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Research Article

Approximation by Half Terms of Baskakov-Kantorovich Sequence of Linear Positive Operators

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Abstract. This paper modified the Baskakov-Kantorovich sequence by taking the half terms of the sequence. Firstly, the convergence of this sequence was shown using Korovkin's theorem. Secondly, the Voronovskaja-type asymptotic theorem for this sequence is established. Finally, this sequence gives two numerical examples to approximate two test functions. Then, the numerical outcomes will be contrasted with those of the corresponding classical sequence. Compared to the classical sequence, the modified sequence produces superior numerical results.

Keywords. Baskakov operators, Korovkin theorem, Voronovskaja-type asymptotic theorem, Numerical experiments

Mathematics Subject Classification (2020). 41A10, 41A25, 41A36

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1. Introduction

Approximation by linear positive operator is one of the most important subjects in approximation theory. It aims to simplify complex functions with a simpler one to study them. Korovkin [10] was the first to use this technique by Korovkin's theorem to prove the convergence of sequences which was a great boost for many researchers to develop and extend. Baskakov [5] introduced the sequence of linear positive operators

$$M_n(g;y)=\sum_{w=0}^{\infty} q_{w,n}(y)\,g\left(\frac{w}{n}\right), \tag{1.1}$$

for a function $g \in C_\lambda[0,\infty) := \{g \in C[0,\infty) : g(t) = O((1+t)^\lambda)\}$, for some $\lambda > 0$, denoted to the space of unbounded functions of polynomial growth. This space is normed by the norm

$$\|g\|_{C_\lambda} = \sup_{t\in[0,\infty)} |g(t)|(1+t)^{-\lambda},$$

where

$$q_{w,n}(y) = \binom{n-1+w}{w} y^w (1+y)^{-n-w}, \quad y \in [0,\infty) \text{ and } n \in \mathbb{N} := \{1,2,\dots\}. \tag{1.2}$$

In [6], Ditzian defined the Baskakov-Kantorovich sequence for a function $g \in C_\lambda[0,\infty)$ and $n \in \mathbb{N}$ as

$$Q_n(g;y) = n \sum_{w=0}^{\infty} q_{w,n}(y) \int_{\frac{w}{n}}^{\frac{1+w}{n}} g(t)dt. \tag{1.3}$$

Abel and Gupta [1] study the Baskakov-Kantorovich sequence with the Bezier variant. Mohammad and Hassan [11] study the approximation properties of partial sums for summation integral Baskakov-type sequence. Agrawal and Goyal [2] introduced the generalized Baskakov-Kantorovich sequence depending on new weight functions. Ilarslan *et al*. [8] defined the generalization of the Kantorovich sequence depending on the non-negative parametric α-Baskakov sequence. Yilmaz *et al*. [16] generalized the Baskakov-Kantorovich sequence that preserved the function e^{-x}. They also explored this generalization's uniform convergence using the continuity modulus. Mohammad *et al*. [13] study the new family of Baskakov sequences depending on the parameter $s > -\frac{1}{2}$. Mohammad and Samad [12] introduced the r-th powers of the rational Bernstein sequence. Mohiuddin *et al*. [14] combined the Stancu sequence with the Baskakov-Kantorovich sequence to Approximate functions about unbounded intervals dependent on the parameter $\alpha > 0$. Goyal and Agrawal [7] generalized the Baskakov-Kantorovich sequence to two-dimensional and evaluated the degree of this approximation. Jasim and Mohammad [9] introduced the generalized of the multiple sums of sequence dependent on parameter $s > -\frac{1}{2}$.

This paper is defined and study a modification of the Baskakov-Kantorovich sequence (1.3), which is defined by

$$G_n(g;y) = 4n \sum_{w=0}^{\infty} q_{2w,2n}(y) \int_{\frac{w}{n}}^{\frac{2w+1}{2n}} g(t)dt. \tag{1.4}$$

This modification is called the half of the Baskakov-Kantorovich sequence.

2. Primary Results

Some definitions and lemmas are given here, which support the paper's main findings.

Lemma 2.1. *For the function $q_{2w,2n}(y)$, the following fact holds*

$$y(y+1)q'_{2w,2n}(y) = 2(w-ny)q_{2w,2n}(y). \tag{2.1}$$

Proof. By using the derivative of the function $q_{2w,2n}(y)$ and multiplication of both sides by (y^2+y), one gets

$$\begin{aligned}(y^2+y)q'_{2w,2n}(y) &= \binom{2n-1+2w}{2w} y^{2w}(1+y)^{-(2n+2w)}(2w(y+1)-(2n+2w)y)\\ &= 2(w-ny)q_{2w,2n}(y).\end{aligned}$$

□

Lemma 2.2. *The function*

$$R_{m,n,\beta}(y) = \sum_{w=0}^{\infty} q_{2w,2n}(y)\left(\frac{2w+\beta}{2n} - y\right)^m, \tag{2.2}$$

has the following properties:

(i) $R_{0,n,\beta}(y) = 1 + (2y+1)^{-2n}$,

(ii) $R_{1,n,\beta}(y) = \frac{\beta}{2n} + \frac{-4y^2n+(2\beta-4n)y+\beta}{2n(2y+1)^{2n+1}}$,

(iii) $R_{m+1,n,\beta}(y) = \frac{1}{2n}((y+x^2)(R'_{m,n,\beta}(y) + mR_{m-1,n,\beta}(y)) + \beta R_{m,n,\beta}(y))$.

Proof. The proof of properties (i) and (ii) are obtained directly, and the proof of (iii) can be obtained below

$$\begin{aligned} R'_{m,n,\beta}(y) &= \sum_{w=0}^{\infty}\left(q'_{2w,2n}(y)\left(\frac{2w+\beta}{2n} - y\right)^m - mq_{2w,2n}(y)\left(\frac{2w+\beta}{2n} - y\right)^{m-1}\right) \\ &= \sum_{w=0}^{\infty}\left(q'_{2w,2n}(y)\left(\frac{2w+\beta}{2n} - y\right)^m\right) - mR_{m-1,n,\beta}(y). \end{aligned}$$

Now, by multiplication of both sides by (y^2+y) and using Lemma 2.1, one gets

$$\begin{aligned} (y^2+y)(R'_{m,n,\beta}(y)) &= \sum_{w=0}^{\infty}\left(2(w-ny)q_{2w,2n}(y)\left(\frac{2w+\beta}{2n} - y\right)^m\right) - (y^2+y)mR_{m-1,n,\beta}(y) \\ &= \sum_{w=0}^{\infty}\left(2n\left(\frac{w}{n} - y + \frac{\beta}{2n} - \frac{\beta}{2n}\right)q_{2w,2n}(y)\left(\frac{2w+\beta}{2n} - y\right)^m\right) - (y^2+y)mR_{m-1,n,\beta}(y) \\ &= 2nR_{m+1,n,\beta}(y) - \beta R_{m,n,\beta}(y) - (y^2+y)mR_{m-1,n,\beta}(y), \end{aligned}$$

then

$$R_{m+1,n,\beta}(y) = \frac{1}{2n}((y+y^2)(R'_{m,n,\beta}(y) + mR_{m-1,n,\beta}(y)) + \beta R_{m,n,\beta}(y)). \qquad \square$$

The m-th order moments for the sequence $G_n(g;y)$ is defined by

$$\Omega_{m,n}(y) = G_n((t-y)^m;y) = 4n\sum_{w=0}^{\infty} q_{2w,2n}(y)\int_{\frac{w}{n}}^{\frac{2w+1}{2n}}(t-y)^m dt. \tag{2.3}$$

Lemma 2.3. *The function $\Omega_{m,n}(y)$ has the following properties:*

(i) $\Omega_{0,n}(y) = (1+2x)^{-2n} + 1$,

(ii) $\Omega_{1,n}(y) = \frac{1}{4n}\left(1 + \frac{-8ny^2+(2-8n)y+1}{(2y+1)^{2n+1}}\right)$,

(iii) $\Omega_{2,n}(y) = \frac{y^2+y}{2n} + \frac{1}{12n^2} + \frac{48n^2y^4+(96n^2-24n)y^3+(48n^2-42n+4)y^2-(18n-4)y+1}{12n^2(2y+1)^{2n+2}}$,

(iv) $\Omega_{m,n}(y) = \frac{2n}{m+1}(R_{m+1,n,1}(y) - R_{m+1,n,0}(y))$.

Proof. The proof of properties (i), (ii) and (iii) are obtained directly, and the proof of (iv) can be obtained as the following:

$$\begin{aligned} \Omega_{m,n}(y) &= 4n\sum_{w=0}^{\infty} q_{2w,2n}(y)\int_{\frac{w}{n}}^{\frac{2w+1}{2n}}(t-y)^m dt \\ &= \frac{4n}{m+1}\sum_{w=0}^{\infty} q_{2w,2n}(y)\left(\left(\frac{2w+1}{2n} - y\right)^{m+1} - \left(\frac{w}{n} - y\right)^{m+1}\right) \end{aligned}$$

by using Lemma 2.2, one gets

$$\Omega_{m,n}(y) = \frac{2n}{m+1}(R_{m+1,n,1}(y) - R_{m+1,n,0}(y)).$$ □

Lemma 2.4. *The function $\Omega_{m,n}(y)$, has the following facts*

(i) $\lim_{n\to\infty} n\Omega_{1,n}(y) = \frac{1}{4}$,

(ii) $\lim_{n\to\infty} n\Omega_{2,n}(y) = \frac{y+y^2}{2}$,

(iii) $\lim_{n\to\infty} n\Omega_{m,n}(y) = 0$, *where* $m \geq 3$.

Proof. By direct evaluation, this result holds. □

3. Main Results

This section introduced the convergence theorem and Voronovskaja-type asymptotic theorem for sequence $G_n(g;y)$.

Theorem 3.1 (Convergence theorem)**.** *The sequence $G_n(g;y)$ is converges to the function g as $n\to\infty$, where $g \in C_\lambda[0,\infty)$.*

Proof. Using Korovkin's theorem [10], to proof this theorem,

(i) $G_n(1;y) = 4n\sum_{w=0}^{\infty} q_{2w,2n}(y)\int_{\frac{w}{n}}^{\frac{2w+1}{2n}} dt = 1 + (2y+1)^{-2n}$,

(ii) $G_n(t;y) = 4n\sum_{w=0}^{\infty} q_{2w,2n}(y)\int_{\frac{k}{n}}^{\frac{2w+1}{2n}} t\,dt = y + \frac{1}{4n}\left(1 + \frac{(2-4n)y+1}{(2y+1)^{2n+1}}\right)$,

(iii) $G_n(t^2;y) = 4n\sum_{w=0}^{\infty} q_{2w,2n}(y)\int_{\frac{w}{n}}^{\frac{2w+1}{2n}} t^2 dt = \frac{2n+1}{2n}y^2 + \frac{y}{n} + \frac{1}{12n^2}\left(1 + \frac{(12n^2-18n+4)y^2+(4-12n)y+1}{(2y+1)^{2n+2}}\right)$.

Hence, $G_n(g;y) \to g(y)$ as $n\to\infty$. □

Definition 3.1. For $\alpha > 0$. The modulus of continuity [3], $\omega_\epsilon(g)$ is defined by

$$\omega_\epsilon(g) = \sup_{|t-y|\leq\epsilon} |g(t)-g(y)|, \quad \text{where } t,y \in [0,\infty). \tag{3.1}$$

Theorem 3.2. *For $g \in C_\lambda[0,\infty)$, then*

$$|G_n(g;y) - g(y)| \leq 2\omega_\epsilon(g), \tag{3.2}$$

where $\epsilon = \sqrt{\Omega_{2,n}\Omega_{0,n}}$.

Proof. By applying the property of modulus of continuity [3], then

$$|g(t)-g(y)| \leq \omega_\epsilon(g)\left(\frac{|t-y|}{\epsilon}+1\right).$$

Now, by taking the sequence $G_n(\cdot;y)$ for two both sides, one gets

$$\begin{aligned}|G_n(g(t);y)-g(y)| &\leq \omega_\epsilon(g)\left(\frac{G_n(|t-y|;y)}{\epsilon}+1\right)\\ &= \omega_\epsilon(g)\left(\left(\frac{4n}{\epsilon}\sum_{w=0}^{\infty} q_{2w,2n}^{\frac{1}{2}+\frac{1}{2}}(y)\int_{\frac{w}{n}}^{\frac{1+2w}{2n}} |t-y|\,dt\right)+1\right).\end{aligned}$$

By using the Cauchy-Schwarz inequality [15] and Lemma 2.2, one has

$$|G_n(g(t);y)-g(y)| \le \frac{\omega_\epsilon(g)}{\epsilon}\left(4n\sum_{w=0}^{\infty} q_{2w,2n}(y)\int_{\frac{w}{n}}^{\frac{1+2w}{2n}}(t-y)^2dt\right)^{\frac{1}{2}}\left(4n\sum_{w=0}^{\infty} q_{2w,2n}(y)\int_{\frac{w}{n}}^{\frac{1+2w}{2n}}dt\right)^{\frac{1}{2}}+\omega_\epsilon(g)$$
$$=\omega_\epsilon(g)\left(\frac{1}{\epsilon}(\Omega_{2,n}(y))^{\frac{1}{2}}\left(\Omega_{0,n}(y)\right)^{\frac{1}{2}}+1\right).$$

Since, $\epsilon=\sqrt{\Omega_{2,n}\Omega_{0,n}}$.
Hence,

$$|G_n(g;y)-g(y)| \le 2\omega_\epsilon(g). \qquad \square$$

Theorem 3.3. *For* $g\in C_\lambda[0,\infty)$ *and* g'' *exists and continuous then,*

$$\lim_{n\to\infty} n(G_n(g(t);y)-g(y)) = \frac{g'(y)+y(y+1)g''(y)}{4}. \tag{3.3}$$

Proof. At the point y, Taylor's expansion [2] of $g(t)$ is provided as follows:

$$g(t)=g(y)+(t-y)g'(y)+\frac{(t-y)^2g''(y)}{2}+(t-y)^2\zeta(t,y),$$

where $\zeta(t,y)\to 0$ as $t\to y$.
Now, by taking the sequence $G_n(\cdot;y)$ for two both sides and using Lemma 2.3, one has

$$G_n(g(t);y)=g(y)\Omega_{0,n}(y)+g'(y)\Omega_{1,n}(y)+\frac{1}{2}g''(y)\Omega_{2,n}(y)+G_n((t-y)^2\zeta(t,y);y).$$

Then,

$$\lim_{n\to\infty} n(G_n(g(t);y)-g(y)) = g(y)\lim_{n\to\infty}\left(\frac{n}{(2y+1)^{2n}}\right)+g'(y)\lim_{n\to\infty}(n\Omega_{1,n}(y))$$
$$+\frac{1}{2}g''(y)\lim_{n\to\infty}(n\Omega_{2,n}(y))+\lim_{n\to\infty}(nG_n((t-y)^2\zeta(t,y);y)).$$

By using Lemma 2.4, one has

$$\lim_{n\to\infty} n(G_n(g(t);y)-g(y)) = \frac{g'(y)+y(y+1)g''(y)}{4}+\lim_{n\to\infty}(nG_n((t-y)^2\zeta(t,y);y)).$$

Now, if it shows that

$$\lim_{n\to\infty}(nG_n((t-y)^2\zeta(t,y);y))=0,$$

then the proof is done.
Hence,

$$G_n(n(t-y)^2\zeta(t,y);y)=4n^2\sum_{w=0}^{\infty} q_{2w,2n}(y)\int_{\frac{w}{n}}^{\frac{2w+1}{2n}}(t-y)^2\zeta(t,y)dt$$
$$=4n^2\sum_{w=0}^{\infty} q_{2w,2n}^{\frac{1}{2}+\frac{1}{2}}(y)\int_{\frac{w}{n}}^{\frac{2w+1}{2n}}(t-y)^2\zeta(t,y)dt.$$

By Cauchy-Schwarz inequality [15], one has

$$G_n(n(t-y)^2\zeta(t,y);y)\le 4n^2\left(\sum_{w=0}^{\infty} q_{2w,2n}(y)\int_{\frac{w}{n}}^{\frac{2w+1}{2n}}(t-y)^4dt\right)^{\frac{1}{2}}\left(\sum_{w=0}^{\infty} q_{2w,2n}(y)\int_{\frac{w}{n}}^{\frac{2w+1}{2n}}\zeta^2(t,y)dt\right)^{\frac{1}{2}}.$$

Then,

$$\lim_{n\to\infty}(nG_n((t-y)^2\zeta(t,y);y)) \le \lim_{n\to\infty}\sqrt{n}(n\Omega_{4,n}(y))^{\frac{1}{2}}(G_n(\zeta^2(t,y);y))^{\frac{1}{2}}.$$

By using Korovkin's theorem [10] and Lemma 2.4, then,

$$G_n(\zeta^2(t,y);y) \to \zeta^2(y,y) = 0 \quad \text{as} \quad n\to\infty \quad \text{and} \quad \lim_{n\to\infty} n\Omega_{4,n}(y) = 0.$$

Then,

$$\lim_{n\to\infty}(nG_n((t-y)^2\zeta(t,y);y)) = 0.$$

Hence,

$$\lim_{n\to\infty} n(G_n(g(t);y)-g(y)) = \frac{g'(y)+y(y+1)g''(y)}{4}. \qquad \square$$

4. Numerical Example

A two-selected test functions approximation for the sequence $G_n(g;y)$. The results show that the approximation of half of the Baskakov-Kantorovich sequence yields numerical results superior to those of the classical sequence. For the modification sequence described here to outperform the classical one numerically. This property is explained using the test function graphs with approximations (the Baskakov-Kantorovich sequences, both classical and modified) and the tables of numerical values of the partition of the interval $[0,4]$ that are provided for both types of sequences.

Example 4.1. $g(y) = \cos^3\left((2y-1)\frac{\pi}{2}\right) + \sin(y)$, $y \in [0,4]$.

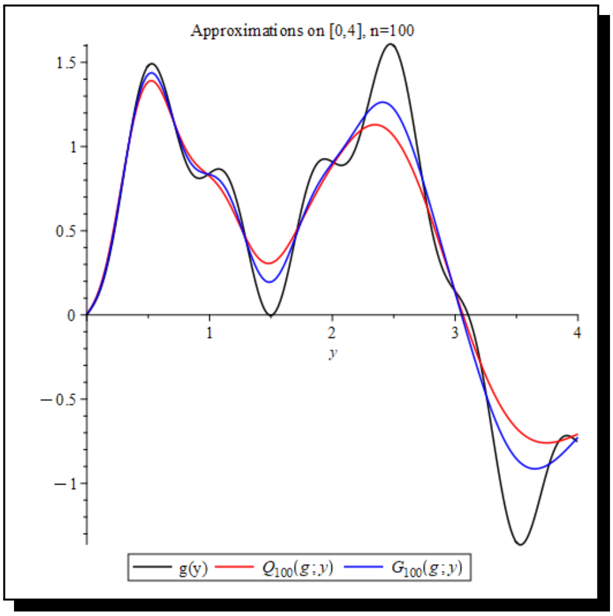

Figure 1. The numerical convergence of the sequence Q_n and G_n to the function $g(y)$ in Example 4.1, with $n = 100$

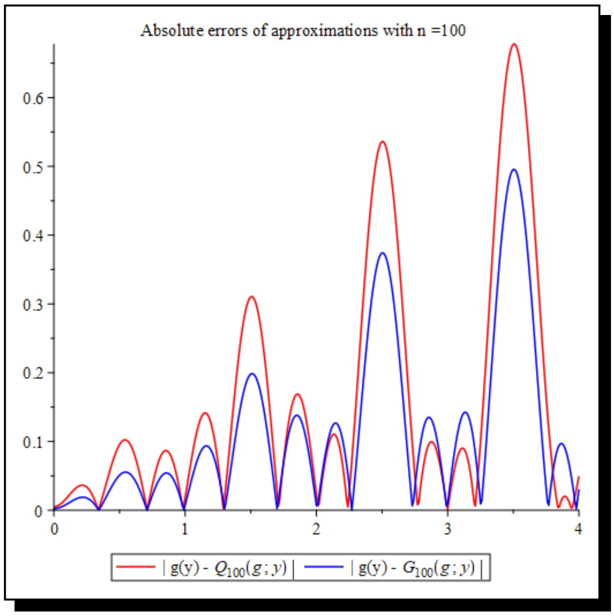

Figure 2. The absolute error for the sequence Q_n and G_n in Example 4.1, with $n = 100$ and $h = 0.01$

Table 1. The average absolute errors of the sequence Q_n and G_n for Example 4.1 with $h = 0.1$

n	$\frac{\sum_{l=1}^{z} \lvert Q_n(g;y_l)-g(y_l)\rvert}{z}$	$\frac{\sum_{l=1}^{z} \lvert G_n(g;y_l)-g(y_l)\rvert}{z}$
25	0.258185113	0.184856331
53	0.263758828	0.184357408
100	0.190060218	0.130292508

Table 2. Some numerical values of the sequence G_n and Q_n with $n = 100$ in Example 4.1

y_i	$g(y_i)$	$Q_n(g;y_i)$	$G_n(g;y_i)$
0	0	0	0
0.5	1.479425539	1.3839121520	1.4285566180
1	0.8414709848	0.8234390717	0.8339796036
1.5	−0.0025050134	0.3078044271	0.1955738612
2	0.9092974268	0.8872949643	0.9026941160
2.5	1.5984721440	1.0622868470	1.2242814240
3	0.1411200081	0.1395713431	0.1318274728
3.5	−1.3507832280	−0.6731662179	−0.8552615580
4	−0.7568024953	−0.7074687077	−0.7262173372

Example 4.2. $g(y) = \sin^3(4y)e^{-\frac{y}{2}} + \sin^3(2y)$, $y \in [0,4]$.

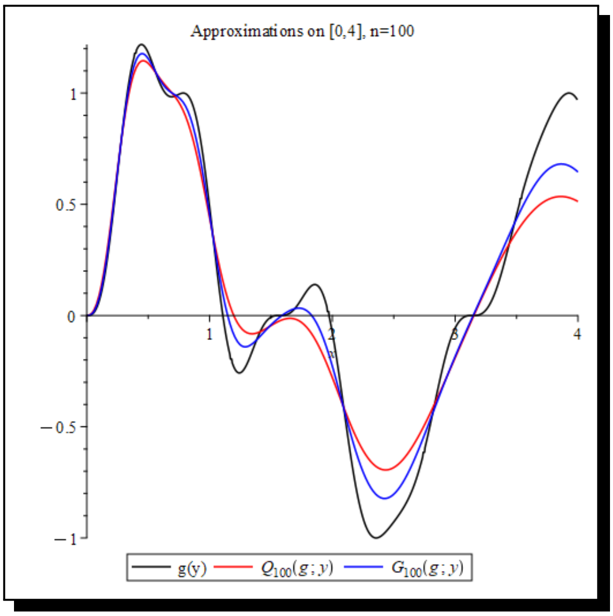

Figure 3. The numerical convergence of the sequence Q_n and G_n to the function $g(y)$ in Example 4.2, with $n = 100$

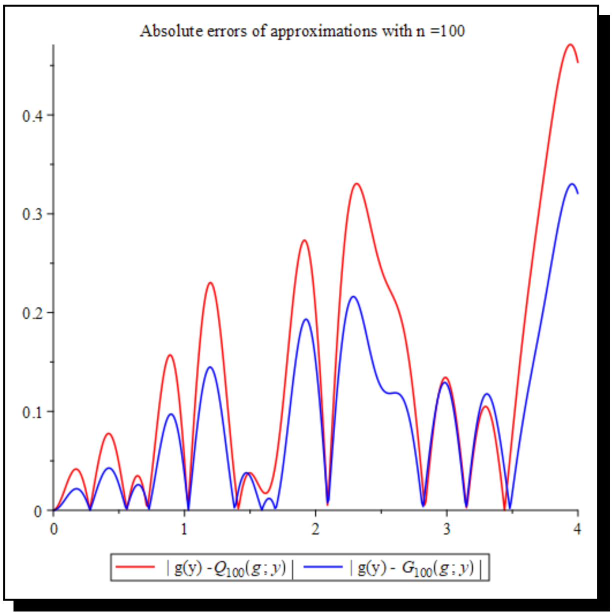

Figure 4. The absolute error for the sequence Q_n and G_n in Example 4.2, with $n = 100$ and $h = 0.01$

Table 3. The average absolute errors of the sequence Q_n and G_n for Example 4.2 with $h = 0.1$

n	$\frac{\sum_{l=1}^{z} \lvert Q_n(g;y_l)-g(y_l)\rvert}{z}$	$\frac{\sum_{l=1}^{z} \lvert G_n(g;y_l)-g(y_l)\rvert}{z}$
30	0.367909606	0.271367483
55	0.193534483	0.132923549
100	0.140426254	0.092166565

Table 4. Some numerical values of the sequence G_n and Q_n with $n = 100$ in Example 4.2

y_i	$g(y_i)$	$Q_n(g;y_i)$	$G_n(g;y_i)$
0	0	0	0
0.5	1.181346650	1.133859623	1.155161496
1	0.488920988	0.439056203	0.452959150
1.5	−0.007494202	−0.045384306	−0.042160552
2	−0.077199307	−0.279332813	−0.228082432
2.5	−0.927894709	−0.683565913	−0.803453225
3	−0.056285069	−0.190049704	−0.184140004
3.5	0.452499240	0.375653673	0.434646511
4	0.965183658	0.512607755	0.645168561

5. Conclusions

The modified sequence becomes faster and more accurate based on the numerical data. The sequence is, therefore, more efficient than the classical Baskakov-Kantorovich. We recommend using this sequence instead of the classical Baskakov-Kantorovich sequence in the related applications.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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