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Research Article

Stochastic Modelling by Combining a Random Contraction and a Random Dilation for Decision Making

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Abstract. Random contractions and dilations of positive variables are essential probabilistic tools in the discipline of stochastic modelling. The present paper establishes two stochastic models that are formulated via the combination of the random contraction of a random variable with the random dilation of random variable. The theoretical contribution is based on the computation of the corresponding characteristic function, while the practical contribution is attained through the application of the proposed stochastic models in decision making within the field of financial management and risk management.

Keywords. Stochastic model, Contraction, Dilation, Characteristic function, Decision making

Mathematics Subject Classification (2020). 60E10, 90B50, 91B06

1. Introduction

Random contractions and dilations are considered powerful analytical tools in the discipline of probability theory. They constitute types of stochastic models that have valuable applications in various disciplines, including statistics, engineering, economics and systemics (see, Akiba *et al.* [2], Artikis and Artikis [5, 6], Artikis [3], Beutner and Kamps [8], Fagnani and Zampieri [10], Gupta *et al.* [12]). In the context of stochastic modelling, the concepts of contraction and dilation describe a modification or operation performed on a random variable (see, Artikis [3], Hashorva *et al.* [14], Kawano and Hosoe [17], Kumar *et al.* [18], Letac [19]). In stochastic modelling, random contractions and dilations of random variables serve a significant role in investigating extreme variations of random variables and their effects (see, Belock and Dobric [7], Hasegawa *et al.* [13], Pal *et al.* [22]).

The main contribution of the paper involves the formulation of two stochastic models through the incorporation of a random contraction of a positive random variable and the random dilation of a positive random variable. The theoretical contribution is demonstrated by calculating the corresponding characteristic functions, while the practical applicability arises from the interpretation within the disciplines of investment decision making, liquidity management, and risk management.

The structure of the paper is outlined as follows. Section 2 presents relevant prior research in the area of stochastic modelling, concentrating on the utilization of random contractions and dilations of random variables. The structural elements and the formulation of the two stochastic models are presented in Section 3. The process for determining the corresponding characteristic function of the formulated stochastic models is presented in Section 4. Section 5 covers the applicability of the two stochastic models in investment decision-making, liquidity management, and risk management operations. Section 6 provides a simulation of the proposed stochastic models and a presentation of the simulation outcomes. Section 7 consists of final remarks and topics for further investigation.

2. Previous Research

A significant number of studies have been conducted in the discipline of stochastic modelling, focusing on random contractions, random dilations, and their application in numerous practical disciplines. This section provides research work that is significant to the present work.

Hashorva *et al.* [14] analyze the asymptotic behavior of random contractions and their practical use in insurance and finance. Kawano and Hosoe [17] conducted a study on contraction analysis of discrete time stochastic systems and established an innovative contraction framework to analyze the stability of discrete time nonlinear systems. A model predictive control approach for stochastic nonlinear discrete time systems that relies on contractions was introduced by Wang and Yan [25]. Artikis [3] focuses on the development, theoretical analysis, and practical application of a stochastic model that utilizes a random dilation for crisis management. Artikis and Artikis [5] contributed to the theoretical investigation of several stochastic models that incorporate random contractions and their practical interpretation in global risk management practices.

3. Principal Components and Formulation of the Stochastic Model

The current section presents the key components and the process by which the proposed stochastic models are formulated. The stochastic models are formulated by utilizing four independent, continuous, positive random variables.

We suppose that X is a continuous, positive random variable and U is a random variable with values in the interval $(0,1)$. If the random variables X and U are independent then the random variable

$$L = XU$$

is considered a contraction of the random variable X via the random variable U (Hashorva *et al.* [14]).

We also suppose that Y is a continuous, positive random variable and S is a random variable with values in the interval $(1,\infty)$. If the random variables Y and S are independent then the random variable

$$K = YS$$

is considered a dilation of the random variable Y via the random variable S (Artikis [3]).

Incorporating the aforementioned considerations, we formulate the stochastic model

$$H = \frac{L}{K}$$

and

$$J = L - K$$

or equivalently

$$H = \frac{XU}{YS}$$

and

$$J = XU - YS.$$

The paper focuses on analyzing the probabilistic factors and determining practical applications in investment decision making, cash flow and liquidity management, and risk management of the stochastic models H and J.

4. Calculating the Characteristic Function of the Stochastic Model

The characteristic function is a powerful mathematical tool in order to analyze and comprehend the probabilistic characteristics of random variables and their distributions, in probability theory. Considering the characteristic function of a random variable enables the estimation of its distribution, and vice versa. Characteristic functions are essential in stochastic modelling due to their crucial role in analyzing and validating models. Characteristic functions are a fundamental tool for comprehending the structural elements of stochastic models and are crucial for stochastic modelling methods (Artikis [4], Artikis and Artikis [6], Mun [20]). As indicated, the present section establishes the necessary conditions for evaluating the corresponding characteristic function of the formulated stochastic models.

Theorem. *Let X be a positive random variable with characteristic function $\varphi_X(u)$ and U a positive random variable taking values in the set $(0,1)$ with distribution function $F_U(v)$. We also suppose that Y is a positive random variable with distribution function $F_Y(y)$, and S a positive random variable taking values in the set $(1,\infty)$ with distribution function $F_S(s)$.*

We set the random variable

$$L = XU$$

and the random variable

$$K = YS.$$

If X, U, Y and S are independent then characteristic function of the stochastic models

$$H = \frac{L}{K}$$

and

$$J = L - K$$

or equivalently

$$H = \frac{XU}{YS} \quad \text{and} \quad J = XU - YS$$

are given by

$$\varphi_H(u) = \int_0^\infty \left[\int_0^1 \varphi_X\left(\frac{uv}{k}\right) dF_U(v)\right] d\left[\int_1^\infty F_Y\left(\frac{k}{s}\right) dF_S(s)\right]$$

and

$$\varphi_J(u) = \left(\int_0^1 \varphi_X(uv) dF_U(v)\right)\left(\int_1^\infty \varphi_Y(-us) dF_S(s)\right).$$

respectively.

Proof. The independence of X, U, Y and S implies the independence of X and U and the independence of Y and S. It easily follows that the characteristic functions of $L = XU$ and $K = YS$ are given by

$$\begin{aligned}
\varphi_L(u) &= E(e^{iuL}) \\
&= E(e^{iuXU}) \\
&= E(E(e^{iuXU} \mid U = v)) \\
&= \int_0^1 E(e^{iuXU} \mid U = v) dF_U(v) \\
&= \int_0^1 E(e^{iuvX} \mid U = v) dF_U(v) \\
&= \int_0^1 E(e^{iuvX}) dF_U(v) \\
&= \int_0^1 \varphi_X(uv) dF_U(v)
\end{aligned} \tag{4.1}$$

and

$$\begin{aligned}
\varphi_K(u) &= E(e^{iuK}) \\
&= E(e^{iuYS}) \\
&= E(E(e^{iuYS} \mid S = s)) \\
&= \int_1^\infty (e^{iuYS} \mid S = s) dF_S(s)
\end{aligned}$$

$$
\begin{aligned}
&= \int_1^{\infty} E(e^{iusY} \mid S = s) dF_S(s) \\
&= \int_1^{\infty} E(e^{iusY}) dF_S(s) \\
&= \int_1^{\infty} \varphi_Y(us) dF_S(s). \qquad (4.2)
\end{aligned}
$$

Moreover, the distribution function of $K = YS$ is given by

$$
\begin{aligned}
F_K(k) &= P(K \le k) \\
&= P(YS \le k) \\
&= \int_1^{\infty} P(YS \le k \mid S = s) dF_S(s) \\
&= \int_1^{\infty} P(Ys \le k \mid S = s) dF_S(s) \\
&= \int_1^{\infty} P\left(Y \le \frac{k}{s}\right) dF_S(s) \\
&= \int_1^{\infty} F_Y\left(\frac{k}{S}\right) dF_S(s). \qquad (4.3)
\end{aligned}
$$

The independence of X, U, Y and S implies the independence of K and L. If $\varphi_H(u)$ is the characteristic function of the random variable H then it can be written in the form

$$
\begin{aligned}
\varphi_H(u) &= E(e^{iuH}) \\
&= E(e^{iu\frac{L}{K}}) \\
&= E(e^{iu\frac{L}{K}}) \\
&= E[E(e^{iu\frac{L}{K}} \mid K = k)] \\
&= \int_1^{\infty} E(e^{iu\frac{L}{K}} \mid K = k) dF_k(k) \\
&= \int_1^{\infty} E(e^{iu\frac{L}{k}} \mid K = k) dF_k(k) \\
&= \int_1^{\infty} E(e^{i\frac{u}{k}L}) dF_k(k) \\
&= \int_1^{\infty} \varphi_L\left(\frac{u}{k}\right) dF_K(k), \qquad (4.4)
\end{aligned}
$$

and if $\varphi_J(u)$ is the characteristic function of the random variable J then it can be written in the form

$$
\begin{aligned}
\varphi_J(u) &= E(e^{iuJ}) \\
&= E(e^{iu(L-K)}) \\
&= E(e^{iuL - iuK}) \\
&= E(e^{iuL} e^{iuK}) \\
&= E(e^{iuL}) E(e^{-iuK}) \\
&= \varphi_L(u) \varphi_K(-u). \qquad (4.5)
\end{aligned}
$$

From (4.1), (4.3) and (4.4) we get

$$\varphi_H(u) = \int_1^\infty \left[\int_0^1 \varphi_X\left(\frac{uv}{k}\right) dF_U(v) \right] d\left[\int_1^\infty F_Y\left(\frac{k}{s}\right) dF_S(s) \right]$$

and from (4.1), (4.2) and (4.5) we get

$$\varphi_J(u) = \left(\int_0^1 \varphi_X(uv) dF_U(v) \right) \left(\int_1^\infty \varphi_Y(-us) dF_S(s) \right). \qquad \square$$

5. Practical Implementation in Decision Making and Management

Stochastic models that incorporate random contractions and dilations of random variables enable the modification of random variables in order to consider their extreme variability and the effects on complex systems. As a result, the utilization of random contractions and dilations of random variables in stochastic modelling offers a method to examine and comprehend extreme value events in various practical disciplines ([3,5]). This section provides practical applications that illustrate how the formulated stochastic models can be utilized in business decision making and financial management, such as investment evaluation, liquidity management and risk management.

Stochastic models can be employed in order to evaluate the outcomes or profitability of an investment. As a result, stochastic modelling can be used to quantify the return of an investment regarding to its cost (see, Abel [1], de Freitas *et al.* [11]). We suppose that X represents the revenue generated by an investment. We also suppose that Y represents the cost associated to implement the investment. Therefore, $L = XU$ and $K = YS$ represent the random contraction via the random variable U and the random dilation via the random variable S of the income and the cost generated by the investment, respectively. As a result, the stochastic models H and J serve as significant indicators of investment assessment, by providing crucial insights about the performance of the investment under conditions of high uncertainty and fluctuation in terms of revenue and costs.

Effective liquidity management is crucial for the sustainability of an organization. Organizations can improve their ability to withstand challenges, adapt to changes, and maintain their viability by successfully managing liquidity. Stochastic models are widely acknowledged to offer organizations beneficial techniques for analyzing, predicting, and efficiently managing liquidity. By integrating stochasticity and ambiguity into financial decision making processes, these models provide organizations optimal liquidity management procedures, enhancing adaptability to financial turbulence, and promoting business viability and profitability (see, He and Lin [15], Routledge and Zin [23]). We suppose that X represents the cash reserves that an organization maintains for use in unforeseen circumstances, primarily to fulfil unexpected costs or expenses. We also suppose that Y represents unexpected costs or expenses that the organization may be obligated to fulfill. Therefore $L = XU$ and $K = YS$ represent the random contraction via the random variable U and random dilation via the random variable S of the cash reserves retained by the organization and the unexpected costs obliged to cover, respectively. As a result, the stochastic models H and J serve as significant liquidity indicators, providing crucial information concerning the ability of the organization to maintain sufficient funds for situations of significant expenditures and cost demands.

Stochastic modelling is essential in risk management. Stochastic models offer an effective framework for comprehending, measuring, and controlling risks in various situations, supporting organizations in making accurate decisions when confronted with uncertainty ([3,5,21]). We suppose that X represents the capital reserve held by an organization in order to protect against losses that may occur after a risk occurrence. We also suppose that Y represents unexpected losses occurring after the risk occurrence. Therefore, $L = XU$ and $K = YS$ represent the random contraction via the random variable U and random dilation via the random variable S of the capital reserves held by the organization and the unexpected losses after the risk occurrence, respectively. As a result, the stochastic models H and J provide vital information regarding the financial requirements of the organization to withstand significant losses in the event of a risk occurrence that threatens its long-term sustainability.

6. Implementation and Interpretation of the Simulation

The following section presents the simulation results of the formulated stochastic models. Simulation of stochastic models provides an opportunity to assess various techniques or strategies across different scenarios. The simulation was executed through the MATLAB programming and computational environment, version 8.5.0.197613 [9,16,24]. The process involved an overall number of 500 iterations. Furthermore, the simulation of the stochastic model was executed utilizing the below assumptions.

We assume that X follows the normal distribution with parameters $\mu = 100.000$ and $\sigma = 5.000$ and U follows the continuous uniform distribution with parameters $a = 0$ and $b = 1$. We also assume that Y follows the normal distribution with parameters $\mu = 60.000$ and $\sigma = 10.000$ and S follows the continuous uniform distribution with parameters $\gamma = 1$ and $\delta = 2$. After performing 500 iterations, the frequency table and descriptive statistics for the random variables X, U, Y, S, L, K, H and J are displayed in Tables 1 to 8, respectively.

Table 1. Frequency table and descriptive statistic of X

BinStart	BinEnd	Frequency
80.000	900.000	9
90.000	1,00e+05	232
1,00e+05	1,10e+05	249
1,10e+05	1,20e+05	10
Mean: 100207,154		
Median: 100180,5		
Standard Deviation: 4839,1195		
Variance: 23417077,7818		
Minimum: 86144		
Maximum: 116331		
Range: 30187		
Interquartile Range (IQR): 6445		

Table 2. Frequency table and descriptive statistic of U

BinStart	BinEnd	Frequency
0	0,2	101
0,2	0,4	94
0,4	0,6	90
0,6	0,8	102
0,8	1	113
Mean: 0,5142		
Median: 0,52009		
Standard Deviation: 0,2938		
Variance: 0,086319		
Minimum: 0,0014632		
Maximum: 0,99977		
Range: 0,99831		

Table 3. Frequency table and descriptive statistics of Y

BinStart	BinEnd	Frequency
20.000	30.000	1
30.000	40.000	10
40.000	50.000	75
50.000	60.000	178
60.000	70.000	160
70.000	80.000	65
80.000	90.000	10
90.000	1,00e+05	1
Mean: 59143,1714		
Median: 58966,6457		
Standard Deviation: 10112,1055		
Variance: 102254677,4033		
Minimum: 22777,8875		
Maximum: 93255,3583		
Range: 70477,4707		
Interquartile Range (IQR): 13114,5369		

Table 4. Frequency table and descriptive statistics of S

BinStart	BinEnd	Frequency
1	1,2	105
1,2	1,4	96
1,4	1,6	91
1,6	1,8	108
1,8	2	100
Mean: 1,5009		
Median: 1,5103		
Standard Deviation: 0,29057		
Variance: 0,084433		
Minimum: 1,0003		
Maximum: 1,9969		
Range: 0,99655		
Interquartile Range (IQR): 0,50541		

Table 5. Frequency table and descriptive statistics of L

BinStart	BinEnd	Frequency
0	20.000	102
20.000	40.000	87
40.000	60.000	100
60.000	80.000	100
80.000	1,00e+05	101
1,00e+05	1,20e+05	10
Mean: 51535,674		
Median: 51797		
Standard Deviation: 29489,4607		
Variance: 869628293,6791		
Minimum: 141		
Maximum: 108134		
Range: 107993		
Interquartile Range (IQR): 53705,5		

Table 6. Frequency table and descriptive statistics of K

BinStart	BinEnd	Frequency
35.000	50.000	8
50.000	65.000	76
65.000	80.000	122
80.000	95.000	105
95.000	1,10e+05	93
1,10e+05	1,25e+05	57
1,25e+05	1,40e+05	20
1,40e+05	1,55e+05	17
1,55e+05	1,70e+05	2
Mean: 88858,478		
Median: 85600		
Standard Deviation: 23959,6625		
Variance: 574065428,4143		
Minimum: 37615		
Maximum: 166389		
Range: 128774		
Interquartile Range (IQR): 33160		

Table 7. Frequency table and descriptive statistics of H

BinStart	BinEnd	Frequency
0	0,5	209
0,5	1	206
1	1,5	73
1,5	2	11
2	2,5	0
2,5	3	1
Mean: 0,62206		
Median: 0,5955		
Standard Deviation: 0,4076		
Variance: 016613		
Minimum: 0,001		
Maximum: 2,607		
Range: 2,606		
Interquartile Range (IQR): 0,6195		

Table 8. Frequency table and descriptive statistics of J

BinStart	BinEnd	Frequency
−140.000	−119.800	9
−119.800	−99.600	18
−99.600	−79.400	42
−79.400	−59.200	73
−59.200	−39.000	87
−39.000	−18.800	111
−18.800	1.400	71
1.400	21.600	64
21.600	41.800	21
41.800	62.000	4
Mean: −37322,8233		
Median: −36047,405		
Standard Deviation: 37563,0874		
Variance: 1410985538,5925		
Minimum: −140957		
Maximum: 60444.85		
Range: 201401,85		
Interquartile Range (IQR): 54223595		

The corresponding cumulative distribution functions of the stochastic models H and J are displayed in Figure 1 and Figure 2, respectively.

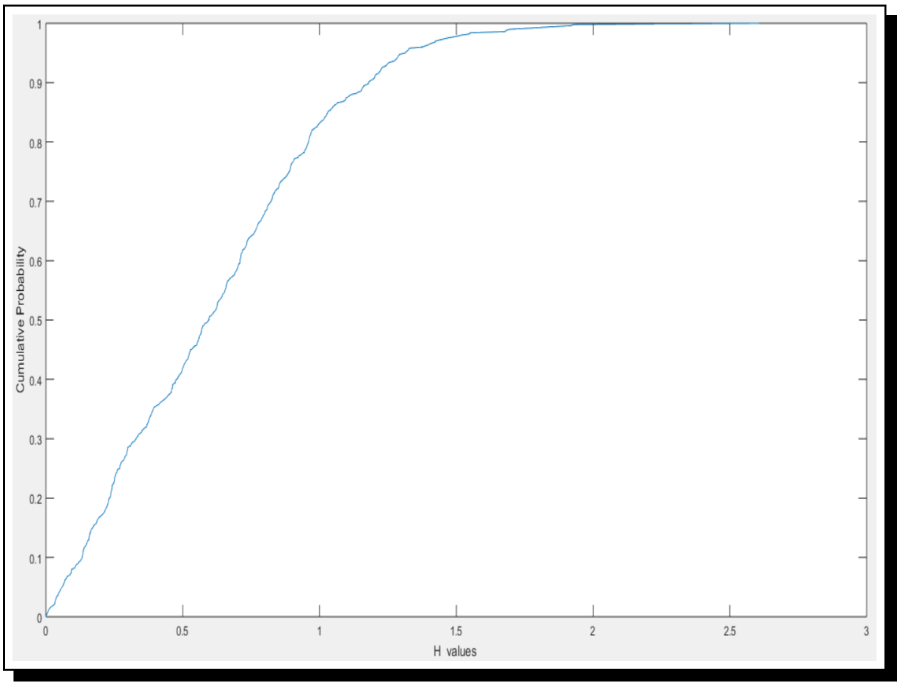

Figure 1. Cumulative distribution function of H

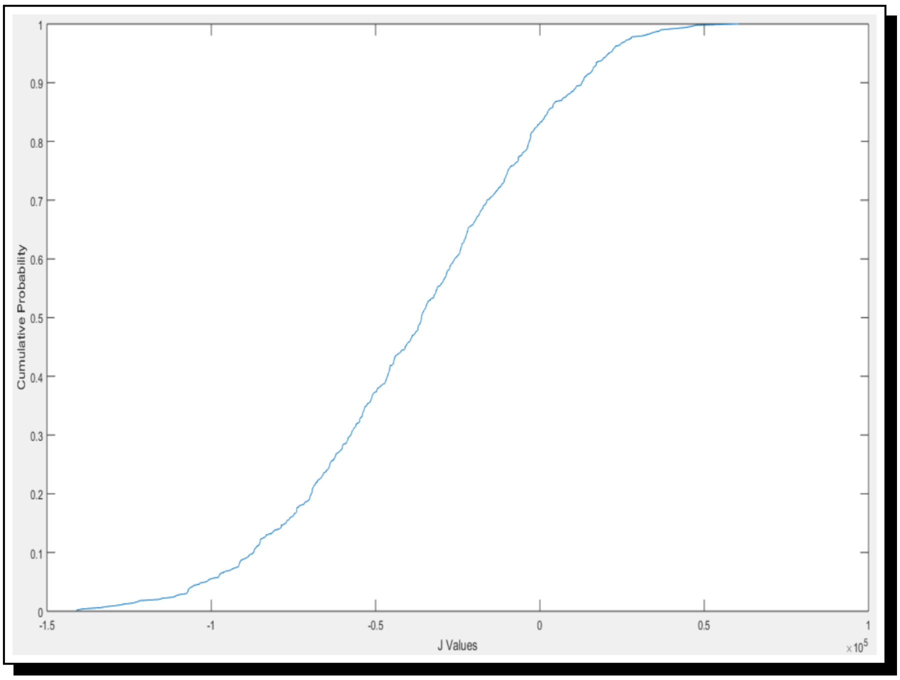

Figure 2. Cumulative distribution function of J

As it follows from the cdf shown in Figure 1 and Figure 2 there is a probability of approximately eighty five percent that the random variable $H = \frac{L}{K}$ is less than or equal to 1, and the random variable $J = L - K$ is less than or equal to 0, i.e., $P(H \leq 1) = P(J \leq 0) = 0,85$. The interpretation based on the aforementioned applications of the proposed stochastic models is the following, there is a probability of approximately eighty five percent that the cost in order to implement the investment will be greater than or equal to the revenue produced by the investment, the unexpected expenses will be greater than or equal to the available cash reserves, and the losses after the manifestation of the risk will be greater than or equal to the available capital reserve.

7. Conclusion

Stochastic models incorporating random contractions and dilations, constitute an essential component in examining and analyzing extreme uncertainty in intricate systems and processes. The present paper introduces two stochastic models that integrate both a random contraction of a random variable and a random dilation of a random variable. The incorporation of these two probabilistic transformations of random variables in stochastic modelling offers a thorough examination of the effects under conditions of extreme variability of the structural random variables and allows for comparative analysis of the outcomes under normal conditions.

The computation of the corresponding characteristic function establishes the theoretical significance of the formulated stochastic models and their interpretation in decision making and business operations management provides the practical implementation and utilization. Furthermore, it would be an interesting scientific endeavor to develop stochastic models with more intricate structural components, enabling the investigation and assessment of complex processes under critical fluctuation and extreme circumstances.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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