Communications in Mathematics and Applications
Vol. 6, No. 1, pp. 17–20, 2015
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications

http://www.rgnpublications.com

On Weak Essential Ideals of Semiring

Research Article

Kishor Pawar

Department of Mathematics, School of Mathematical Sciences,
North Maharashtra University, Jalgaon 425001, India
***Corresponding author:** kfpawar@nmu.ac.in

Abstract. In this paper we investigate a class of ideals that lies between essential ideals and semi-essential ideals, we call these class of ideals as weak essential ideals.

Keywords. Semirings; Ideals; Essential ideal; Semi-essential ideal; Weak-essential ideal; Semi-prime ideal

MSC. 16Y60

Received: June 30, 2015 **Accepted:** August 6, 2015

1. Introduction

The notion of an essential ideal for semirings was given by Dutta and Das in [2]. Pawar and Deore proved proved some important results of essential ideals for semirings in [4]. Some results of essential subsemimodules and semi-essential subsemimodules of [5]-[6] has been extended to the weak essential ideals for semirings. M.A. Ahmed in [1] introduced and studied the Weak Essential Submodules. In this paper we generalized the proofs of some results on the line of [1] to semirings. Hence we investigate a class of ideals that lies between essential ideals and semi-essential ideals, and we call this class of ideals, *weak essential ideals*.

2. Preliminaries

Definition 2.1 ([3])**.** A semiring is a set R together with two binary operations called addition $(+)$ and multiplication $(\cdot)$ such that $(R,+)$ is a commutative monoid with identity element 0_R; $(R,\cdot)$ is a monoid with identity element 1; multiplication distributes over addition from either side and 0 is multiplicative absorbing, that is, $a\cdot 0 = 0\cdot a = 0$ for each $a \in R$.

Definition 2.2 ([3])**.** A semiring R is said to have a unity if there exists $1_R \in R$ such that $1_R \cdot a = a \cdot 1_R = a$ for each $a \in R$.

E.g., the set $\mathbb{N}$ of non-negative integers with the usual operations of addition and multiplication of integers is a semiring with unity, $1_{\mathbb{N}}$.

Definition 2.3 ([3])**.** An ideal I of a semiring R will be called subtractive (k-ideal) if for $a \in I$, $a+b \in I$, $b \in R$ imply $b \in I$.

Definition 2.4 ([3])**.** An ideal I of a semiring R is called prime (completely prime) if whenever $aRb \subseteq I$ ($ab \in I$) where $a, b \in R$ implies $a \in I$ or $b \in I$.

Definition 2.5 ([3])**.** An ideal I of a semiring R is called irreducible if and only if for ideal H and K of R, we have $I = H \cap K$ only when $I = H$ or $I = K$.

Definition 2.6 ([3])**.** An ideal I of a semiring R is semiprime if and only if, for any ideal H of R, we have $H^2 \subseteq I$ only when $H \subseteq I$.

Note 2.7 ([3])**.** *Prime ideals are surely semiprime ideals.*

Note 2.8 ([3])**.** *Every semiprime ideal of a semiring R is semisubtractive.*

3. Essential and Semi-essential Ideal

Definition 3.1 ([2])**.** An ideal I of a semiring R is said to be an essential ideal of R if $I \cap K \neq 0$, for every nonzero ideal K of R.

Notation. We shall denote an essential ideal I of a semiring R by $I \lhd \cdot R$.

Definition 3.2. A nonzero ideal I of a semiring R is said to be semi-essential ideal of R if $I \cap K \neq 0$, for each nonzero prime ideal K of R.

Notation. We shall denote semi-essential ideal I of a semiring R by by $I \lhd_s \cdot R$.

Lemma 3.3. *Let R be a semiring. If I is an essential ideal of J and J is an essential ideal of R then I is an essential in R.*

Proposition 3.4. *Let R be a semiring and suppose that $I_1, I_2, \ldots, I_k$ are ideals of R. Then $\bigcap_{i=1}^{k} I_i \lhd \cdot R$ if and only if each I_i is an essential ideal in R i.e. $I_i \lhd \cdot R$, for all i.*

4. Weak Essential Ideal

Definition 4.1. A nonzero ideal I of a semiring R is called weak essential if $I \cap J \neq (0)$, for each nonzero semiprime ideal J of R.

Notation. We shall denote an Weak Essential Ideal I of a semiring R by $I \lhd_w \cdot R$.

Proposition 4.2. *Let R be a semiring. A non-zero ideal I of a semiring R is weak essential if and only if for each non-zero semiprime ideal K of R there exists $x \in K$ and $r \in R$, such that $(0) \neq rx \in I$.*

Proof. Suppose that for each non-zero semi-prime ideal K of R, there exists $x \in K$ and $r \in R$ such that $(0) \neq rx \in I$. Note that $rx \in K$, therefore $(0) \neq rx \in I \cap K$. Thus $I \cap K \neq (0)$, that is K is a weak essential ideal of R. Conversely, suppose that W is a weak essential ideal of R. Then $K \cap I \neq (0)$ for each semiprime ideal I of R, thus there exists $(0) \neq x \in K \cap I$. This implies that $x \in I$ and hence $(0) \neq 1 \cdot x \in I$. □

Lemma 4.3. *Let I be an irreducible ideal of a semiring R. Then I is semi-prime if and only if I is prime ideal of R.*

Proposition 4.4. *Let R be a semiring such that every semi-prime ideal of R is irreducible. If an ideal I of R is semi-essential then I is a weak essential ideal of R.*

Proof. Let S be a non-zero semi-prime ideal of R with $I \cap S = (0)$. Since S is irreducible ideal then by above Lemma 4.3, S is prime ideal. But I is semi-essential ideal of R, therefore $S = (0)$. □

Lemma 4.5. *Let R be a semiring and let I_1 and I_2 be any two ideals of R such that $I_1 \subseteq I_2$. If I_1 is a weak essential ideal of R then I_2 is weak essential ideal of R.*

Lemma 4.6. *Let R be a semiring and let I_1 and I_2 be any two ideals of R such that $I_1 \cap I_2$ is a weak essential ideal of R, then both I_1 and I_2 are weak essential ideals of R.*

Proposition 4.7. *Let R be a semiring and let I_1 and I_2 be any two ideals of R such that I_1 is an essential ideal of R and I_2 is weak essential ideal of R. Then $I_1 \cap I_2$ is weak essential ideal of R.*

Proof. Since I_2 is a weak essential ideal of R, then $I_2 \cap S \neq (0)$, for each non-zero semi-prime ideal S of R. But I_1 is an essential ideal of R, so $I_1 \cap (I_2 \cap S) \neq (0)$, this implies that $(I_1 \cap I_2) \cap S \neq (0)$, and hence the proof. □

Proposition 4.8. *Let R be a semiring and let I_1 and I_2 be any two ideals of R such that one of them does not contained in any semi-prime ideal of R. If I_1 and I_2 are weak essential ideal of R, then $I_1 \cap I_2$ is weak essential ideal of R.*

Proof. Since I_2 is a weak essential ideal of R, then $I_2 \cap S \neq (0)$, for each non-zero semi-prime ideal S of R. But I_1 is an essential ideal of R, so $I_1 \cap (I_2 \cap S) \neq (0)$, this implies that $(I_1 \cap I_2) \cap S \neq (0)$, and hence the proof. □

5. Conclusion

In this paper we have investigated a class of ideals that lies between essential ideals and semiessential ideals. This class of ideals we called as weak essential ideals. Also we have verified some basic properties of weak essential ideals.

Competing Interests

I declare that I have no significant competing financial, professional or personal interests that might have influenced the performance or presentation of the work described in this manuscript.

Authors' Contributions

The author has wrote this article. The author has read and approved the final manuscript.

References

[1] M.A. Ahmed, Weak essential submodules, *Um-Salama Science Journal*, **6**(1) (2009), 214–220.

[2] T.K. Dutta and M.L. Das, On strongly prime semiring, *Bull. Malays Math. Sci. Soc.*, **30**(2) (2007), 135–141.

[3] J.S. Golan, *The Theory of Semirings with Applications in Mathematics and Theoretical Computer Science*, Pitman Monogr. Surveys Pure Appl. Math., Longman Scientific and Technical Harlow (1992).

[4] K.F. Pawar and R.P. Deore, On essential ideal and radical class, *Int. J. of Pure and Appl. Math. Sci.*, **5**(1) (2012), 1–5.

[5] K. Pawar, A note on essential subsemimodule, *New Trends in Mathematical Sciences*, **1**(2) (2013), 18–21.

[6] K. Pawar and P. Gujarathi, On semi-essential subsemimodules, *Int. Res. J. of Pure Algebra*, **5**(5) (2015), 85–87.