Communications in Mathematics and Applications
Vol. 16, No. 1, pp. 301–313, 2025
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v16i1.2929

http://www.rgnpublications.com

Research Article

On Distribution of the Stock Market Risk with a Maximum Drawdown of a Wiener Process

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Received: October 26, 2024 **Revised:** December 12, 2024 **Accepted:** January 7, 2025

Abstract. In this article, we investigate the Wiener stock market risk's maximum drawdown distribution. The danger of stochastic volatility in stock prices can be reduced by taking into account the most reliable and accurate decisions using this distribution. We extract various significant dependability aspects of this distribution, including the hazard and inverted hazard rate functions, in addition to presenting the closed-form pricing formula, which demonstrated the precise maximum drawdown distribution of the price path from the perspective of mathematical analysis. Additionally, a Wiener stock market risk's estimated value is calculated. This predicted value can be used to forecast future risk. Furthermore, we present a multivariate distribution of a maximum drawdown for an m-dimensional Wiener process and its key reliability aspects when this risk depends on the n separate and distinct primary stock market risks.

Keywords. Statistical physics distributions, Multivariate distribution, Maximum drawdown distribution of a Wiener process, Basic statistical properties, Stock market risk

Mathematics Subject Classification (2020). 60E05

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1. Introduction

A stochastic process's probability distribution indicates the likelihood that certain states or outcomes may materialize at a particular moment or during a predetermined period of time. Mathematical models known as stochastic processes are employed to explain stochastic events that change over time, like weather patterns, interest rates, and stock values. Making predictions, assessing risk, and creating efficient control techniques all depend on understanding the probability distribution of a stochastic process, which offers important insights into the uncertainty and variability related to the activity. Some interesting real-world applications, depending on some stochastic processes and probability distribution, have been provided in some recent works, like studying the influence of antimicrobial resistance on the probability distribution of densities for synchronized growth of different kinds of bacteria (see El-Hadidy and Alraddadi [9]. Also, a probability distribution that showed the diffusion of the random Walk Microorganisms cells on a planar surface has been discussed in El-Hadidy [4]. These distributions presented many benefits in physical studies of the particles motion on the fluid, as in El-Hadidy and Alzulaibani [7]. They provided an interesting distribution that predicted the position of a Brownian particle at any time t. These distributions are also used to study the water pollution densities when the pollutants are stochastically jumped from one position to another in the fluid, see El-Hadidy [5].

The maximum drawdown distribution of a Wiener Process is one of the most important distributions in physics and economics, as is the Wiener range distribution. One of the main concerns in economic science is making the most stable and true decisions to reduce the risk of stochastic volatility in stock prices. However, by virtue of the nature of this instrument and its correlation to expectations, it has become a risk in itself, which exacerbated the global financial crisis in 2008. Most of the strong and weak economies have been affected by this crisis, which has had a negative impact. This leads the economists to create multiple models to predict the stock market's value to avoid these failures. Many studies have been carried out to discuss the random fluctuations of stock prices, but none of them has been studied to get the expected value of their risk. This important subject is the main idea of our study. To do this, we need to get the distribution of the maximum drawdown to measure the risk of the stock price, which follows a particular Wiener process.

The economists consider the range of a Weiner process useful to express the range between the highest and lowest value of the stock. Thus, we deal with the distribution of the Wiener process range in a bounded domain, which has been studied before in Teamah *et al*. [21], El-Hadidy [3] and El-Hadidy and Alfreedi [6]. They found the distribution of the range at the end of the day if the current time is the start of the day. But with previous highs and lows being hit, this adds much complication to the problem. This distribution is an extension of the one obtained earlier by Feller [10]. He used the method of images to derive the probability density function of the range. Based on the probability density function of this range, Withers and Nadarajah [23] provided its quantiles and the cumulative distribution function.

On the other hand, there are many published papers concerned with the forecasting of the stock price in the future in continuous and discrete cases; for example, see Nowakowska [20]. She constructed a discrete dynamic model to forecast the price of energy. Furthermore, earlier studies by Wang and Hui [22], Nagahara [18], Nguyen *et al*. [19], and Liu *et al*. [13] focused on supplying conditional distributions of two different kinds of random variables that expressed the change in price of two stocks that were connected. This model is used to forecast the demand

and the amount of energy produced by generators. In addition, the price for the next period can be calculated. Also, this model takes into consideration some basic concepts, such as demand and supply changes and their influence on the market. Furthermore, we take into account a multivariate distribution of n independent major stock prices that are affected by various variables. These variables, which include variations in supply and demand as well as the rate of inflation, have an impact on how the stock price moves. There is no investigated model that can give the expected value of the risk of the stock market. We think that the problem is fairly complicated, so there may not be a closed formula for the expected risk value. In this work, we aim to obtain the statistical distribution of stock market risk. Also, we aim to get the expected value of this risk. This will be important in studying the prediction of stock price risk in the future.

The paper is organized as follows: In Section 2, we present the statistical distribution of stock market risk by giving the accurate distribution of the maximum drawdown of a Wiener process. We study some different statistical properties for this distribution in Section 3. In Section 4, we obtain the expected value of this risk. Section 5 provides a multivariate distribution of the maximum drawdown for the m-dimensional Wiener process and some of its reability properties. Section 6 discusses the concluding remarks and future works.

2. The Statistical Distribution of Wiener's Stock Market Risk

The unpredictable swings in asset prices seen in financial markets can be understood and modeled mathematically with the help of Wiener's stock market risk. Analysts and investors can better comprehend the risks and uncertainties involved in investing in financial assets and create methods to manage and minimize these risks by adding Wiener's technique into financial models. Goldman *et al*. [11] presented the closed-form pricing formula using Wiener's process. They showed the exact distribution of the maximum and the minimum of the price path from the point of view of mathematical analysis. All earlier papers considered the log-normal distribution to obtain the closed-form pricing formula for the arithmetic average options; see, for example, Bergman [1], Kemna and Vorst [12]. Right now, we are assuming the maximum drawdown of a Wiener process $W(t)$, $0 \le t \le T$. Then, as in Magdon-Ismail *et al*. [15], the maximum drawdown random variable $D = \sup_{t\in[0,T]}\left[\sup_{s\in[0,t]} W(s) - W(t)\right]$ has the following distribution function:

$$G_{D(T)}(h) = P[D \ge h] = \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{n+\frac{1}{2}}\left(1-\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h^2}\right\}\right), \quad h>0, \tag{2.1}$$

where n is the number of maximum drawdown points in the time interval $[0,T]$. Really, that is not the correct distribution function of the maximum drawdown points of the Wiener process. Based on this error and the *Probability Density Function* (PDF),

$$g_{D(T)}(h) = \frac{dG_{D(T)}(h)}{dh} = \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\left(n+\frac{1}{2}\right)\pi^2 T}{h^3}\left(\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h^2}\right\}\right), \quad h>0. \tag{2.2}$$

Since $G_{D(T)}(h) = P[D < h] = 1 - P[D \geq h]$, then the corrected distribution function of the maximum drawdown of a Wiener stock market risk (see Figure 1) should be

$$F_{D(T)}(h) = P[D < h] = 1 - P[D \geq h] = 1 - \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{n+\frac{1}{2}}\left(1 - \exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h^2}\right\}\right), \quad h > 0. \quad (2.3)$$

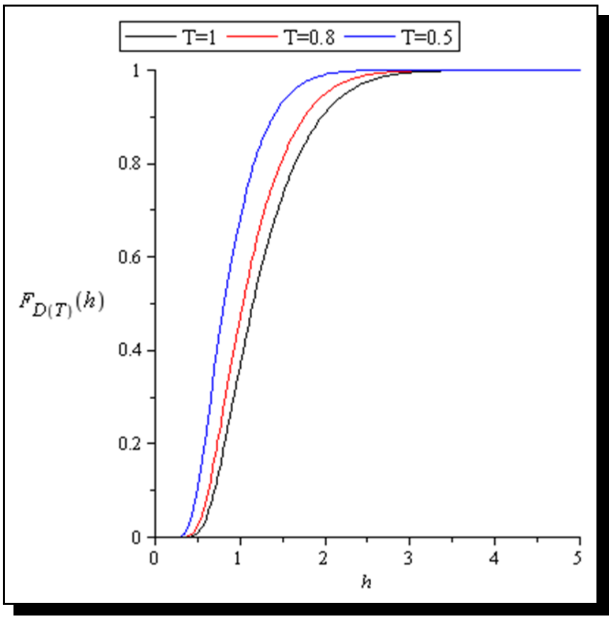

Figure 1. The correct distribution function of a Wiener stock market risk

This leads to, the correct PDF of the Wiener stock market risk is,

$$f_{D(T)}(h) = \frac{dF_{D(T)}(h)}{dh} = \frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\left(n+\frac{1}{2}\right)\pi^2 T}{h^3}\left(\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h^2}\right\}\right), \quad h > 0 \quad (2.4)$$

(see, Figure 2). Also, one can get

$$\int_0^{\infty} f_{D(T)}(h)dh = \frac{2}{\pi}\int_0^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n\left(n+\frac{1}{2}\right)\pi^2 T}{h^3}\left(\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h^2}\right\}\right).$$

Since $\frac{1}{2}\left(n+\frac{1}{2}\right)^2\pi^2 T > 0$, then the series $\sum_{n=0}^{\infty}\frac{(-1)^n\left(n+\frac{1}{2}\right)\pi^2 T}{h^3}\left(\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h^2}\right\}\right)$ is uniformly convergence. Thus,

$$\begin{aligned}\int_0^{\infty} f_{D(T)}(h)dh &= \frac{2}{\pi}\sum_{n=0}^{\infty}\int_0^{\infty}\frac{(-1)^n\left(n+\frac{1}{2}\right)\pi^2 T}{h^3}\left(\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h^2}\right\}\right)dh \\ &= \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^{n+1}\lim_{h\to 0^+}\exp\left\{-\frac{(2n+1)^2\pi^2 T}{8h^2}\right\} + (-1)^n}{2n+1} \\ &= \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^{n+1}\exp\{-\infty\} + (-1)^n}{2n+1} \\ &= \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}\end{aligned}$$

$$= \frac{4}{\pi} \times \frac{\pi}{4} = 1.$$

This confirms that, (2.4) is the correct PDF of the Wiener stock market risk.

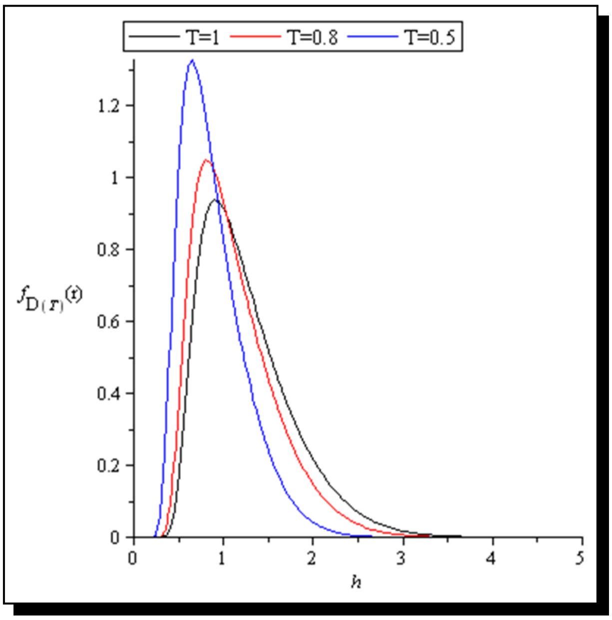

Figure 2. The correct PDF of a Wiener stock market risk

Now, it is easy to get the survival function $S_{D(T)}(\cdot) = 1 - F_{D(T)}(h)$ is decreasing by increasing the value of h, see Figure 3.

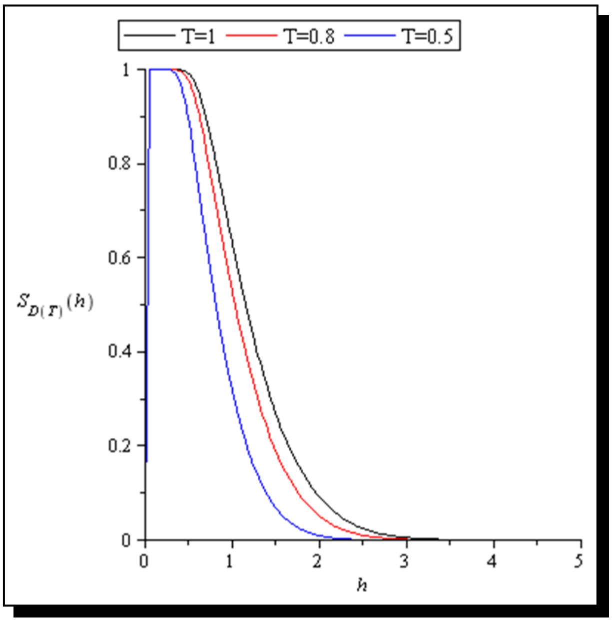

Figure 3. Survival function of a Wiener stock market risk

3. Basic Properties of a Wiener Stock Market Distribution

We aim to get some basic measures of the risk of a Wiener stock market over known time intervals. In this section, we study some various statistical properties of the distribution of a Wiener stock market risk, including reliability properties.

3.1 Reliability Properties

The hazard rate (risk rate) is one of the important applicable measures in reliability theory and economic sciences. During the time period $(0,T)$, the risk rate is influenced by the swings between the rise and fall of much of the stock market value. Thus, the risk rate function of a Wiener stock market is given by $H_{D(T)}(h) = \frac{f_{D(T)}(h)}{1-F_{D(T)}(h)}$, and it is represented in Figure 4.

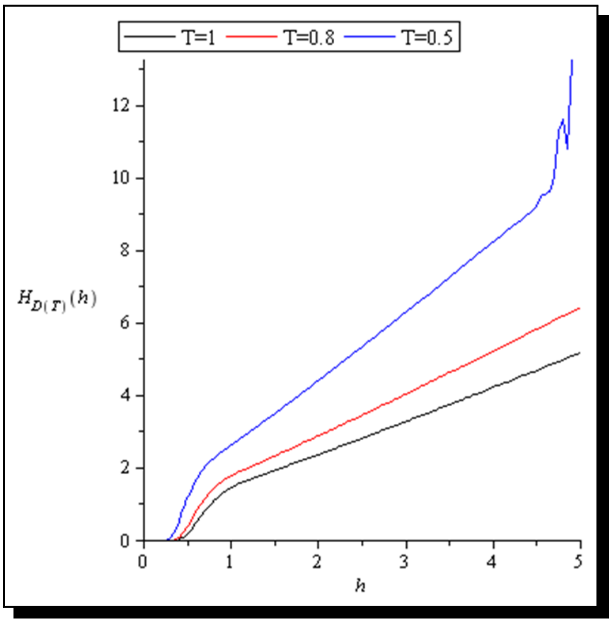

Figure 4. Hazard rate function of a Wiener stock market risk

Also, Figure 5 gives the reversed hazard rate function of the Wiener stock market risk, which is calculated from $H_{D(T)}(h) = \frac{f_{D(T)}(h)}{F_{D(T)}(h)}$.

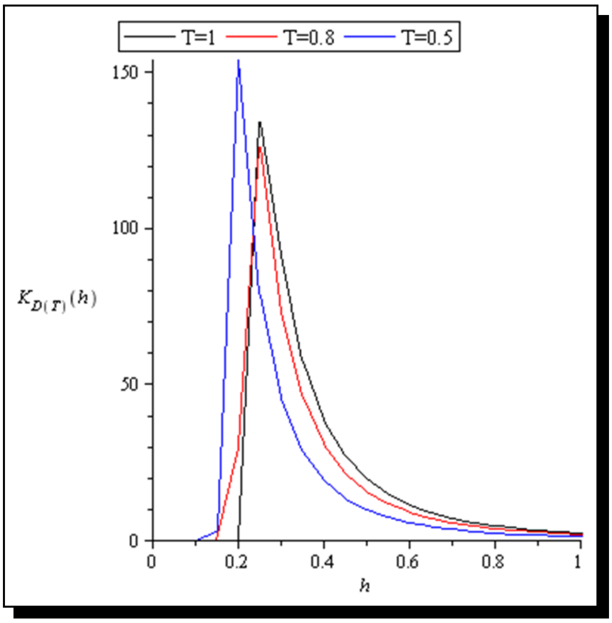

Figure 5. Reversed hazard rate function of a Wiener stock market risk

4. The Expected Value of a Wiener Stock Market Risk

We aim to get the expected risk value of a maximum drawdown of a Weiner stock. For example, if we have a stock ABC that trades from 9 am to 4 pm, that has a volatility of supposing that at 11 am, the stock has a price of 30 dollars, and, so far during the day, it has had a low of 28 and a high of 31. Then, we are trying to find a closed formula for the expected value of a maximum drawdown of a Weiner stock at the end of the day (the largest drop from a peak to a low). We know that the maximum drawdown at the end of the day will be at least 3 dollars because the stock has already gotten too low at 28 dollars and too high at 31 dollars. Also, we know that the current price affects the maximum drawdown at the end of the day because if the current price is close to a high or low, then the stock is more likely to reach a new high or low by the end of the day. I think that the problem is fairly complicated, so there may not be a closed formula for the expected maximum drawdown by the end of the day (T).

From (2.4), the expected risk value of a maximum drawdown of a Weiner stock is given by:

$$\begin{aligned}
E_{D(T)}(h) &= \int_0^{\infty} h f_{D(T)}(h)dh \\
&= \int_0^{\infty} \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n \left(n+\frac{1}{2}\right)\pi^2 T}{h^2} \left(\exp\left\{ -\frac{\left(n+\frac{1}{2}\right)^2 \pi^2 T}{2h^2} \right\} \right) dh \\
&= \frac{2}{\pi} \frac{1}{\sqrt{2}} \sum_{n=0}^{\infty} (-1)^n \pi^{\frac{3}{2}} \sqrt{T} \left(\lim_{r\to 0^+} \operatorname{erf}\left(\frac{\sqrt{2T}\pi(2n+1)}{4h} \right) \right) \\
&= \sqrt{2\pi T} \sum_{n=0}^{\infty} (-1)^n \left(\lim_{h\to 0^+} \operatorname{erf}\left(\frac{\sqrt{2T}\pi(2n+1)}{4h} \right) \right).
\end{aligned}$$

Clearly, the expected risk value of a maximum drawdown of a Weiner stock depends on the number of maximum drawdown points in the time interval $[0,T]$.

5. Multivariate Distribution of a Maximum Drawdown for m-Dimensional Wiener Process

Maximum drawdown analysis is a flexible tool that has many uses in finance and investment management. It offers insightful information on risk management, portfolio performance, and investment risk. A multivariate distribution maximum drawdown for m-dimensional Wiener process can be used to simulate the combined behavior of numerous independent stock market hazards when it comes to economic risk. Investors and portfolio managers can apply risk management strategies to minimize possible losses and maximize portfolio performance in a variety of market scenarios by identifying and comprehending some independent primary stock market risks. Effective risk management and mitigation strategies include active risk monitoring, hedging, and diversification. Thus, if m independent primary stock market risks with m independent Wiener process $W(t) = (W_1(t), W_2(t), \dots, W_m(t))$, $0 \le t \le T$. Maximum drawdowns exist, then we have an independent vector of maximum drawdown random variables,

$$[D_1 \geq h_1, D_2 \geq h_2, \ldots, D_m \geq h_m] = \left[\sup_{t\in[0,T]}\left[\sup_{s\in[0,t]} W_1(s)-W_1(t)\right], \sup_{t\in[0,T]}\left[\sup_{s\in[0,t]} W_2(s)-W_2(t)\right], \ldots, \sup_{t\in[0,T]}\left[\sup_{s\in[0,t]} W_m(s)-W_m(t)\right]\right].$$

From the independence principle and using (2.4) the joint PDF of these random variables is,

$$\widetilde{f}_{D_1(T),D_2(T),\ldots,D_m(T)}(h_1,h_2,\ldots,h_m) = \left(\frac{2}{\pi}\right)^m \prod_{i=1}^{m}\left(\sum_{n=0}^{\infty}\frac{(-1)^n\left(n+\frac{1}{2}\right)\pi^2 T}{h_i^3}\left(\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h_i^2}\right\}\right)\right), \quad h_i>0,\ \forall\ i=1,2,\ldots,m \tag{5.1}$$

and the joint cumulative distribution function is,

$$\widetilde{F}_{D_1(T),D_2(T),\ldots,D_n(T)}(h_1,h_2,\ldots,h_m) = \prod_{i=1}^{m}\left(1-\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{n+\frac{1}{2}}\left(1-\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h_i^2}\right\}\right)\right), \quad h_i>0,\ \forall\ i=1,2,\ldots,m. \tag{5.2}$$

Using statistical techniques, one may determine the expected risk value of a multivariate maximum drawdown by utilizing the joint PDF (5.1) of the variables. It is given by,

$$\widetilde{E}_{D_1(T),D_2(T),\ldots,D_n(T)}(h_1,h_2,\ldots,h_m) = \prod_{i=1}^{m}[E_{D_i(T)}(h_i)] = (\sqrt{2\pi T})^m \prod_{i=1}^{m}\sum_{n=0}^{\infty}\left((-1)^n\left(\lim_{h_i\to 0^+}\operatorname{erf}\left(\frac{\sqrt{2T}\pi(2n+1)}{4h_i}\right)\right)\right).$$

This entails taking into account the combined distribution of several assets or portfolios and the matching maximum drawdowns in the context of financial markets.

5.1 Bonferroni and Lorenz Curves

For the purpose of examining the distribution of risk values, especially those connected to multivariate maximum drawdowns, the Bonferroni curve and the Lorenz curve are helpful resources. These curves help with risk assessment and portfolio management decisions by offering insightful information about the distribution and concentration of risk values.The Lorenz curve for this multivariate distribution can be obtained as in Lorenz [14]:

$$L(\widetilde{F}_{D_1(T),\ldots,D_m(T)}(h_1,\ldots,h_m)) = \prod_{i=1}^{m}\left(\frac{\int_0^{h_i} h_i f_{D_i(T)}(h_i)dh_i}{E_{D_i(T)}(h_i)}\right) = \prod_{i=1}^{m}\left(\sum_{n=0}^{\infty}(-1)^n\left(\frac{(-1)^n 4\sqrt{2\pi}\sqrt{\left(n+\frac{1}{2}\right)^2 T}\left(-\sqrt{\left(n+\frac{1}{2}\right)^2 T}\,erf\left(\frac{\pi\sqrt{2T}(2n+1)}{4h_i}\right)+\left(n+\frac{1}{2}\right)\sqrt{T}\right)}{\sqrt{2\pi}T(2n+1)^2}\right)\right). \tag{5.3}$$

Although they are both graphical tools for analyzing value distribution, the Lorenz and Bonferroni curves have diverse uses and are utilized in various situations. The Lorenz and Bonferroni curves do not directly relate to one another, but they can both be utilized in various ways to learn more about how risk values are distributed.

The Bonferroni curve is thus obtained as in Bonferroni [2] for the nonnegative and continuous independent random variables $D_1 \geq h_1$, $D_2 \geq h_2, \dots, D_m \geq h_m$ with the joint cumulative distribution function (5.2):

$$\begin{aligned} B(F_{D_1(T),D_2(T),\dots,D_m(T)}(h_1,h_2,\dots,h_m)) &= \frac{L(F_{D_1(T),D_2(T),\dots,D_m(T)}(h_1,h_2,\dots,h_m))}{F_{D_1(T),D_2(T),\dots,D_m(T)}(h_1,h_2,\dots,h_m)} \\ &= \prod_{i=1}^{m} \frac{\left(\sum_{n=0}^{\infty}(-1)^n\left(\frac{(-1)^n 4\sqrt{2\pi}\sqrt{\left(n+\frac{1}{2}\right)^2 T}\left(-\sqrt{\left(n+\frac{1}{2}\right)^2 T}\,erf\left(\frac{\pi\sqrt{2T}(2n+1)}{4h_i}\right)+\left(n+\frac{1}{2}\right)\sqrt{T}\right)}{\sqrt{2\pi}T(2n+1)^2}\right)\right)}{\left(1-\frac{2}{\pi}\sum_{n-0}^{\infty}\frac{(-1)^n}{n+\frac{1}{2}}\left(1-\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h_i^2}\right\}\right)\right)}. \end{aligned} \tag{5.4}$$

5.2 Bivariate Distribution of Maximum Drawdown of a Wiener Process and Its Properties

A bivariate distribution of maximum drawdowns is the joint distribution of drawdowns for two distinct assets, portfolios, or risk variables in the context of finance and risk management. It enables the investigation of the two variables' combined behavior under various conditions and sheds light on the relationship between their drawdowns. Therefore, the joint PDF (5.1) and the joint cumulative distribution function (5.2) will have the following shapes (see, Figures 6 and 7) if we have two distinct stock market risks with two independent random variables, $D_1 \geq h_1$ and $D_2 \geq h_2$,

$$f_{1D_1(T),D_2(T)}(h_1,h_2) = \left(\frac{2}{\pi}\right)^2 \prod_{i=1}^{2}\left(\sum_{n=0}^{\infty}\frac{(-1)^n\left(n+\frac{1}{2}\right)\pi^2 T}{h_i^3}\left(\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h_i^2}\right\}\right)\right) \tag{5.5}$$

and

$$F_{1D_1(T),D_2(T)}(h_1,h_2) = \prod_{i=1}^{2}\left(1-\frac{2}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{n+\frac{1}{2}}\left(1-\exp\left\{-\frac{\left(n+\frac{1}{2}\right)^2\pi^2 T}{2h_i^2}\right\}\right)\right). \tag{5.6}$$

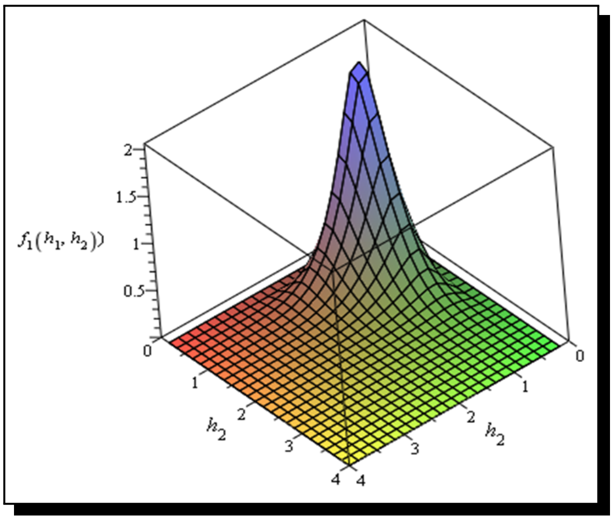

Figure 6. The joint PDF of h_1 and h_2 at $T = 0.8$ and $n = 100$

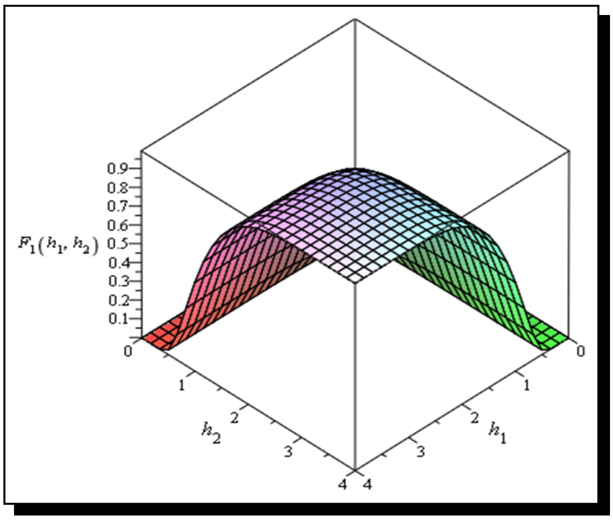

Figure 7. The joint cumulative distribution function of h_1 and h_2 at $T = 0.8$ and $n = 100$

In addition, we can use the definition that has been studied in El-Hadidy and Alraddadi [8] of truncated areas to study the distribution of variables h and g in different intervals. This provides us with a more flexible distribution to understand the successive changes in different stock prices during any time period.

The likelihood that both variables will simultaneously surpass specific drawdown thresholds is represented by the survival function for a bivariate maximum drawdown distribution. Stated differently, it quantifies the probability that neither of the variables encounters a decrease above a given threshold. Therefore, we can utilize the survival function to guide judgments about risk management. Determine threshold values, for instance, at which the survival probability dramatically decreases, indicating a larger likelihood of drawdowns over those thresholds for both variables. Setting risk limits, hedging techniques, and portfolio diversification can all be aided by this information. Consequently, $S_{1D_1(T),D_2(T)}(\cdot) = 1 - F_{1D_1(T),D_2(T)}(h_1,h_2)$ is the survival function of the bivariate model of our distribution, see Figure 8.

On the other hand, reliability qualities can be used to evaluate the stability, consistency, and predictive capability of a bivariate maximum drawdown distribution in relation to its risk value. Analysts and risk managers can appraise the utility and dependability of risk value estimations obtained from a bivariate maximum drawdown distribution by evaluating these reliability properties. This contributes to the accuracy of the distribution's representation of the combined behavior of the maximum drawdowns for the two variables and offers significant information for risk assessment and decision-making. The risk value can be computed from the risk rate function (hazard rate function) by:

$$\Psi_{1D_1(T),D_2(T)}(h_1,h_2) = \frac{f_{1D_1(T),D_2(T)}(h_1,h_2)}{1 - F_{1D_1(T),D_2(T)}(h_1,h_2)}. \tag{5.7}$$

(See, Figure 9.)

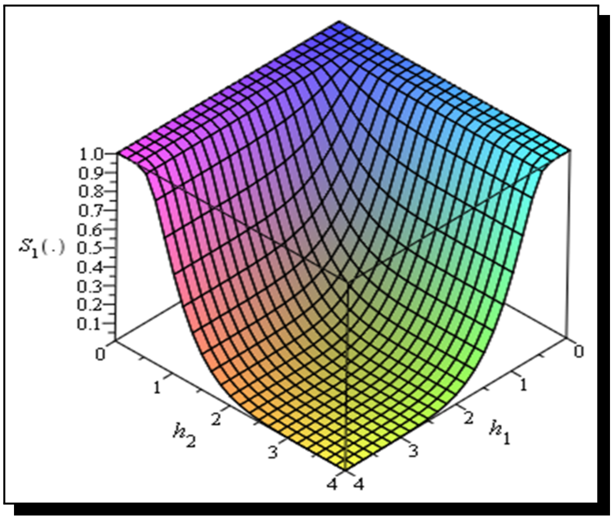

Figure 8. The survival function of h_1 and h_2 at $T = 0.8$ and $n = 100$

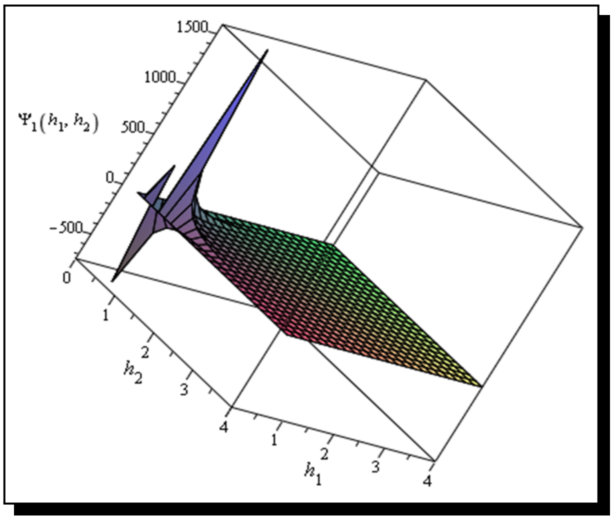

Figure 9. The hazard rate function of h_1 and h_2 at $T = 0.8$ and $n = 100$

On the other hand, the reversed hazard rate function, $\Omega_{1D_1(T),D_2(T)}(h_1,h_2) = \frac{f_{1D_1(T),D_2(T)}(h_1,h_2)}{F_{1D_1(T),D_2(T)}(h_1,h_2)}$ (see Figure 10) can be used to analyze the conditional probability of observing a maximum drawdown exceeding a certain threshold after a specified duration without such an occurrence.

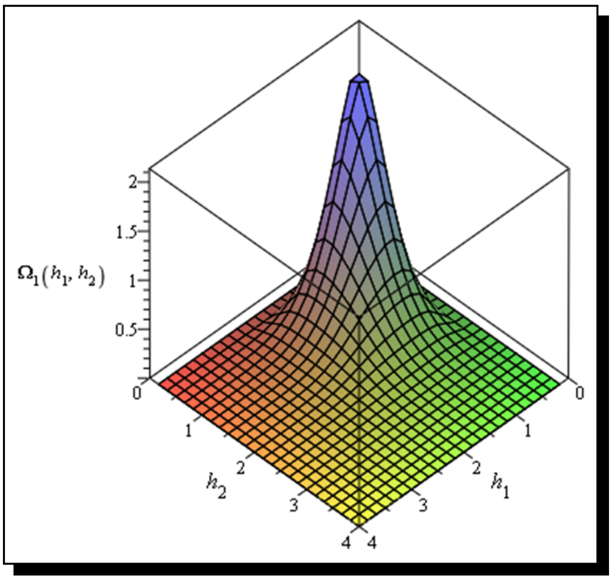

Figure 10. The reversed hazard rate function of h_1 and h_2 at $T = 0.8$ and $n = 100$

6. Concluding Remarks

In this paper, we present the accurate distribution for a stock market risk that follows a particular Wiener process. For any period of time, this distribution is the best for a maximum drawdown of stock market risk. We provided some reliability properties for this model. Furthermore, we consider its expected value, which is useful to predict the risk in the future.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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