transfer along moving porous vertical plates was studied by Animasaun and Oyem [3]. Chou *et al*. [6] investigated natural convection for temperature-dependent fluid viscosity in porous channel. Bhatti and Zeeshan [5] investigated the effect of changing viscosity with heat transmission in a channel for the Jeffery fluid flow model. Hassan [8] investigated the influence of varying viscosity when upper plate is moved with constant velocity and lower plate is static in presence of heat source in couette fluid flow using the Adomian decomposition method. Ajibade and Tafida [2] investigated effects of heat conductivity, changing viscosity on flow of steady fluid in vertical channel using homotopy perturbation technique. The combined effects of heat conductivity, variable viscosity on magnetohydrodynamics were examined by Gbadeyan *et al*. [7]. In an inclined channel, coupled parallel fluid flow with changing viscosity caused by a permeable layer was examined by Zaytoon and Hamdan [22]. Abubakar *et al*. [1] studied magnetic field, changing viscosity, heat, mass transmission effects past vertical plate. The formation of entropy and changing viscosity in circular pipe with MHD porous material were investigated by Rashed [16]. The non-Newtonian fluid flow (Casson) in rectangular conduit between two porous parallel plates was studied by Jalili *et al*. [10]. Changing viscosity, non-uniform heat sink intensity effects on Newtonian viscous fluid heat transfer flow through porous surface were investigated by Samuel and Fayemi [17].

Enthused by paucity of pertinent investigations, theproblem of heat transfer model involving MHD, temperature dependent variable viscosity imperiled to non-uniform temperature in a horizontal channel is studied. The comportment of velocity of the fluid, energy, Nusselt number at walls, skin friction and total volume rate through the conduit for various persuasive parameters are conferred through graphs and tables.

2. Formulation of the Problem

A steady incompressible fluid flow is considered in x direction beneath deed of constant pressure gradient over a closed channel where the length L and width H. A uniform magnetic field of strength B_0 is applied perpendicular to the fluid flow direction. The pattern of the flow model premeditated in this paper is portrayed in Figure 1. The temperature dependent viscosity can be stated by $\mu^* = \mu_0[1-\beta(T^*-T_l)]$, where μ_0 is fluid dynamic viscosity at T_l lower plate temperature.

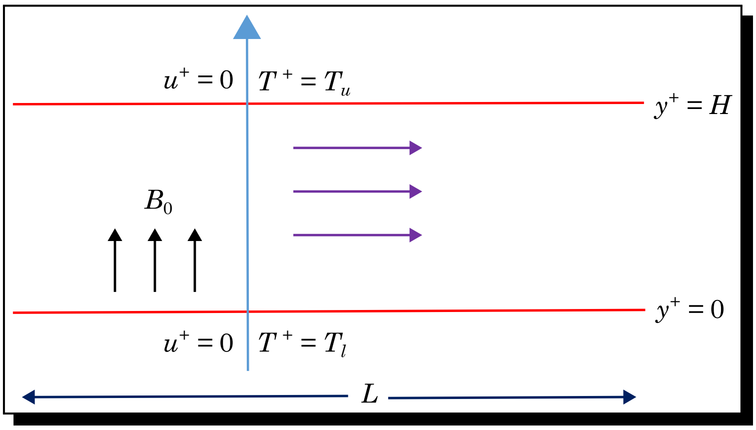

Figure 1. Schematic illustration of problem

The main equation of continuity, momentum and energy governing equations are based on the previous studies [12, 18],

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0, \tag{2.1}$$

$$\varepsilon^2 Re\left(u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right)=-\frac{\partial P^*}{\partial x^*}-\sigma B_0^2u^*+2\varepsilon^2\frac{\partial}{\partial x^*}\left(\mu^*\frac{\partial u^*}{\partial x^*}\right)+\frac{\partial}{\partial y^*}\left[\mu^*\left(\frac{\partial u^*}{\partial y^*}+\varepsilon^2\frac{\partial v^*}{\partial x^*}\right)\right], \tag{2.2}$$

$$\varepsilon^2 Re\left(u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}\right)=-\frac{\partial P^*}{\partial y^*}-\sigma B_0^2u^*+2\varepsilon^2\frac{\partial}{\partial y^*}\left(\mu^*\frac{\partial u^*}{\partial y^*}\right)+\frac{\partial}{\partial x^*}\left[\mu^*\left(\frac{\partial u^*}{\partial y^*}+\varepsilon^2\frac{\partial v^*}{\partial x^*}\right)\right], \tag{2.3}$$

$$\varepsilon^2 Pe\left(u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}\right)=\varepsilon^2\frac{\partial^2T^*}{\partial {x^*}^2}+\frac{\partial^2T^*}{\partial {y^*}^2}+\mu^*\psi, \tag{2.4}$$

where

$$\psi=Br\left[2\varepsilon^2\left(\frac{\partial u^*}{\partial x^*}\right)^2+2\varepsilon^2\left(\frac{\partial v^*}{\partial x^*}\right)^2+\left(\frac{\partial u^*}{\partial y^*}+\varepsilon^2\frac{\partial v^*}{\partial x^*}\right)^2\right]. \tag{2.5}$$

The boundary conditions of flow model are

$$\left.\begin{aligned} y^*=0: u^*=0,\ T^*=T_l\\ y^*=H: u^*=0,\ T^*=T_u\end{aligned}\right\} \tag{2.6}$$

We have imposed below non-dimensional quantities in equations (2.1) to (2.6)

$$\left.\begin{aligned} &y=\frac{y^*}{L\varepsilon},\ x=\frac{x^*}{L},\ u=\frac{u^*}{U\varepsilon},\ \varepsilon=\frac{H}{L},\ \mu=\frac{\mu^*}{\mu_0},\ P=\frac{P^*\varepsilon^2L}{\mu_0U},\ Re=\frac{\rho UL}{\mu_0},\\ &T=\frac{T^*-T_l}{T_u-T_l},\ \alpha=\beta(T_u-T_l),\ Br=\frac{\mu_0U^2}{k(T_u-T_l)},\ Pe=\frac{\rho c_pUL}{k},\ M^2=\frac{\sigma B_0^2L^2}{\mu_0},\end{aligned}\right\} \tag{2.7}$$

where

u^* : axial velocity,
v^* : normal velocity,
ρ : fluid density,
k : thermal conductivity,
T^* : fluid temperature,
T_l : temperature at lower plate,
T_u : temperature at upper plate,
P : pressure,
c_p : specific heat,
Br : Brinkman number,
Re : Reynolds number,
Pe : Peclet number,
α : viscosity variation parameter

On account of the channel is narrow and facet ratio $0 < \varepsilon < 1$, the lubrication approximation established on an asymptotic interpretations of governing equations (2.1) to (2.5) is entreated and we obtain

$$-\frac{\partial P}{\partial x} + \frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y} + \mu\frac{\partial^2 u}{\partial y^2} - M^2 u = 0, \tag{2.8}$$

$$\frac{\partial^2 T}{\partial y^2} + \mu Br\left(\frac{\partial u}{\partial y}\right)^2 = 0, \tag{2.9}$$

where $\mu = 1 - \alpha T$.

Boundary conditions are:

$$\left.\begin{aligned} y = 0 &: u = 0,\ T = 0, \\ y = 1 &: u = 0,\ T = 1. \end{aligned}\right\} \tag{2.10}$$

3. Solution of the Problem:

To solve equations (2.8) and (2.9) subject to boundary conditions (2.10), we adopt variation in fluid viscosity is very small $0 < \alpha < 1$ and strive asymptotic elucidations for fluid velocity and temperature of the form

$$u = u_0 + \alpha u_1 \text{ and } T = T_0 + \alpha T_1. \tag{3.1}$$

By using equation (3.1) in (2.8) and (2.9), we get the following equations

$$\frac{d^2 u_0}{dy^2} = -G + M^2 u_0, \tag{3.2}$$

$$\frac{d^2 T_0}{dy^2} = -Br\left(\frac{du_0}{dy}\right)^2, \tag{3.3}$$

$$\frac{d^2 u_1}{dy^2} = T_0\frac{d^2 u_0}{dy^2} + \frac{du_0}{dy}\frac{dT_0}{dy} + M^2 u_1, \tag{3.4}$$

$$\frac{d^2 T_1}{dy^2} = -Br\left[-T_0\left(\frac{du_0}{dy}\right)^2 + 2\frac{du_0}{dy}\frac{du_1}{dy}\right]. \tag{3.5}$$

Boundary conditions relevant to it are:

$$\left.\begin{aligned} y = 0 &: u_0 = 0, T_0 = 0, u_1 = 0, T_1 = 0, \\ y = 1 &: u_0 = 0, T_0 = 1, u_1 = 0, T_1 = 0, \end{aligned}\right\} \tag{3.6}$$

where $-\frac{\partial P}{\partial x} = G$, the solutions for velocity of fluid and temperature are:

$$\begin{aligned} u(y) &= c_1 e^{My} + c_2 e^{-My} + \frac{G}{M^2} + \alpha(c_8 e^{My} + c_9 e^{-My} + c_{10} y e^{My} + c_{11} y e^{-My}) \\ &\quad + \alpha\left\{ c_{12}\left(y^2 - \frac{y}{M}\right)e^{My} + c_{13}\left(y^2 + \frac{y}{M}\right)e^{-My} + c_{14} e^{3My} + c_{15} e^{-3My} \right. \\ &\quad \left. + c_{16}\left(\frac{y^3}{3} - \frac{y^2}{2M} + \frac{y}{2M^2}\right)e^{My} + c_{17}\left(\frac{y^3}{3} + \frac{y^2}{2M} + \frac{y}{2M^2}\right)e^{-My} \right\}, \end{aligned} \tag{3.7}$$

$$\begin{aligned} T(y) &= c_3 + c_4 y + c_5 e^{2My} + c_6 e^{-2My} + c_7 y^2 + \alpha(c_{18} + c_{19} y + c_{20} e^{2My} + c_{21} y^2 + c_{25} y^3) \\ &\quad + \alpha\left\{ c_{22}\left(y - \frac{1}{M}\right)e^{2My} + c_{23}\left(y^2 - \frac{3y}{M} + \frac{5}{2M^2}\right)e^{2My} + c_{24}\left(\frac{y^4}{12} + \frac{y^3}{6M}\right) + c_{28} e^{-2My} \right. \\ &\qquad + c_{26}\left(\frac{y^3}{3} - \frac{3y^2}{2M} + \frac{3y}{M^2} - \frac{9}{4M^3}\right)e^{2My} + c_{27}\left(\frac{y^5}{60} + \frac{y^4}{24M} + \frac{y^3}{12M^2}\right) + c_{33} e^{4My} \\ &\qquad + c_{29}\left(y + \frac{1}{M}\right)e^{-2My} + c_{30}\left(\frac{y^4}{12} - \frac{y^3}{6M}\right) + c_{31}\left(y^2 + \frac{3y}{M} + \frac{5}{2M^2}\right)e^{-2My} + c_{34} e^{-4My} \\ &\qquad + c_{32}\left(\frac{y^3}{3} + \frac{3y^2}{2M} + \frac{3y}{M^2} + \frac{9}{4M^3}\right)e^{-2My} + c_{35}\left(y^2 - \frac{2y}{M} + \frac{3}{2M^2}\right)e^{2My} + c_{37} y^4 \\ &\qquad + c_{36}\left(y^2 + \frac{2y}{M} + \frac{3}{2M^2}\right)e^{-2My} + c_{38}\left(\frac{y^5}{60} - \frac{y^4}{24M} + \frac{y^3}{12M^2}\right) + c_{39}\left(y^2 - \frac{3y}{M} + \frac{3}{M^2}\right)e^{2My} \\ &\qquad \left. + c_{40}\left(y^2 + \frac{3y}{M} + \frac{3}{M^2}\right)e^{-2My} + c_{41}\left(\frac{y^4}{12} + \frac{y^3}{6M} + \frac{y^2}{4M^2}\right) + c_{42}\left(\frac{y^4}{12} - \frac{y^3}{6M} + \frac{y^2}{4M^2}\right) \right\}. \end{aligned} \tag{3.8}$$

Volume Flow Rate: The volume flow rate through channel:

$$\begin{aligned} Q &= \int_0^1 u(y)dy \\ &= \frac{c_1}{M}(e^M - 1) + \frac{c_2}{M}(1 - e^{-M}) + \frac{G}{M^2} + \alpha\left[\frac{c_8}{M}(e^M - 1) + \frac{c_9}{M}(1 - e^{-M}) + \frac{c_{10}}{M^2}(Me^M - e^M + 1)\right] \\ &\quad + \alpha\left[\frac{c_{11}}{M^2}(-Me^{-M} - e^{-M} + 1) + \frac{c_{12}}{M^3}(M^2 e^M - 3Me^M + 3e^M - 3) + \frac{c_{14} M}{3}(e^{3M} - 1) \right. \\ &\qquad + \frac{c_{13}}{M^3}(-M^2 e^{-M} - 3Me^{-M} - 3e^{-M} + 3) + \frac{c_{15} M}{3}(e^{-3M} - 1) \\ &\qquad + \frac{c_{16}}{6M^4}(2M^3 e^M - 9M^2 e^M + 21Me^M - 21e^M + 21) \\ &\qquad \left. + \frac{c_{17}}{6M^4}(-2M^3 e^{-M} - 9M^2 e^{-M} + 21Me^{-M} - 21e^{-M} + 21)\right]. \end{aligned} \tag{3.9}$$

Skin Friction Coefficient: The skin friction coefficient at lower and upper plates:

$$\tau = \left(\mu\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } \tau = \left(\mu\frac{\partial u}{\partial y}\right)_{y=1}. \tag{3.10}$$

Nusselt Number: The Nusselt number at lower and upper plates:

$$Nu = \left(\frac{\partial T}{\partial y}\right)_{y=0} \text{ and } Nu = \left(\frac{\partial T}{\partial y}\right)_{y=1}. \tag{3.11}$$

4. Results and Discussion

This section comprises variation of fluid velocity, temperature profiles andconversation of altered values of magnetic (M), pressure gradient (G), viscosity variation parameters (α), Brinkman number (Br). Also, conferred variation of volume flow rate (Q), skin friction coefficient (τ) and Nusselt number (Nu) at both walls through tables. Figure 2 interprets that by raising magnetic field, Lorentz force becomes sturdier, which generates resistive strength in fluid flow by causing fluid speed to slow down.

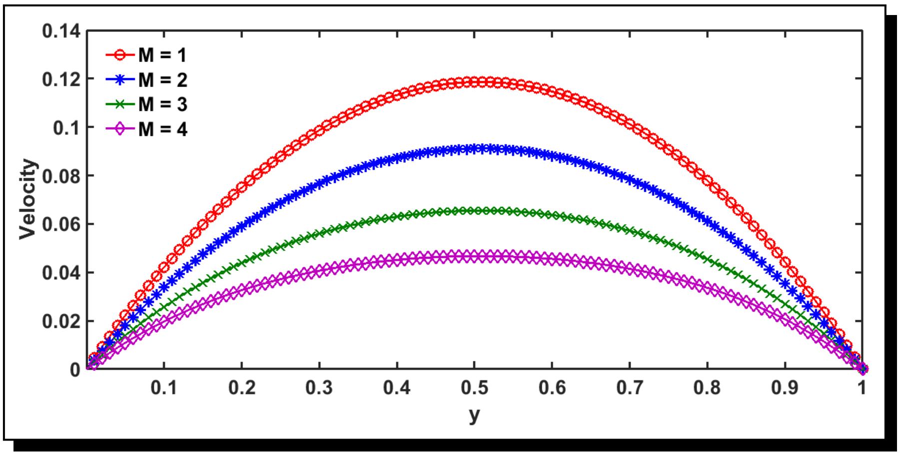

Figure 2. Velocity profile for M when $G = 1$, $\alpha = 0.1$ and $Br = 10$

From Figures 3-5, it is noticed that fluid velocity intensifications, by increasing values of pressure gradient parameter, Brinkman number and viscosity variation parameter. Generally parabolic velocity profile is observed along centreline in channel maximum value and at walls minimum. By increasing the values of α, viscosity of fluid decreases and viscous heating rises, due to that velocity accelerates. The fluid temperature variations are shown in Figures 6-8. It is pragmatic that fluid temperature increases by increasing values of Brinkman number, pressure gradient,viscosity variation parameter. Interim, least temperature is observed at lower plate, followed by transverse intensification to its maximum value, then decreases slowly to prescribed value at upper plate.

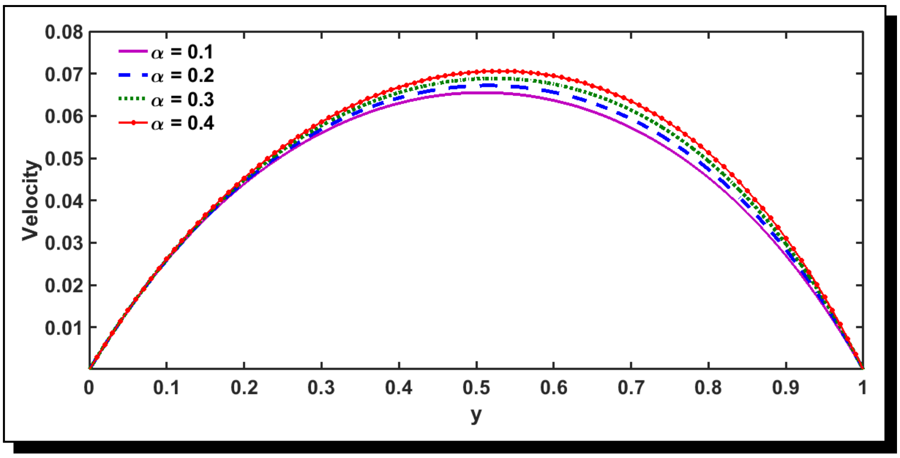

Figure 3. Velocity profile for α when $G = 1$, $M = 3$ and $Br = 10$

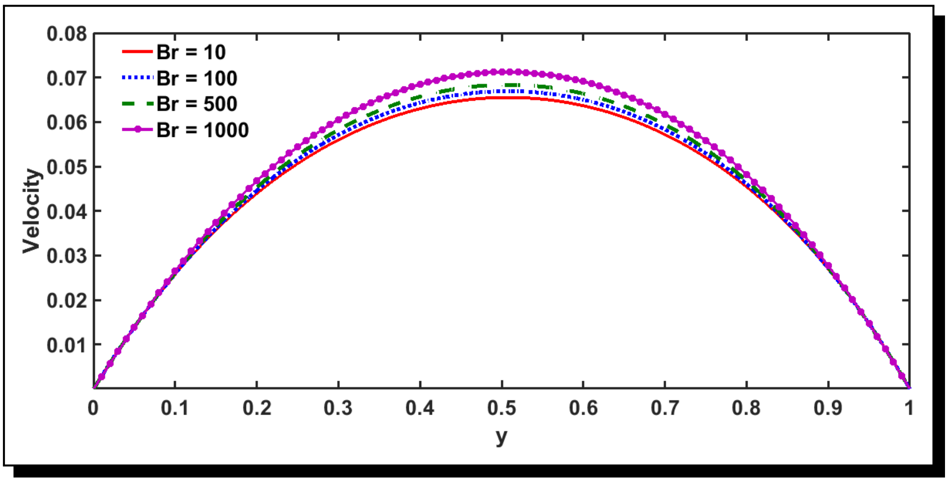

Figure 4. Velocity profile for Br when $G = 1$, $M = 3$ and $\alpha = 0.1$

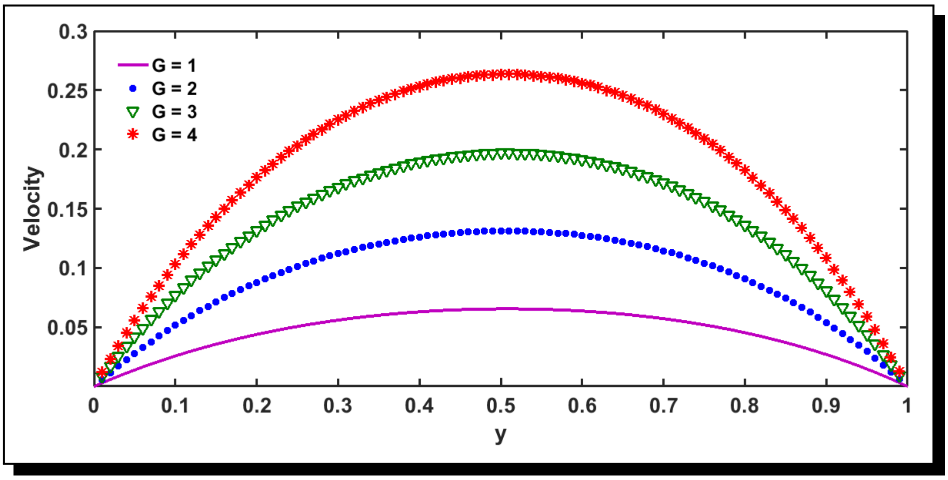

Figure 5. Velocity profile for G when $Br = 10$, $M = 3$ and $\alpha = 0.1$

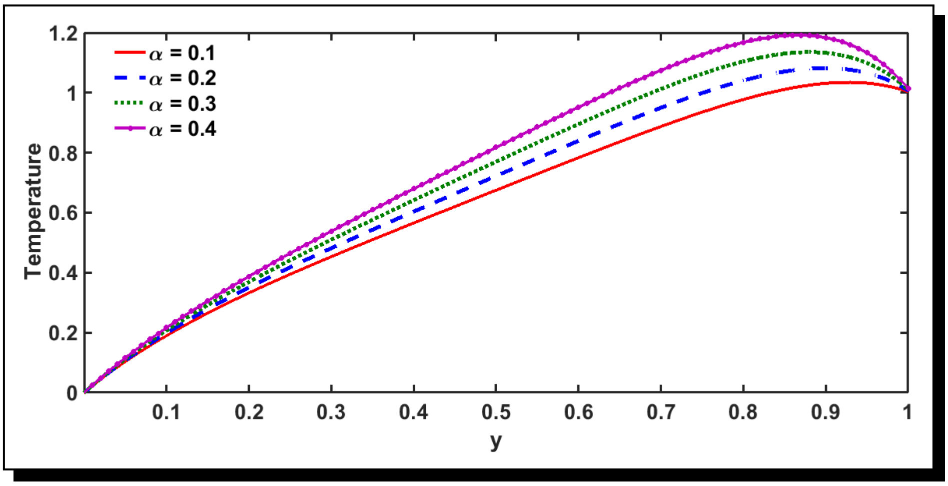

Figure 6. Temperature profile for α when $G = 1$, $M = 3$ and $Br = 100$

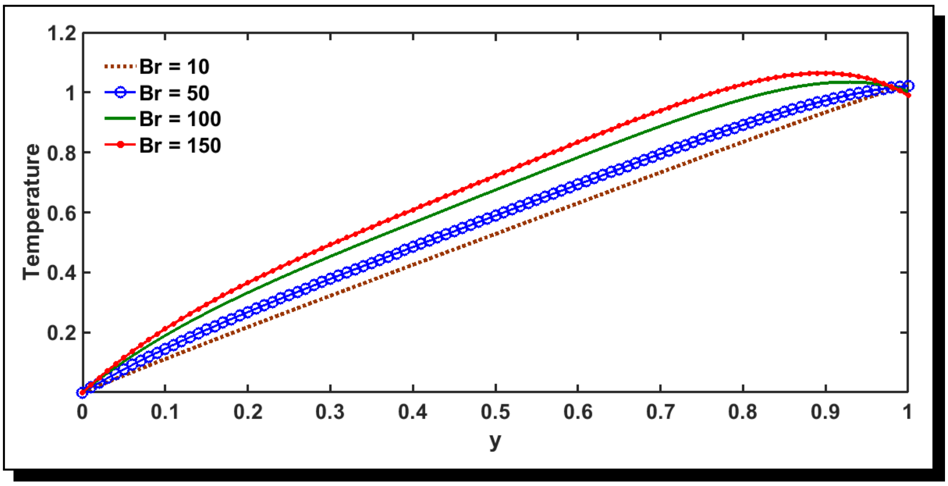

Figure 7. Temperature profile for Br when $G = 1$, $M = 3$ and $\alpha = 0.1$

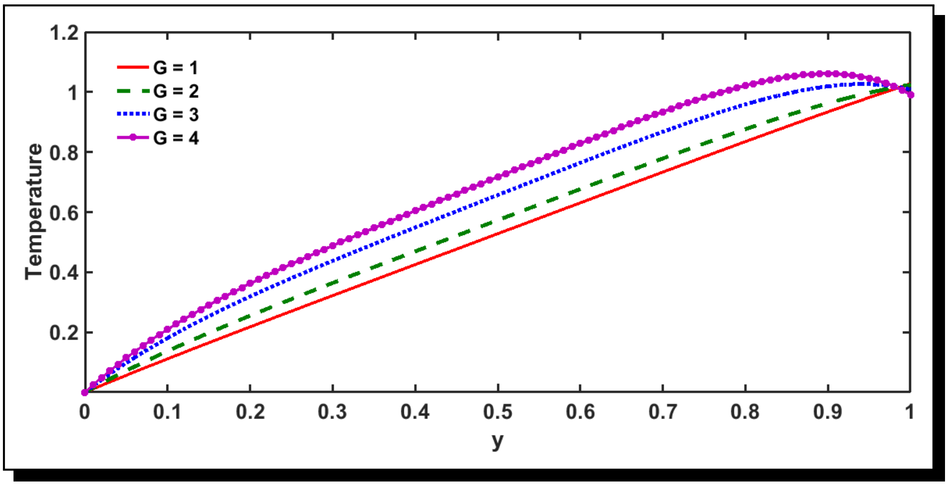

Figure 8. Temperature profile for G when $Br = 10$, $M = 3$ and $\alpha = 0.1$

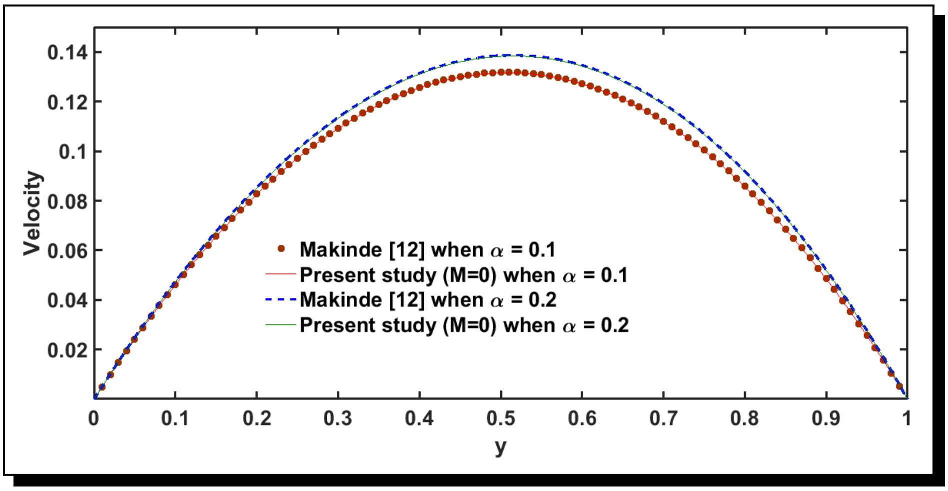

Figure 9. Comparison for velocity profile by viscosity variation parameter

Table 1 epitomizes the verdicts for the volume flow rate by varying magnetic, viscosity, pressure gradient parameters, Brinkman number. From this table it is noticed that while viscosity, pressure gradient parameter, Brinkman number rise, so does heat fabrication from viscous dissipation. This creates volume flow rate goes up across channel. But opposite phenomenon is occurred in case of magnetic parameter that is the volume flow rate decelerated if magnetic effect increased. The shear tension $\tau$and the momentum of warmth transmission in spans of Nusselt number Nu on walls $y = 0$ and $y = 1$ have existed for different variation of the parameters M, α, G and Br are presented in Tables 2 and 3. Table 2 show that skin friction decreases at lower barrier and increases at upper barrier, increasing magnetic parameter or Brinkman number. Increasing viscosity variation or pressure gradient parameter represses the skin friction at the upper plate but it improves at the lower plate. Table 3 shows that Nusselt number raises at upper barrier and decreases at lower barrier on increasing pressure gradient parameter or Brinkman number. The heat transfer rate reduces by improving magnetic parameter and viscosity variation parameter at both plates. The present investigation in absence of magnetic field validates earlier published work of Makinde [12]. The results are same and presented in Figure 9.

Table 1. Volume flow rate

M	α	G	Br	Q
1	0.1	1	10	0.079376168
2	0.1	1	10	0.061796361
3	**0.1**	**1**	**10**	**0.045253715**
3	**0.2**	1	10	0.046444338
3	**0.3**	1	10	0.047634961
3	0.1	**2**	10	0.090605743
3	0.1	**3**	10	0.136154395
3	0.1	1	**15**	0.045261908
3	0.1	1	**20**	0.045270101

Table 2. Skin friction at the plates $y = 0$ and $y = 1$

M	α	G	Br	τ at $y = 0$	τ at $y = 1$
1	0.1	1	10	0.468137798	−0.498697749
2	0.1	1	10	0.383123712	−0.407770552
3	**0.1**	**1**	**10**	**0.301955252**	**−0.320932919**
3	**0.2**	1	10	0.302194419	−0.340149753
3	**0.3**	1	10	0.302433587	−0.359366588
3	0.1	**2**	10	0.603468099	−0.641423433
3	0.1	**3**	10	0.904096136	−0.961029137
3	0.1	1	**15**	0.301918385	−0.320896052
3	0.1	1	**20**	0.301881518	−0.320859185

Table 3. Nusselt number at the plates $y = 0$ and $y = 1$

M	α	G	Br	Nu at $y = 0$	Nu at $y = 1$
1	0.1	1	10	−0.363716097	4.859874358
2	0.1	1	10	−1.067026963	−0.284011969
3	**0.1**	**1**	**10**	**−1.149302809**	**−0.806961823**
3	**0.2**	1	10	−1.181181862	−0.831739981
3	**0.3**	1	10	−1.213060913	−0.856518139
3	0.1	**2**	10	−1.508678156	−0.045353869
3	0.1	**3**	10	−2.090304566	1.554526351
3	0.1	1	**15**	−1.209740342	−0.690356339
3	0.1	1	**20**	−1.269961217	−0.569619186

5. Conclusions

The effect of temperature dependent viscosity on MHD fluid flow has been studied between two horizontal plates. The effects of pertinent parameters on arenas of velocity, temperature, volume flow rate, skin friction coefficient and Nusselt number are determined as follows:

- Fluid velocity, temperature accelerates with intensification in viscosity parameter, Brinkman number and pressure gradient.
- Fluid velocity and volume flow rate decreases when magnetic effect rises across the channel.
- The volume flow rate through channel increases as pressure gradient parameter, Brinkman number and viscosity variation parameter increase.
- At the upper plate, skin friction coefficient escalations as magnetic parameter and Brinkman number increases. But at lower plate it declines as magnetic parameter and Brinkman number rises.
- Nusselt number across the channel decreases with Hartman number and viscosity variation parameter proliferation.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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