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Research Article

Application of MBJ-Neutrosophic in BRK-Algebra

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Abstract. This paper introduces the concept of MBJ-neutrosophic ideals and subalgrbras in BRK-algebras. We study various properties regarding these concepts. Also, a relationship between MBJ-neutrosophic ideals and MBJ-neutrosophic subalgrbras is presented. Moreover, various characterisations for MBJ-neutrosophic ideals are proved.

Keywords. MBJ-neutrosophic, BRK-algebra, Ideal, Subalgebra

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1. Introduction

Fuzzy sets was introduced by Zadeh [7] in order to manage uncertainties in numerous situations in different aspects. Then several attempts were made to generalize the notation of classical sets and fuzzy sets. Neutrosophic set was introduced and developed by Smarandache [5].

These concepts are applied to several algebraic structure such as BCK-algebra and BCI-algebra. BRK-algebra is a generalization of BCK-algebra and BCI-algebra introduced by Bandaru [1]. Then many papers have discussed some concepts in BRK-algebras (see, El-Gendy [2, 3], and Hayat *et al*. [4]).

MBJ-neutrosophic structure was introduced by Takallo *et al*. [6] and applied in BCK-algebra and BCI-algebra. We aim to generalize and applied MBJ-neutrosophic structure on BRK-algebra.

This paper introduces the concept of MBJ-neutrosophic ideals and subalgrbras in BRK-algebras. We study various properties regarding these concepts. Also, a relationship between MBJ-neutrosophic ideals and MBJ-neutrosophic subalgrbras is presented. Moreover, various characterisations for MBJ-neutrosophic ideals are proved.

Throught this paper, we write MBJ-NT to denote MBJ-neutrosophic.

2. Preliminaries

Recall that a BRK-algebra, P, is a non-empty set with a binary operation $\circledast$ and a constant 0 that satisfies the following criteria [1]:

(i) $s_{01} \circledast 0 = s_{01}$, for all $s_{01} \in P$.

(ii) $(s_{01} \circledast s_{02}) \circledast s_{01} = 0 \circledast s_{02}$, for all $s_{01}, s_{02} \in P$.

A partial ordered relation $\leq$ on P is defined as follows [1]:

$$s_{01} \leq s_{02} \Leftrightarrow s_{01} * s_{02} = 0.$$

Any BRK-algebra satisfies the following properties [1]:

(i) $s_{01} * s_{01} = 0$, for all $s_{01} \in P$.

(ii) $0 \circledast (s_{01} \circledast s_{02}) = (0 \circledast s_{01}) \circledast (0 \circledast s_{02})$, for every $s_{01}, s_{02} \in P$.

3. MBJ-Neutrosophic Ideals

Definition 3.1 ([5]). Let A be a non-empty set. An MBJ-NT set in A can be defined by:

$$\Upsilon := \{\langle a; M_\Upsilon(a), \widetilde{B}_\Upsilon(a), J_\Upsilon(a)\rangle : a \in A\},$$

in which M_Υ is a truth membership function, J_Υ is a false membership function and $\widetilde{B}_\Upsilon$ is an indeterminate interval-valued membership function.

Now, we are able to apply this concept to BRK-algebras.

Definition 3.2 ([5]). Let P be a BRK-algebra. An MBJ-NT set, $\Upsilon = (M_\Upsilon, \widetilde{B}_\Upsilon, J_\Upsilon)$, in $P$is called an MBJ-NT ideal if:

(i) For all $s_{01} \in P$,

$$\left.\begin{array}{l} M_\Upsilon(0) \geq M_\Upsilon(s_{01}), \\ \widetilde{B}_\Upsilon(0) \succeq \widetilde{B}_\Upsilon(s_{01}), \\ J_\Upsilon(0) \leq J_\Upsilon(s_{01}). \end{array}\right\} \tag{3.1}$$

(ii) For all $s_{01}, s_{02}, s_{03} \in P$,

$$\left.\begin{array}{l} M_\Upsilon(0 \circledast s_{01}) \geq \min\{M_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), M_\Upsilon(0 \circledast s_{02})\}, \\ \widetilde{B}_\Upsilon(0 \circledast s_{01}) \succeq r\min\{\widetilde{B}_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), \widetilde{B}_\Upsilon(0 \circledast s_{02})\}, \\ J_\Upsilon(0 \circledast s_{01}) \leq \max\{J_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), J_\Upsilon(0 \circledast s_{02})\}. \end{array}\right\} \tag{3.2}$$

Theorem 3.1. *The intersection of MBJ-NT ideals of a BRK-algebra is also an MBJ-NT ideal.*

Proof. Let $\Upsilon_1, \Upsilon_2, \dots, \Upsilon_n$ be MBJ-NT ideals. Then

(i)
$$\begin{aligned} M_{\cap\Upsilon_i}(0) &= \min\{M_{\Upsilon_i}(0)\} \\ &\geq \min\{M_{\Upsilon_i}(s_{01})\} \\ &= M_{\cap\Upsilon_i}(s_{01}), \end{aligned}$$

$$
\begin{aligned}
\widetilde{B}_{\cap\Upsilon_i}(0) &= r\min\{\widetilde{B}_{\Upsilon_i}(0)\} \\
&\geq r\min\{\widetilde{B}_{\Upsilon_i}(s_{01})\} \\
&= \widetilde{B}_{\cap\Upsilon_i}(s_{01}), \\
J_{\cap\Upsilon_i}(0) &= \max\{J_{\Upsilon_i}(0)\} \\
&\leq \max\{J_{\Upsilon_i}(s_{01})\} \\
&= J_{\cap\Upsilon_i}(s_{01}),
\end{aligned}
$$

(ii)
$$
\begin{aligned}
M_{\cap\Upsilon_i}(0 \circledast s_{01}) &= \min\{M_{\Upsilon_i}(0 \circledast s_{01})\} \\
&\geq \min\{\min\{M_{\Upsilon_i}(0 \circledast (s_{01} \circledast s_{02})), M_{\Upsilon_i}(0 \circledast s_{02})\}\} \\
&= \min\{\min\{M_{\Upsilon_i}(0 \circledast (s_{01} \circledast s_{02}))\}, \min\{M_{\Upsilon_i}(0 \circledast s_{02})\}\} \\
&= \min\{M_{\cap\Upsilon_i}(0 \circledast (s_{01} \circledast s_{02})), M_{\cap\Upsilon_i}(0 \circledast s_{02})\}, \\
\widetilde{B}_{\cap\Upsilon_i}(0 \circledast s_{01}) &= r\min\{\widetilde{B}_{\Upsilon_i}(0 \circledast s_{01})\} \\
&\geq r\min\{r\min\{\widetilde{B}_{\Upsilon_i}(0 \circledast (s_{01} \circledast s_{02})), \widetilde{B}_{\Upsilon_i}(0 \circledast s_{02})\}\} \\
&= r\min\{r\min\{\widetilde{B}_{\Upsilon_i}(0 \circledast (s_{01} \circledast s_{02}))\}, r\min\{\widetilde{B}_{\Upsilon_i}(0 \circledast s_{02})\}\} \\
&= r\min\{\widetilde{B}_{\cap\Upsilon_i}(0 \circledast (s_{01} \circledast s_{02})), \widetilde{B}_{\cap\Upsilon_i}(0 \circledast s_{02})\}, \\
J_{\cap\Upsilon_i}(0 \circledast s_{01}) &= \max\{J_{\Upsilon_i}(0 \circledast s_{01})\} \\
&\leq \max\{\max\{J_{\Upsilon_i}(0 \circledast (s_{01} \circledast s_{02})), J_{\Upsilon_i}(0 \circledast s_{02})\}\} \\
&= \max\{\max\{J_{\Upsilon_i}(0 \circledast (s_{01} \circledast s_{02}))\}, \max\{J_{\Upsilon_i}(0 \circledast s_{02})\}\} \\
&= \max\{J_{\cap\Upsilon_i}(0 \circledast (s_{01} \circledast s_{02})), J_{\cap\Upsilon_i}(0 \circledast s_{02})\}.
\end{aligned}
$$

Thus $\cap\Upsilon_i$ is an MBJ-NT ideal as required. □

A complement of an MBJ-NT set, Υ, is an MBJ-NT set defined by:

$$\Upsilon^c = (M_\Upsilon^c, \widetilde{B}_\Upsilon^c, J_\Upsilon^c),$$

where

$$M_\Upsilon^c = 1 - M_\Upsilon, \quad \widetilde{B}_\Upsilon^c = 1 - \widetilde{B}_\Upsilon, \quad J_\Upsilon^c = 1 - J_\Upsilon.$$

We now prove the following theorem:

Theorem 3.2. *A subset of a BRK-algebra is an MBJ-NT ideal if and only if its complement is an anti MBJ-NT ideal.*

Proof. Let Υ be an MBJ-NT ideal, this implies

(i)
$$
\begin{aligned}
M_\Upsilon(0) &\geq M_\Upsilon(s_{01}), \\
1 - M_\Upsilon(0) &\leq 1 - M_\Upsilon(s_{01}), \\
M_\Upsilon^c(0) &\leq M_\Upsilon^c(s_{01}).
\end{aligned}
$$

Also, since

$$
\begin{aligned}
\widetilde{B}_\Upsilon(0) &= [B_\Upsilon^-(0), B_\Upsilon^+(0)] \\
&\geq [B_\Upsilon^-(s_{01}), B_\Upsilon^+(s_{01})] \\
&= \widetilde{B}_\Upsilon(s_{01}),
\end{aligned}
$$

then

$$\widetilde{B}_\Upsilon^c(0) = 1 - \widetilde{B}_\Upsilon(0)$$

$$= [1 - B_\Upsilon^+(0), 1 - B_\Upsilon^-(0)]$$
$$\le [1 - B_\Upsilon^+(s_{01}), 1 - B_\Upsilon^-(s_{01})]$$
$$= \widetilde{B}_\Upsilon^c(s_{01}).$$

In addition,

$$J_\Upsilon(0) \le J_\Upsilon(s_{01}),$$
$$1 - J_\Upsilon(0) \ge 1 - J_\Upsilon(s_{01}),$$
$$J_\Upsilon^c(0) \ge J_\Upsilon^c(s_{01}),$$

(ii)
$$M_\Upsilon(0 \circledast s_{01}) \ge \min\{M_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), M_\Upsilon(0 \circledast s_{02})\}$$
$$1 - M_\Upsilon(0 \circledast s_{01}) \le 1 - \min\{M_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), M_\Upsilon(0 \circledast s_{02})\}$$
$$M_\Upsilon^c(0 \circledast s_{01}) \le \max\{1 - M_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), 1 - M_\Upsilon(0 \circledast s_{02})\}$$
$$= \max\{M_\Upsilon^c(0 \circledast (s_{01} \circledast s_{02})), M_\Upsilon^c(0 \circledast s_{02})\}.$$

Also, since

$$\widetilde{B}_\Upsilon(0 \circledast s_{01}) \ge r\min\{\widetilde{B}_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), \widetilde{B}_\Upsilon(0 \circledast s_{02})\}$$
$$= [\min\{B_\Upsilon^-(0 \circledast (s_{01} \circledast s_{02})), B_\Upsilon^-(0 \circledast s_{02})\},$$
$$\min\{B_\Upsilon^+(0 \circledast (s_{01} \circledast s_{02})), B_\Upsilon^+(0 \circledast s_{02})\}],$$

we obtain

$$\widetilde{B}_\Upsilon^c(0 \circledast s_{01}) = 1 - \widetilde{B}_\Upsilon(0 \circledast s_{01})$$
$$\le [1 - \min\{B_\Upsilon^+(0 \circledast (s_{01} \circledast s_{02})), B_\Upsilon^+(0 \circledast s_{02})\},$$
$$1 - \min\{B_\Upsilon^-(0 \circledast (s_{01} \circledast s_{02})), B_\Upsilon^-(0 \circledast s_{02})\}]$$
$$= [\max\{1 - B_\Upsilon^+(0 \circledast (s_{01} \circledast s_{02})), 1 - B_\Upsilon^+(0 \circledast s_{02})\},$$
$$\max\{1 - B_\Upsilon^-(0 \circledast (s_{01} \circledast s_{02})), 1 - B_\Upsilon^-(0 \circledast s_{02})\}]$$
$$= [\max\{{B_\Upsilon^-}^c(0 \circledast (s_{01} \circledast s_{02})), {B_\Upsilon^-}^c(0 \circledast s_{02})\},$$
$$\max\{{B_\Upsilon^+}^c(0 \circledast (s_{01} \circledast s_{02})), {B_\Upsilon^+}^c(0 \circledast s_{02})\}]$$
$$= r\max\{\widetilde{B}_\Upsilon^c(0 \circledast (s_{01} \circledast s_{02})), \widetilde{B}_\Upsilon^c(0 \circledast s_{02})\}.$$

In addition,

$$J_\Upsilon(0 \circledast s_{01}) \le \max\{J_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), J_\Upsilon(0 \circledast s_{02})\}$$
$$1 - J_\Upsilon(0 \circledast s_{01}) \ge 1 - \max\{J_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), J_\Upsilon(0 \circledast s_{02})\}$$
$$J_\Upsilon^c(0 \circledast s_{01}) \ge \min\{1 - J_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), 1 - J_\Upsilon(0 \circledast s_{02})\}$$
$$= \min\{J_\Upsilon^c(0 \circledast (s_{01} \circledast s_{02})), J_\Upsilon^c(0 \circledast s_{02})\}.$$

Thus Υ^c is an anti MBJ-NT ideal. Conversely, suppose that Υ^c is an anti MBJ-NT ideal. Then

(i)
$$M_\Upsilon^c(0) \le M_\Upsilon^c(s_{01}),$$
$$1 - M_\Upsilon(0) \le 1 - M_\Upsilon(s_{01}),$$
$$M_\Upsilon(0) \ge M_\Upsilon(s_{01}).$$

Note that

$$\widetilde{B}_\Upsilon^c(0) \le \widetilde{B}_\Upsilon^c(s_{01}).$$

Thus

$$1-\widetilde{B}_Y(0)\le 1-\widetilde{B}_Y(s_{01}),$$
$$\widetilde{B}_Y(0)\ge \widetilde{B}_Y(s_{01}).$$

Also,

$$J^c_Y(0)\le J^c_Y(s_{01}),$$
$$1-J_Y(0)\le 1-J_Y(s_{01}),$$
$$J_Y(0)\ge J_Y(s_{01}).$$

(ii)
$$\begin{aligned}M^c_Y(0\circledast s_{01})&\le \max\{M^c_Y(0\circledast(s_{01}\circledast s_{02})),M^c_Y(0\circledast s_{02})\}\\&=\max\{1-M_Y(0\circledast(s_{01}\circledast s_{02})),1-M_Y(0\circledast s_{02})\},\\1-M_Y(0\circledast s_{01})&\le 1-\min\{M_Y(0\circledast(s_{01}\circledast s_{02})),M_Y(0\circledast s_{02})\},\\M_Y(0\circledast s_{01})&\ge \min\{M_Y(0\circledast(s_{01}\circledast s_{02})),M_Y(0\circledast s_{02})\}.\end{aligned}$$

We also have

$$\begin{aligned}\widetilde{B}^c_Y(0\circledast s_{01})&\le r\max\{\widetilde{B}^c_Y(0\circledast(s_{01}\circledast s_{02})),\widetilde{B}^c_Y(0\circledast s_{02})\}\\&=[\max\{B^{-c}_Y(0\circledast(s_{01}\circledast s_{02})),B^{-c}_Y(0\circledast s_{02})\},\\&\quad\max\{B^{+c}_Y(0\circledast(s_{01}\circledast s_{02})),B^{+c}_Y(0\circledast s_{02})\}]\\&=[\max\{1-B^{+}_Y(0\circledast(s_{01}\circledast s_{02})),1-B^{+}_Y(0\circledast s_{02})\},\\&\quad\max\{1-B^{-}_Y(0\circledast(s_{01}\circledast s_{02})),1-B^{-}_Y(0\circledast s_{02})\}]\\&=[1-\min\{B^{+}_Y(0\circledast(s_{01}\circledast s_{02})),B^{+}_Y(0\circledast s_{02})\},\\&\quad 1-\min\{B^{-}_Y(0\circledast(s_{01}\circledast s_{02})),B^{-}_Y(0\circledast s_{02})\}].\end{aligned}$$

Then

$$\begin{aligned}1-\widetilde{B}_Y(0\circledast s_{01})&\le[1-\min\{B^{+}_Y(0\circledast(s_{01}\circledast s_{02})),B^{+}_Y(0\circledast s_{02})\},\\&\qquad 1-\min\{B^{-}_Y(0\circledast(s_{01}\circledast s_{02})),B^{-}_Y(0\circledast s_{02})\}]\\\widetilde{B}_Y(0\circledast s_{01})&\ge[\min\{B^{-}_Y(0\circledast(s_{01}\circledast s_{02})),B^{-}_Y(0\circledast s_{02})\},\\&\qquad\min\{B^{+}_Y(0\circledast(s_{01}\circledast s_{02})),B^{+}_Y(0\circledast s_{02})\}]\\&=r\min\{\widetilde{B}_Y(0\circledast(s_{01}\circledast s_{02})),\widetilde{B}_Y(0\circledast s_{02})\}.\end{aligned}$$

Moreover,

$$\begin{aligned}J^c_Y(0\circledast s_{01})&\ge \min\{J^c_Y(0\circledast(s_{01}\circledast s_{02})),J^c_Y(0\circledast s_{02})\}\\&=\min\{1-J_Y(0\circledast(s_{01}\circledast s_{02})),1-J_Y(0\circledast s_{02})\},\\1-J_Y(0\circledast s_{01})&\ge 1-\max\{J_Y(0\circledast(s_{01}\circledast s_{02})),J_Y(0\circledast s_{02})\},\\J_Y(0\circledast s_{01})&\le \max\{J_Y(0\circledast(s_{01}\circledast s_{02})),J_Y(0\circledast s_{02})\}.\end{aligned}$$

□

Theorem 3.3. *Every MBJ-NT ideal in a BRK-algebra P satisfies the property:*
If $s_{01}\le s_{02}$, then

$$M_Y(0\circledast s_{01})\ge M_Y(0\circledast s_{02}),$$
$$\widetilde{B}_Y(0\circledast s_{01})\succeq \widetilde{B}_Y(0\circledast s_{02}),$$
$$J_Y(0\circledast s_{01})\le J_Y(0\circledast s_{02}).$$

Proof. Suppose that $s_{01}, s_{02} \in P$. Let Υ be an MBJ-NT ideal. Since $s_{01} \le s_{02}$, then $s_{01} \circledast s_{02} = 0$. Thus

$$\begin{aligned} M_\Upsilon(0 \circledast s_{01}) &\ge \min\{M_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), M_\Upsilon(0 \circledast s_{02})\} \\ &= \min\{M_\Upsilon(0 \circledast 0), M_\Upsilon(0 \circledast s_{02})\} \\ &= \min\{M_\Upsilon(0), M_\Upsilon(0 \circledast s_{02})\} \\ &= M_\Upsilon(0 \circledast s_{02}), \\ \widetilde{B}_\Upsilon(0 \circledast s_{01}) &\succeq r\min\{\widetilde{B}_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), \widetilde{B}_\Upsilon(0 \circledast s_{02})\} \\ &= r\min\{\widetilde{B}_\Upsilon(0 \circledast 0), \widetilde{B}_\Upsilon(0 \circledast s_{02})\} \\ &= r\min\{\widetilde{B}_\Upsilon(0), \widetilde{B}_\Upsilon(0 \circledast s_{02})\} \\ &= \widetilde{B}_\Upsilon(0 \circledast s_{02}), \\ J_\Upsilon(0 \circledast s_{01}) &\le \max\{J_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), J_\Upsilon(0 \circledast s_{02})\} \\ &= \max\{J_\Upsilon(0 \circledast 0), J_\Upsilon(0 \circledast s_{02})\} \\ &= \max\{J_\Upsilon(0), J_\Upsilon(0 \circledast s_{02})\} \\ &= J_\Upsilon(0 \circledast s_{02}) \end{aligned}$$

as required. □

Proposition 1. *Let P be a BRK-algebra. If Υ is an MBJ-NT ideal and*

$$s_{01} \circledast s_{02} \le s_{03},$$

for some $s_{01}, s_{02}, s_{03} \in \Upsilon$, then

$$\left.\begin{aligned} M_\Upsilon(0 \circledast s_{01}) &\ge \min\{M_\Upsilon(0 \circledast s_{02}), M_\Upsilon(0 \circledast s_{03})\}, \\ \widetilde{B}_\Upsilon(0 \circledast s_{01}) &\succeq \min\{\widetilde{B}_\Upsilon(0 \circledast s_{02}), \widetilde{B}_\Upsilon(0 \circledast s_{03})\}, \\ J_\Upsilon(0 \circledast s_{01}) &\le \max\{J_\Upsilon(0 \circledast s_{02}), J_\Upsilon(0 \circledast s_{03})\}. \end{aligned}\right\} \tag{3.3}$$

Proof. Suppose that $s_{01} \circledast s_{02} \le s_{03}$. Then $(s_{01} \circledast s_{02}) \circledast s_{03} = 0$ and so

$$0 \circledast (s_{01} \circledast s_{02}) = 0 \circledast s_{03},$$

that Υ is an MBJ-NT ideal implies

$$\left.\begin{aligned} M_\Upsilon(0 \circledast s_{01}) &\ge \min\{M_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), M_\Upsilon(0 \circledast s_{02})\}, \\ \widetilde{B}_\Upsilon(0 \circledast s_{01}) &\succeq r\min\{\widetilde{B}_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), \widetilde{B}_\Upsilon(0 \circledast s_{02})\}, \\ J_\Upsilon(0 \circledast s_{01}) &\le \max\{J_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), J_\Upsilon(0 \circledast s_{02})\}. \end{aligned}\right\}$$

But note that

$$\begin{aligned} M_\Upsilon(0 \circledast (s_{01} \circledast s_{02})) &\ge \min\{M_\Upsilon(0 \circledast ((s_{01} \circledast s_{02}) \circledast s_{03}), M_\Upsilon(0 \circledast s_{03})\} \\ &= \min\{M_\Upsilon(0), M_\Upsilon(0 \circledast s_{03})\} \\ &= M_\Upsilon(0 \circledast s_{03}), \\ \widetilde{B}_\Upsilon(0 \circledast (s_{01} \circledast s_{02})) &\succeq r\min\{\widetilde{B}_\Upsilon(0 \circledast ((s_{01} \circledast s_{02}) \circledast s_{03}), \widetilde{B}_\Upsilon(0 \circledast s_{03})\} \\ &= r\min\{\widetilde{B}_\Upsilon(0), \widetilde{B}_\Upsilon(0 \circledast s_{03})\} \\ &= \widetilde{B}_\Upsilon(0 \circledast s_{03}), \\ J_\Upsilon(0 \circledast (s_{01} \circledast s_{02})) &\le \max\{J_\Upsilon(0 \circledast ((s_{01} \circledast s_{02}) \circledast s_{03}), J_\Upsilon(0 \circledast s_{03})\} \end{aligned}$$

$$= \max\{J_\Upsilon(0), J_\Upsilon(0 \circledast s_{03})\}$$
$$= J_\Upsilon(0 \circledast s_{03}).$$

Hence

$$M_\Upsilon(0 \circledast s_{01}) \geq \min\{M_\Upsilon(0 \circledast s_{02}), M_\Upsilon(0 \circledast s_{03})\},$$
$$\widetilde{B}_\Upsilon(0 \circledast s_{01}) \succeq \min\{\widetilde{B}_\Upsilon(0 \circledast s_{02}), \widetilde{B}_\Upsilon(0 \circledast s_{03})\},$$
$$J_\Upsilon(0 \circledast s_{01}) \leq \max\{J_\Upsilon(0 \circledast s_{02}), J_\Upsilon(0 \circledast s_{03})\}.$$

□

Theorem 3.4. *Every MBJ-NT set in a BRK-algebra satisfying condition* (3.1) *and condition* (3.3) *is an MBJ-NT ideal.*

Proof. Let Υ be an MBJ-NT set satisfying (3.1) and (3.3). Since $s_{01} \circledast s_{02} \leq s_{01} \circledast s_{02}$, replacing $s_{03} = s_{01} \circledast s_{02}$ in condition (3.3), we obtain

$$M_\Upsilon(0 \circledast s_{01}) \geq \min\{M_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), M_\Upsilon(0 \circledast s_{02})\},$$
$$\widetilde{B}_\Upsilon(0 \circledast s_{01}) \succeq r\min\{\widetilde{B}_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), \widetilde{B}_\Upsilon(0 \circledast s_{02})\},$$
$$J_\Upsilon(0 \circledast s_{01}) \leq \max\{J_\Upsilon(0 \circledast (s_{01} \circledast s_{02})), J_\Upsilon(0 \circledast s_{02})\}.$$

Hence Υ is an MBJ-NT ideal. □

Definition 3.3. Let P be a BRK-algebra. The set

$$B(P) = \{s_{01} \in P : 0 \circledast s_{01} = 0\}$$

is called a p-radical of P.

Proposition 2. *A p-radical of a BRK-algebra is an MBJ-NT ideal.*

Proof. It follows from the definition of a p-radical that the coditions of an MBJ-NT ideal are fulfilled. □

4. Subalgebra

Let P be a BRK-algebra. An MBJ-NT set $\Upsilon = (M_\Upsilon, \widetilde{B}_\Upsilon, J_\Upsilon)$ in P is called an MBJ-NT subalgebra if:

$$M_\Upsilon(s_{01} \circledast s_{02}) \geq \min\{M_\Upsilon(s_{01}), M_\Upsilon(s_{02})\},$$
$$\widetilde{B}_\Upsilon(s_{01} \circledast s_{02}) \succeq r\min\{\widetilde{B}_\Upsilon(s_{01}), \widetilde{B}_\Upsilon(s_{02})\},$$
$$J_\Upsilon(s_{01} \circledast s_{02}) \leq \max\{J_\Upsilon(s_{01}), J_\Upsilon(s_{02})\},$$

for every $s_{01}, s_{02} \in P$.

Theorem 4.1. *Let P be a BRK-algebra and Υ an MBJ-NT subalgebra. If Υ satisfies condition* (3.2)*, then Υ is an MBJ-NT ideal.*

Proof. Let $s_{01} \in \Upsilon$. Then

$$M_\Upsilon(0) = M_\Upsilon(s_{01} \circledast s_{01})$$
$$\geq \min\{M_\Upsilon(s_{01}), M_\Upsilon(s_{01})\}$$
$$= M_\Upsilon(s_{01}),$$
$$\widetilde{B}_\Upsilon(0) = \widetilde{B}_\Upsilon(s_{01} \circledast s_{01})$$

$$
\begin{aligned}
&\geq r\min\{\widetilde{B}_\Upsilon(s_{01}),\widetilde{B}_\Upsilon(s_{01})\}\\
&= r\min\{[B_\Upsilon^-(s_{01}),B_\Upsilon^+(s_{01})],[B_\Upsilon^-(s_{01}),B_\Upsilon^+(s_{01})]\}\\
&= [B_\Upsilon^-(s_{01}),B_\Upsilon^+(s_{01})]\\
&= \widetilde{B}_\Upsilon(s_{01}),\\
J_\Upsilon(0) &= J_\Upsilon(s_{01}\circledast s_{01})\\
&\leq \max\{J_\Upsilon(s_{01}),J_\Upsilon(s_{01})\}\\
&= J_\Upsilon(s_{01}).
\end{aligned}
$$

□

Definition 4.1. Let P be a BRK-algebra. The set

$$G(P)=\{s_{01}\in P: 0\circledast s_{01}=s_{01}\}$$

is called the G-part of P.

Theorem 4.2. *Let P be a BRK-algebra and Υ an MBJ-NT ideal. Then $\Upsilon\cap G(P)$ is an MBJ-NT subalgebra.*

Proof. Suppose that $s_{01},s_{02},s_{03}\in\Upsilon\cap G(P)$. Then

$$
\begin{aligned}
M_\Upsilon(s_{01}\circledast s_{02}) &= M_\Upsilon(0\circledast(s_{01}\circledast s_{02}))\\
&\geq \min\{M_\Upsilon(0\circledast((s_{01}\circledast s_{02})\circledast s_{03})),M_\Upsilon(0\circledast s_{03})\}\\
&= \min\{M_\Upsilon((s_{01}\circledast s_{02})\circledast s_{03}),M_\Upsilon(s_{03})\}\\
&= \min\{M_\Upsilon((s_{01}\circledast s_{02})\circledast s_{01}),M_\Upsilon(s_{01})\}\\
&= \min\{M_\Upsilon(0\circledast s_{02}),M_\Upsilon(s_{01})\}\\
&= \min\{M_\Upsilon(0\circledast s_{02}),M_\Upsilon(s_{01})\}\\
&= \min\{M_\Upsilon(s_{01}),M_\Upsilon(s_{02})\},\\
\widetilde{B}_\Upsilon(s_{01}\circledast s_{02}) &= \widetilde{B}_\Upsilon(0\circledast(s_{01}\circledast s_{02}))\\
&\geq r\min\{\widetilde{B}_\Upsilon(0\circledast((s_{01}\circledast s_{02})\circledast s_{03})),\widetilde{B}_\Upsilon(0\circledast s_{03})\}\\
&= r\min\{\widetilde{B}_\Upsilon((s_{01}\circledast s_{02})\circledast s_{03}),\widetilde{B}_\Upsilon(s_{03})\}\\
&= r\min\{\widetilde{B}_\Upsilon((s_{01}\circledast s_{02})\circledast s_{01}),\widetilde{B}_\Upsilon(s_{01})\}\\
&= r\min\{\widetilde{B}_\Upsilon(0\circledast s_{02}),\widetilde{B}_\Upsilon(s_{01})\}\\
&= r\min\{\widetilde{B}_\Upsilon(0\circledast s_{02}),\widetilde{B}_\Upsilon(s_{01})\}\\
&= r\min\{\widetilde{B}_\Upsilon(s_{01}),\widetilde{B}_\Upsilon(s_{02})\},\\
J_\Upsilon(s_{01}\circledast s_{02}) &= J_\Upsilon(0\circledast(s_{01}\circledast s_{02}))\\
&\leq \max\{J_\Upsilon(0\circledast((s_{01}\circledast s_{02})\circledast s_{03})),J_\Upsilon(0\circledast s_{03})\}\\
&= \max\{J_\Upsilon((s_{01}\circledast s_{02})\circledast s_{03}),J_\Upsilon(s_{03})\}\\
&= \max\{J_\Upsilon((s_{01}\circledast s_{02})\circledast s_{01}),J_\Upsilon(s_{01})\}\\
&= \max\{J_\Upsilon(0\circledast s_{02}),J_\Upsilon(s_{01})\}\\
&= \max\{J_\Upsilon(0\circledast s_{02}),J_\Upsilon(s_{01})\}\\
&= \max\{J_\Upsilon(s_{01}),J_\Upsilon(s_{02})\}.
\end{aligned}
$$

Therefore, $\Upsilon\cap G(P)$ is an MBJ-NT subalgebra. □

5. Conclusion

This paper introduces the notions of MBJ-neutrosophic ideals and subalgebras within the framework of BRK-algebras. Various properties of these structures are investigated. In addition, the relationship between MBJ-neutrosophic ideals and MBJ-neutrosophic subalgebras is explored. Several characterizations of MBJ-neutrosophic ideals are also established.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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