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Research Article

Common Fixed Point Results for Three Pairs of Self-Maps in Generalized E-Fuzzy b-Metric Space

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Abstract. In this article, we introduce a new notion of generalized E-fuzzy b-metric space and proceed to establish common fixed point theorems for three pairs of self-mappings in this space. These theorems extend and improve specific existing fixed point theorems found in the literature.

Keywords. Fixed point, Fuzzy metric space, E-fuzzy b-metric space

Mathematics Subject Classification (2020). 47H10, 54H25

1. Introduction and Preliminaries

In 1906, Fréchet [5] firstly introduced metric space. A set and its corresponding metric consisting the notion of metric space. A function that defines the idea of distance between any two members is a metric. Theory of metric space was significantly further given by German mathematician Hausdroff, who introduced the term metric space. In 1922, Banach [2] gave a contraction principle to obtain a fixed point theorem in complete metric space as Banach contraction principle. This principal is crucial to enables the investigation of the existence and uniqueness of solutions to various problems in pure and applied mathematics, which results in the building of mathematical models using systems of nonlinear differential, functional and integral equations. In 1965, the foundation of fuzzy mathematics was laid by Zadeh [18]. The most fascinating and active field of nonlinear analysis study and development is thought to be fixed point theory. In 1994, George and Veeramani [6] modified the concept of fuzzy metric space which was introduced by Kramosil and Michalek [10]. This can be regarded as a generalization of statistical metric space. There are still a lot of metric extensions and different generalized fuzzy metric space concepts in addition to fuzzy metric space. Many researchers have proved various fixed point theorems in these spaces, see e.g., Dhage [4], Hooda *et al*. [7], Jose [8], Mustafa and Sims [12], Sedghi and Shobe [14], and Sukanya and Jose [16]. In 1989, Bakhtin [1] introduced a space where a weaker condition was observed in place of triangle inequality and called as b-metric space.

Further in 1993, Czerwik [3] proved some results for contraction mappings in b-metric space. In 2006, Mustafa and Sims [12] furnish a way to interrupt distance using three points rather than of two points. They term their new approach as G-metric space and establish some fixed point results for various mappings. In 2006, Jungck and Rhoades [9] defined weakly compatible mappings, which are more general than commuting and weakly commuting mappings. In 2012, Sedghi and Shobe [14] explored the relation between b-metric and fuzzy metric spaces and leading to the introduction of fuzzy b-metric space. In 2018, Sukanya and Jose [16], innovated a concept of E-fuzzy metric space by expanding on the idea of fuzzy metric space and validating fixed point outcomes, those results enhance and generalized existing results by Manro [11].

Now, in this article we introduce new concept of generalized E-fuzzy b-metric space and proceeds to establish common fixed point theorems for three pairs of self-mappings under weakly compatible condition. These theorems extend and enhance certain existing fixed point results.

In 1965, Zadeh [18] introduced the fuzzy set in the following way:

Definition 1.1 ([18])**.** Let $\check{\mathfrak{A}}$ be an arbitrary set. A fuzzy set $\widetilde{A}$ on $\check{\mathfrak{A}}$ is defined by following object $\widetilde{A} = \{(\varpi, \beth_{\widetilde{A}}(\varpi)) \mid \varpi \in \check{\mathfrak{A}}\}$. Here, $\beth_{\widetilde{A}}(\varpi)$ is membership function as generalization of characteristic function that implies following:

$\beth_{\widetilde{A}}(\varpi) = 1 \Leftrightarrow \varpi \in \widetilde{A}$,

$\beth_{\widetilde{A}}(\varpi) = 0 \Leftrightarrow \varpi \notin \widetilde{A}$,

$0 < \beth_{\widetilde{A}}(\varpi) < 1 \Leftrightarrow \varpi$ partially belongs to $\widetilde{A}$.

A subset $\widetilde{A}$ of universe $\check{\mathfrak{A}}$ with the membership function $\beth_{\widetilde{A}}(\varpi)$ which may take any value in the interval $[0,1]$ is called fuzzy set.

Definition 1.2 ([13])**.** A t-norm is a binary operation $\widehat{\mathfrak{S}} : [0,1] \times [0,1] \to [0,1]$ that satisfies following conditions for all $\eth_1, \eth_2, \eth_3 \in [0,1]$:

(i) $\widehat{\mathfrak{S}}$ is continuous,

(ii) $\widehat{\mathfrak{S}}$ is commutative, i.e., $\widehat{\mathfrak{S}}(\eth_1, \eth_2) = \widehat{\mathfrak{S}}(\eth_2, \eth_1)$,

(iii) $\widehat{\mathfrak{S}}$ is associative, i.e., $\widehat{\mathfrak{S}}(\eth_1, \widehat{\mathfrak{S}}(\eth_2, \eth_3)) = \widehat{\mathfrak{S}}(\widehat{\mathfrak{S}}(\eth_1, \eth_2), \eth_3)$,

(iv) $\widehat{\mathfrak{S}}(\eth_1, 1) = \eth_1$.

A triangular norm is a type of binary operation is used in the metric space to generalize triangular inequality.

Definition 1.3 ([13])**.** A t-conorm is a binary operation $\widehat{\mathfrak{S}}:[0,1]\times[0,1]\to[0,1]$ which satisfies the following conditions: for all $\eth_1,\eth_2,\eth_3,\eth_4\in[0,1]$:

(i) $\widehat{\mathfrak{S}}$is continuous,

(ii) $\widehat{\mathfrak{S}}$ is commutative and associative,

(iii) $\widehat{\mathfrak{S}}(\eth_1,1)=\eth_1$,

(iv) $\widehat{\mathfrak{S}}$ is monotonic, i.e., $\widehat{\mathfrak{S}}(\eth_1,\eth_2)\le\widehat{\mathfrak{S}}(\eth_3,\eth_4)$ whenever $\eth_1\le\eth_2$ and $\eth_3\le\eth_4$.

Example 1.4 ([13])**.** Following binary operations are examples of t-conorms:

(i) $\widehat{\mathfrak{S}}(\eth_1,\eth_2)=\max\{\eth_1,\eth_2\}$,

(ii) $\widehat{\mathfrak{S}}(\eth_1,\eth_2)=\eth_1+\eth_2-\eth_1\cdot\eth_2$,

(iii) $\widehat{\mathfrak{S}}(\eth_1,\eth_2)=\min\{\eth_1+\eth_2+1\}$.

In 1989, Bakhtin [1] introduced b-metric spaces where instead of triangle inequality a weak condition was observed by multiplying a constant $b\ge1$ in metric space defined as follows:

Definition 1.5 ([1])**.** Let $\check{\mathfrak{A}}$ ($\neq\{\emptyset\}$) be set and $b\ge1$ be a real number.The ordered pair $(\check{\mathfrak{A}},\widetilde{\wp})$ is said to be b-metric space if for a function $\widetilde{\wp}:\check{\mathfrak{A}}\times\check{\mathfrak{A}}\to[0,\infty)$ satisfies the following axioms for each $\varpi,w,\xi\in\check{\mathfrak{A}}$:

($_b$M1) $\widetilde{\wp}(\varpi,w)=0$ iff $\varpi=w$,

($_b$M2) $\widetilde{\wp}(\varpi,w)=\widetilde{\wp}(w,\varpi)$,

($_b$M3) $\widetilde{\wp}(\varpi,w)\le b[\widetilde{\wp}(\varpi,\xi)+\widetilde{\wp}(\xi,w)]$.

In 1975, Kramosil and Michalek [10] firstly introduced the concept of fuzzy metric space. Later, in 1994, George and Veeramani [6] reconstructed the concept of FM space in the following way:

Definition 1.6 ([6])**.** The 3-tuple $(\check{\mathfrak{A}},\acute{\mathbb{M}},\widehat{\mathfrak{S}})$ is known as fuzzy metric space (shortly, FM-space) if $\check{\mathfrak{A}}$ is an arbitrary set, $\widehat{\mathfrak{S}}$ is a continuous t-norm and $\acute{\mathbb{M}}$ is a fuzzy set in $\check{\mathfrak{A}}\times\check{\mathfrak{A}}\times[0,\infty)$ satisfying the following conditions for every $\varpi,w,\xi\in\check{\mathfrak{A}}$ and $s,t>0$:

(FM$_1$) $\acute{\mathbb{M}}(\varpi,w,t)>0$,

(FM$_2$) $\acute{\mathbb{M}}(\varpi,w,t)=1$ iff $\varpi=w$,

(FM$_3$) $\acute{\mathbb{M}}(\varpi,w,t)=\acute{\mathbb{M}}(w,\varpi,t)$,

(FM$_4$) $\widehat{\mathfrak{S}}(\acute{\mathbb{M}}(\varpi,w,t),\acute{\mathbb{M}}(w,\xi,s))\le\acute{\mathbb{M}}(\varpi,\xi,t+s)$,

(FM$_5$) $\acute{\mathbb{M}}(\varpi,w,\bullet):[0,\infty)\to[0,1]$ is continuous.

$\acute{\mathbb{M}}(\varpi,w,t)$ denotes the degree of nearness between ϖ and w with respect to t.

Example 1.7 ([6])**.** Let $\check{\mathfrak{A}}=\mathbb{R}$ and $\acute{\mathbb{M}}=e^{\left(\frac{|\varpi-w|}{t}\right)^{-1}}$, for all $\varpi,w\in\check{\mathfrak{A}}$, $t\in(0,\infty)$ and $\eth_1,\eth_2\in[0,1]$:

(i) $\widehat{\mathfrak{S}}(\eth_1,\eth_2)=\eth_1\eth_2$,

(ii) $\widehat{\mathfrak{S}}(\eth_1,\eth_2)=\min\{\eth_1,\eth_2\}$,

then $\acute{\mathbb{M}}$ is a fuzzy metric on $\check{\mathfrak{A}}$ with a continuous t-norm $\widehat{\mathfrak{S}}$.

In 2006, Mustafa and Sims [12] introduced the notion of G-metric space in the following way:

Definition 1.8 ([12]). The pair $(\check{\mathfrak{A}},\overline{\mathbb{G}})$ is called a G-metric space or more specifically $\overline{\mathbb{G}}$ is a G-metric on $\check{\mathfrak{A}}$ if $\check{\mathfrak{A}}$ is a non-empty set and defined $\overline{\mathbb{G}}:\check{\mathfrak{A}}\times\check{\mathfrak{A}}\times\check{\mathfrak{A}}\to[0,\infty)$ such that following conditions satisfies for every $\varpi,w,\xi,\imath\in\check{\mathfrak{A}}$:

(G_M1) $\overline{\mathbb{G}}(\varpi,w,\xi)=0$ if and only if $\varpi=w=\xi$,

(G_M2) $\overline{\mathbb{G}}(\varpi,\varpi,w)>0$ with $\varpi\neq w$,

(G_M3) $\overline{\mathbb{G}}(\varpi,\varpi,w)\le\overline{\mathbb{G}}(\varpi,w,\xi)$, for all $\varpi,w,\xi\in\check{\mathfrak{A}}$ with $w\neq\xi$,

(G_M4) $\overline{\mathbb{G}}(\varpi,w,\xi)=\overline{\mathbb{G}}(\varpi,\xi,w)=\overline{\mathbb{G}}(w,\varpi,\xi)=\overline{\mathbb{G}}(w,\xi,\varpi)=\overline{\mathbb{G}}(\xi,\varpi,w)=\overline{\mathbb{G}}(\xi,w,\varpi)$,

(G_M5) $\overline{\mathbb{G}}(\varpi,w,\xi)\le\overline{\mathbb{G}}(\varpi,\imath,\imath)+\overline{\mathbb{G}}(\imath,w,\xi)$.

In 2012, Sedghi and Shobe [14] introduced the concept of fuzzy b-metric space as follows:

Definition 1.9 ([14]). The 3-tuple $(\check{\mathfrak{A}},\acute{\mathbb{M}},\widehat{\mathfrak{S}})$ is known as fuzzy b-metric space if $\check{\mathfrak{A}}$ is an arbitrary set, $\widehat{\mathfrak{S}}$ is a continuous t-norm and $\acute{\mathbb{M}}$ is a fuzzy set in $\check{\mathfrak{A}}\times\check{\mathfrak{A}}\times(0,\infty)$ satisfies the following conditions for all $\varpi,w,\xi\in\check{\mathfrak{A}}$, $s,t>0$ and a real number $b\ge1$:

(F_bM-1) $\acute{\mathbb{M}}(\varpi,w,t)>0$,

(F_bM-2) $\acute{\mathbb{M}}(\varpi,w,t)=1$ if and only if $\varpi=w$,

(F_bM-3) $\acute{\mathbb{M}}(\varpi,w,t)=\acute{\mathbb{M}}(w,\varpi,t)$,

(F_bM-4) $\widehat{\mathfrak{S}}(\acute{\mathbb{M}}(\varpi,w,\frac{t}{b}),\acute{\mathbb{M}}(w,\xi,\frac{s}{b}))\le\acute{\mathbb{M}}(\varpi,\xi,t+s)$,

(F_bM-5) $\acute{\mathbb{M}}(\varpi,w,\bullet):(0,\infty)\to[0,1]$ is continuous.

In 2018, Sukanya and Jose [17] introduced the notion of E-fuzzy metric space by using G-metric space (Mustafa and Sims [12]) and generalized fuzzy metric space in the following way:

Definition 1.10 ([17]). The 3-tuple $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ is known as E-fuzzy metric space if $\check{\mathfrak{A}}$ is an arbitrary set, $\widehat{\mathfrak{S}}$ is a continuous t-norm and $\acute{\mathcal{E}}$ is a fuzzy set in $\check{\mathfrak{A}}\times\check{\mathfrak{A}}\times\check{\mathfrak{A}}\times(0,\infty)$ satisfying the following conditions, for all $\varpi,w,\xi,\imath\in\check{\mathfrak{A}}$ and $s,t>0$:

($E_{FM}1$) $\acute{\mathcal{E}}(\varpi,w,\xi,)>0$,

($E_{FM}2$) $\acute{\mathcal{E}}(\varpi,\varpi,w,t)\ge\acute{\mathcal{E}}(\varpi,w,\xi,t)$ with $w\neq\xi$,

($E_{FM}3$) $\acute{\mathcal{E}}(\varpi,w,\xi,t)1$ if and only if $\varpi=w=\xi$,

($E_{FM}4$) $\acute{\mathcal{E}}(\varpi,w,\xi,t)=\acute{\mathcal{E}}(p(\varpi,w,\xi),t)$ (symmetry), where p is a permutation function,

($E_{FM}5$) $\widehat{\mathfrak{S}}(\acute{\mathcal{E}}(\varpi,\imath,\xi,t),\acute{\mathcal{E}}(\imath,w,\xi,s))\le\acute{\mathcal{E}}(\varpi,w,\xi,t+s)$,

($E_{FM}6$) $\acute{\mathcal{E}}(\varpi,w,\xi,\bullet):(0,\infty)\to[0,1]$ is continuous.

E-fuzzy metric space $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ is a generalization of fuzzy metric space.

Example 1.11 ([16]). Let $\check{\mathfrak{A}}=\mathbb{R}$ and $\overline{\mathbb{G}}$ is a G-metric on $\check{\mathfrak{A}}$. Consider $\acute{\mathcal{E}}(\varpi,w,\xi,t)=\left[\exp\left(\frac{\overline{\mathbb{G}}(\varpi,w,\xi)}{t}\right)\right]^{-1}$ satisfies for all $\varpi,w,\xi\in\check{\mathfrak{A}}$, $t\in(0,\infty)$ and t-norm $\widehat{\mathfrak{S}}(\eth_1,\eth_2)=\eth_1\eth_2$, for all $\eth_1,\eth_2\in[0,1]$. Then, $\acute{\mathcal{E}}$ is a E-fuzzy metric space on $\check{\mathfrak{A}}$ with a continuous t-norm $\widehat{\mathfrak{S}}$.

Definition 1.12 ([15])**.** Let $(\breve{\mathfrak{A}}, \acute{\mathcal{E}}, \widehat{\mathfrak{S}})$ be a E-fuzzy metric space, for some $\varpi \in \breve{\mathfrak{A}}$ and $\{\mathfrak{p}_n\}$ be a sequence in $\breve{\mathfrak{A}}$. Then

(i) A sequence $\{\mathfrak{p}_n\}$ is said to converge to ϖ if and only if $\acute{\mathbb{M}}(\mathfrak{p}_n, \varpi, \varpi, t) \to 1$ as $n \to \infty$.

(ii) A sequence $\{\mathfrak{p}_n\}$ is said to be a Cauchy sequence if for any $r \in (0,1)$ and $t > 0$ there exists a natural number n_0 s.t. $\acute{\mathcal{E}}(\mathfrak{p}_m, \mathfrak{p}_n, \mathfrak{p}_l, t) > 1 - r$, for every $m, n, l \geq n_0$.

(iii) A E-fuzzy metric space $(\breve{\mathfrak{A}}, \acute{\mathcal{E}}, \widehat{\mathfrak{S}})$ in which every Cauchy sequence is convergent is said to be complete.

(iv) A E-fuzzy metric space $(\breve{\mathfrak{A}}, \acute{\mathcal{E}}, \widehat{\mathfrak{S}})$ in which every sequence has a convergent subsequence is said to be compact.

Definition 1.13 ([9])**.** Let $\breve{\mathfrak{A}}$ be a set and $\widetilde{\wp}$ and $\check{\mathfrak{h}}$ be self mappings on $\breve{\mathfrak{A}}$. A point ϖ in $\breve{\mathfrak{A}}$ is called a coincidence point of $\widetilde{\wp}$ and $\check{\mathfrak{h}}$ if and only if $\widetilde{\wp}\varpi = \check{\mathfrak{h}}\varpi$. In this case, $\acute{\mathfrak{p}}$, i.e., $\widetilde{\wp}\varpi = \check{\mathfrak{h}}\varpi = \acute{\mathfrak{p}}$ is called point of coincidence of $\widetilde{\wp}$ and $\check{\mathfrak{h}}$.

Definition 1.14 ([9])**.** Let $\breve{\mathfrak{A}}$ be a set. A pair of self-mappings $(\widetilde{\wp}, \check{\mathfrak{h}})$ on $\breve{\mathfrak{A}}$ is said to be weakly compatible if they commute at the coincidence points, i.e., if $\widetilde{\wp}\varpi = \check{\mathfrak{h}}\varpi$, for some $\varpi \in \breve{\mathfrak{A}}$ then $\widetilde{\wp}\check{\mathfrak{h}}\varpi = \check{\mathfrak{h}}\widetilde{\wp}\varpi$.

2. Main Results

We firstly introduce a new notion of generalized E-fuzzy b-metric space in this section. Also, define convergent sequence, Cauchy sequence, completeness and compactness in generalized E-fuzzy b-metric space and then establish some common fixed point theorems related to this generalized space under weakly compatible mappings.

Definition 2.1. The 3-tuple $(\breve{\mathfrak{A}}, \acute{\mathcal{E}}, \widehat{\mathfrak{S}})$ is defined as E-fuzzy b-metric space or generalized E-fuzzy b-metric space if $\breve{\mathfrak{A}}$ is an arbitrary set, $\widehat{\mathfrak{S}}$ is a continuous t-norm and $\acute{\mathcal{E}}$ is a fuzzy set in $\breve{\mathfrak{A}}^3 \times (0,\infty)$ satisfies the following, axioms for all $\varpi, w, \xi, \imath \in \breve{\mathfrak{A}}$, $s, t > 0$ and real number $b \geq 1$:

($E_{FbM}1$) $\acute{\mathcal{E}}\left(\varpi, w, \xi, \frac{t}{b}\right) > 0$,

($E_{FbM}2$) $\acute{\mathcal{E}}\left(\varpi, \varpi, w, \frac{t}{b}\right) \geq \acute{\mathcal{E}}\left(\varpi, w, \xi, \frac{t}{b}\right)$ with $w \neq \xi$,

($E_{FbM}3$) $\acute{\mathcal{E}}\left(\varpi, w, \xi, \frac{t}{b}\right) = 1 \Leftrightarrow \varpi = w = \xi$, for all $t > 0$,

($E_{FbM}4$) $\acute{\mathcal{E}}\left(\varpi, w, \xi, \frac{t}{b}\right) = \acute{\mathcal{E}}\left(p(\varpi, w, \xi), \frac{t}{b}\right)$ (symmetry), where p is a permutation function,

($E_{FbM}5$) $\widehat{\mathfrak{S}}\left(\acute{\mathcal{E}}\left(\varpi, \imath, \xi, \frac{t}{b}\right), \acute{\mathcal{E}}\left(\imath, w, \xi, \frac{s}{b}\right)\right) \leq \acute{\mathcal{E}}\left(\varpi, w, \xi, \frac{t+s}{b}\right)$,

($E_{FbM}6$) $\acute{\mathcal{E}}(\varpi, w, \xi, \bullet) : (0,\infty) \to [0,1]$ is continuous.

If we consider $b = 1$ in generalized E-fuzzy b-metric space then it becomes E-fuzzy metric space.

The following examples satisfy all the six axioms for generalized E-fuzzy b-metric space as follows:

Example 2.2. Let $\breve{\mathfrak{A}} = \mathbb{R}$, $\overline{\mathbb{G}}$ is a G-metric on $\breve{\mathfrak{A}}$ and $\acute{\mathcal{E}}$ is a fuzzy set in $\breve{\mathfrak{A}}^3 \times (0,\infty)$ satisfying: $\acute{\mathcal{E}}\left(\varpi, w, \xi, \frac{t}{b}\right) = \left[\exp\left(\frac{b\overline{\mathbb{G}}(\varpi, w, \xi)}{t}\right)\right]^{-1}$, for all $\varpi, w, \xi \in \breve{\mathfrak{A}}$, $t \in (0,\infty)$, $b \geq 1$ and continuous t-norm $\widehat{\mathfrak{S}}(\eth_1, \eth_2) = \eth_1\eth_2$, for all $\eth_1, \eth_2 \in [0,1]$. Then, $\acute{\mathcal{E}}$ is a generalized E-fuzzy b-metric space on $\breve{\mathfrak{A}}$.

Solution. Axiom (E_{FbM}1), $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right) = \left[\exp\left(\frac{b\overline{\mathbb{G}}(\varpi,w,\xi)}{t}\right)\right]^{-1} > 0$ holds. Now, $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right) = \left[\exp\left(\frac{b\overline{\mathbb{G}}(\varpi,w,\xi)}{t}\right)\right]^{-1} \le \left[\exp\left(\frac{b\overline{\mathbb{G}}(\varpi,\varpi,\xi)}{t}\right)\right]^{-1} = \acute{\mathcal{E}}\left(\varpi,\varpi,w,\frac{t}{b}\right)$ and $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right) = 1$, for every $t \in (0,\infty)$ and real number $b \ge 1$ if and only if $\varpi = w = \xi$. Axioms (E_{FbM}4) and (E_{FbM}6) can be verified easily.

Now, we prove axiom (E_{FbM}5), since

$$\left(\frac{b\overline{\mathbb{G}}(\varpi,\iota,\xi)}{t}\right)+\left(\frac{b\overline{\mathbb{G}}(\iota,w,\xi)}{s}\right) \ge \left(\frac{b\overline{\mathbb{G}}(\varpi,w,\xi)}{(t+s)}\right),$$

$$\left[\exp\left(\frac{b\overline{\mathbb{G}}(\varpi,\iota,\xi)}{t}\right)\right]\left[\exp\left(\frac{b\overline{\mathbb{G}}(\iota,w,\xi)}{s}\right)\right] \ge \left[\exp\left(\frac{b\overline{\mathbb{G}}(\varpi,w,\xi)}{(t+s)}\right)\right],$$

$$\left[\exp\left(\frac{b\overline{\mathbb{G}}(\varpi,\iota,\xi)}{t}\right)\right]^{-1}\left[\exp\left(\frac{b\overline{\mathbb{G}}(\iota,w,\xi)}{s}\right)\right]^{-1} \le \left[\exp\left(\frac{b\overline{\mathbb{G}}(\varpi,w,\xi)}{(t+s)}\right)\right]^{-1}.$$

Thus,

$$\widehat{\mathfrak{S}}\left(\acute{\mathcal{E}}\left(\varpi,\iota,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\iota,w,\xi,\frac{s}{b}\right)\right) \le \acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t+s}{b}\right).$$

Hence, $\acute{\mathcal{E}}$ is a generalized E-fuzzy b-metric space on $\check{\mathfrak{A}}$ with a continuous t-norm $\widehat{\mathfrak{S}}$.

Example 2.3. Let $\check{\mathfrak{A}} = \mathbb{R}$ and $\acute{\mathcal{E}}$ is a fuzzy set in $\check{\mathfrak{A}}^3 \times (0,\infty)$ satisfies: $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right) = \begin{cases} 1, & \text{if } \varpi = w = \xi, \\ \frac{t}{t+b}, & \text{elsewhere,} \end{cases}$ for every $\varpi,w,\xi \in \check{\mathfrak{A}}$, $t \in (0,\infty)$, $b \ge 1$ and continuous t-norm $\widehat{\mathfrak{S}}(\eth_1,\eth_2) = \eth_1\eth_2$, for all $\eth_1,\eth_2 \in [0,1]$. Then, $\acute{\mathcal{E}}$ is a generalized E-fuzzy b-metric space on $\check{\mathfrak{A}}$.

Solution. Axiom (E_{FbM}1), $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right) = \frac{t}{t+b} > 0$ holds. Also, if $\mathfrak{p}_1 = \mathfrak{p}_2 = \mathfrak{p}_3$ or any of two $\mathfrak{p}_1,\mathfrak{p}_2$ and $\mathfrak{p}_3$ are equal then $\acute{\mathcal{E}}\left(\varpi,\varpi,w,\frac{t}{b}\right) \ge \acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)$ holds and $\acute{\mathcal{E}}\left(\varpi,\varpi,w,\frac{t}{b}\right) = \frac{t}{t+b} = \acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)$ if $\varpi \ne w \ne \xi$ satisfies. Axioms (E_{FbM}4) and (E_{FbM}6) can be easily verified. Now, we show Axiom (E_{FbM}5), since

$$\widehat{\mathfrak{S}}\left(\frac{t}{t+b},\frac{s}{s+b}\right) \le \frac{t+s}{t+s+b}.$$

Thus,

$$\widehat{\mathfrak{S}}\left(\acute{\mathcal{E}}\left(\varpi,\iota,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\iota,w,\xi,\frac{s}{b}\right)\right) \le \acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t+s}{b}\right).$$

Hence, $\acute{\mathcal{E}}$ is a generalized E-fuzzy b-metric space on $\check{\mathfrak{A}}$ with a continuous t-norm $\widehat{\mathfrak{S}}$.

Example 2.4. Let $\check{\mathfrak{A}} = \mathbb{R}$, $\overline{\mathbb{G}}$ is a G-metric on $\check{\mathfrak{A}}$ and $\acute{\mathcal{E}}$ is a fuzzy set in $\check{\mathfrak{A}}^3 \times (0,\infty)$ satisfying: $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right) = \frac{t}{t+b\overline{\mathbb{G}}(\varpi,w,\xi)}$, for every $\varpi,w,\xi \in \check{\mathfrak{A}}$, $t \in (0,\infty)$, $b \ge 1$ and continuous t-norm $\widehat{\mathfrak{S}}(\eth_1,\eth_2) = \eth_1\eth_2$, for all $\eth_1,\eth_2 \in [0,1]$. Then, $\acute{\mathcal{E}}$ is a generalized E-fuzzy b-metric space.

Solution. Axiom (E_{FbM}1), $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right) = \frac{t}{t+b\overline{\mathbb{G}}(\varpi,w,\xi)} > 0$ holds. Now, $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right) = \frac{t}{t+b\overline{\mathbb{G}}(\varpi,w,\xi)} \le \frac{t}{t+b\overline{\mathbb{G}}(\varpi,\varpi,\xi)} = \acute{\mathcal{E}}\left(\varpi,\varpi,w,\frac{t}{b}\right)$ and $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right) = 1$ for every $t \in (0,\infty)$ and real number $b \ge 1$ iff $\varpi = w = \xi$. Axioms (E_{FbM}4) and (E_{FbM}6) can be verify easily.

Now, we prove axiom ($E_{FbM}5$), since,

$$\overline{\mathbb{G}}(\varpi,w,\xi) \le \frac{(t+s)}{bt}\overline{\mathbb{G}}(\varpi,\imath,\xi) + \frac{(t+s)}{bs}\overline{\mathbb{G}}(\imath,w,\xi)$$

$$\frac{b\overline{\mathbb{G}}(\varpi,w,\xi)}{(t+s)} \le \frac{b\overline{\mathbb{G}}(\varpi,\imath,\xi)}{t} + \frac{b\overline{\mathbb{G}}(\imath,w,\xi)}{s} + \frac{b\overline{\mathbb{G}}(\imath,w,\xi)\overline{\mathbb{G}}(\varpi,\imath,\xi)}{ts},$$

$$1+\frac{b\overline{\mathbb{G}}(\varpi,w,\xi)}{(t+s)} \le 1+\frac{b\overline{\mathbb{G}}(\varpi,\imath,\xi)}{t} + \frac{b\overline{\mathbb{G}}(\imath,w,\xi)}{s}\left[1+\frac{b\overline{\mathbb{G}}(\varpi,\imath,\xi)}{t}\right],$$

$$\left(\frac{t+s}{t+s+b\overline{\mathbb{G}}(\varpi,w,\xi)}\right) \ge \left(\frac{t}{t+b\overline{\mathbb{G}}(\varpi,\imath,\xi)}\right)\left(\frac{s}{s+b\overline{\mathbb{G}}(\imath,w,\xi)}\right).$$

Thus,

$$\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t+s}{b}\right) \ge \widehat{\mathfrak{S}}\left(\acute{\mathcal{E}}(\varpi,\imath,\xi,\frac{t}{b}),\acute{\mathcal{E}}\left(\imath,w,\xi,\frac{s}{b}\right)\right).$$

Hence, $\acute{\mathcal{E}}$ is a generalized E-fuzzy b-metric space on $\check{\mathfrak{A}}$ with a continuous t-norm $\widehat{\mathfrak{S}}$.

Example 2.5. Let $\check{\mathfrak{A}} = \mathbb{N}$ and $\acute{\mathcal{E}}$ is a fuzzy set in $\check{\mathfrak{A}}^3 \times (0,\infty)$ satisfying: $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right) = \frac{\min\{\varpi,w,\xi\}}{\max\{\varpi,w,\xi\}}$, for every $\varpi,w,\xi \in \check{\mathfrak{A}}$, $t \in (0,\infty)$, $b \ge 1$ and continuous t-norm $\widehat{\mathfrak{S}}(\eth_1,\eth_2) = \eth_1\eth_2$, for all $\eth_1,\eth_2 \in [0,1]$. Then, $\acute{\mathcal{E}}$ is a generalized E-fuzzy b-metric space on $\check{\mathfrak{A}}$. Further, here $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)$ does not depend on t and b, i.e., for each $\varpi,w,\xi \in \check{\mathfrak{A}}$, $t \in (0,\infty)$, $b \ge 1$, $\acute{\mathcal{E}}$ is constant. Hence, $\acute{\mathcal{E}}$ is also said as stationary generalized E-fuzzy b-metric space on $\check{\mathfrak{A}}$.

Definition 2.6. Let $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ be a generalized E-fuzzy b-metric space. A sequence $\{\mathfrak{p}_n\}$ in $\check{\mathfrak{A}}$ is said to be convergent sequence to $\varpi \in \check{\mathfrak{A}}$ if and only if $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\varpi,\varpi,\frac{t}{b}\right) \to 1$ as $n \to \infty$ for every $t > 0$ and $b \ge 1$.

Lemma 2.7. *Let* $\left(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}}\right)$ *be a generalized E-fuzzy b-metric space. A sequence $\{\mathfrak{p}_n\}$ in $\check{\mathfrak{A}}$ converges to $\varpi \in \check{\mathfrak{A}}$. Then,*

(1) $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\mathfrak{p}_n,\varpi,\frac{t}{b}\right) \to 1$ *as* $n \to \infty$*, for every* $t > 0$ *and* $b \ge 1$*,*

(2) $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\mathfrak{p}_m,\varpi,\frac{t}{b}\right) \to 1$ *as* $n,m \to \infty$*, for every* $t > 0$ *and* $b \ge 1$*,*

(3) $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\mathfrak{p}_m,\varpi,\frac{t}{b}\right) > 1-r$*, for every* $r > 0, t > 0$ *and* $b \ge 1$*, there exists a natural number* n_0 *and* $m,n \ge n_0$.

Proof. Since $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ be a generalized E-fuzzy b-metric space and $\{\mathfrak{p}_n\}$ sequence in $\check{\mathfrak{A}}$ converges to $\varpi \in \check{\mathfrak{A}}$. Therefore, $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\varpi,\varpi,\frac{t}{b}\right) \to 1$ as $n \to \infty$ for every $t > 0$ and $b \ge 1$,

(1) $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\mathfrak{p}_n,\varpi,\frac{t}{b}\right) \ge \widehat{\mathfrak{S}}\left(\acute{\mathcal{E}}\left(\mathfrak{p}_n,\varpi,\varpi,\frac{t}{2b}\right),\acute{\mathcal{E}}\left(\varpi,\mathfrak{p}_n,\varpi,\frac{t}{2b}\right)\right)$, since $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\varpi,\varpi,\frac{t}{2b}\right)$ and $\acute{\mathcal{E}}\left(\varpi,\mathfrak{p}_n,\varpi,\frac{t}{2b}\right)$ converges to 1 as $n \to \infty$.

(2) $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\mathfrak{p}_m,\varpi,\frac{t}{b}\right) \ge \widehat{\mathfrak{S}}\left(\acute{\mathcal{E}}\left(\mathfrak{p}_n,\varpi,\varpi,\frac{t}{2b}\right),\acute{\mathcal{E}}\left(\varpi,\mathfrak{p}_m,\varpi,\frac{t}{2b}\right)\right)$, since $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\varpi,\varpi,\frac{t}{2b}\right)$ and $\acute{\mathcal{E}}\left(\varpi,\mathfrak{p}_m,\varpi,\frac{t}{2b}\right)$ converges to 1 as $m,n \to \infty$. Hence, $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\mathfrak{p}_m,\varpi,\frac{t}{b}\right) \to 1$ as $n,m \to \infty$.

(3) $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\mathfrak{p}_m,\varpi,\frac{t}{b}\right) \ge \widehat{\mathfrak{S}}\left(\acute{\mathcal{E}}\left(\mathfrak{p}_n,\varpi,\varpi,\frac{t}{2b}\right),\acute{\mathcal{E}}\left(\varpi,\mathfrak{p}_m,\varpi,\frac{t}{2b}\right)\right)$, since $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\varpi,\varpi,\frac{t}{2b}\right)$ and $\acute{\mathcal{E}}\left(\varpi,\mathfrak{p}_m,\varpi,\frac{t}{2b}\right)$ converges to 1 as $m,n \to \infty$ for every $r > 0, t > 0$ and $b \ge 1$, we can get a $n_0 \in \mathbb{N}$ and $m,n \ge n_0$ such that $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\mathfrak{p}_m,\varpi,\frac{t}{b}\right) > 1-r$. □

Definition 2.8. Let $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ be a generalized E-fuzzy b-metric space. A sequence $\{\mathfrak{p}_n\}$ in $\check{\mathfrak{A}}$ is said to be a Cauchy sequence if and only if for any $r\in(0,1)$, there exists a natural number n_0 such that $\acute{\mathcal{E}}\left(\mathfrak{p}_n,\mathfrak{p}_m,\mathfrak{p}_l,\frac{t}{b}\right)>1-r$, for all $n,m,l \ \geq\ n_0\,t>0$ and $b\geq 1$.

Definition 2.9. A generalized E-fuzzy b-metric space $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ in which every Cauchy sequence is convergent is said to be complete generalized E-fuzzy b-metric space.

Definition 2.10. A generalized E-fuzzy b-metric space $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ in which every sequence has a convergent subsequence is said to be compact generalized E-fuzzy b-metric space.

Lemma 2.11. *If $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ be a generalized E-fuzzy b-metric space then $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)$ is non-decreasing with respect to t, for all $\varpi,w,\xi,\imath\in\check{\mathfrak{A}}$, $s,t>0$ and $b\geq 1$.*

Proof. Let us assume the contrary $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t+s}{b}\right)<\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)$, for all $\varpi,w,\xi\in\check{\mathfrak{A}}$.
Put $\xi=\varpi$ in the inequality, we get

$$\acute{\mathcal{E}}\left(\varpi,\varpi,w,\frac{t+s}{b}\right)<\acute{\mathcal{E}}\left(\varpi,\varpi,w,\frac{t}{b}\right).$$

Now, by taking $\imath=w$ and $\xi=\varpi$ in the axiom ($\mathrm{E_{FbM}}$5), we get

$$\widehat{\mathfrak{S}}\left(\acute{\mathcal{E}}\left(\varpi,\varpi,\varpi,\frac{s}{b}\right),\acute{\mathcal{E}}\left(\varpi,\varpi,w,\frac{t}{b}\right)\right)\leq\acute{\mathcal{E}}\left(\varpi,\varpi,w,\frac{(t+s)}{b}\right).$$

Therefore,

$$\acute{\mathcal{E}}\left(\varpi,\varpi,w,\frac{t}{b}\right)\leq\acute{\mathcal{E}}\left(\varpi,\varpi,w,\frac{t+s}{b}\right),$$

a contradiction. Hence, the result. □

Lemma 2.12. *Let $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ be a generalized E-fuzzy b-metric space and if there exists a constant $0<k<1$, $t\in(0,\infty)$ and $b\geq 1$ satisfying $\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{kt}{b}\right)\geq\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)$, for all $\varpi,w,\xi\in\check{\mathfrak{A}}$ then $\varpi=w=\xi$.*

Proof. Since for every $\varpi,w,\xi\in\check{\mathfrak{A}}$, $0<k<1$, $b\geq 1$ and $t\in(0,\infty)$, we have

$$\begin{aligned}\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{kt}{b}\right)&\geq\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{b}\right)\geq\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{kb}\right)\geq\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{k^2b}\right)\\&\geq\cdots\geq\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{k^{m-1}b}\right)\geq\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{t}{k^mb}\right).\end{aligned}$$

Considering limit as $m\to\infty$, we get

$$\acute{\mathcal{E}}\left(\varpi,w,\xi,\frac{kt}{b}\right)\geq 1.$$

Hence, $\varpi=w=\xi$. □

Theorem 2.13. *Let $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ be a complete generalized E-fuzzy b-metric space and $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ and ζ_6 are six self-maps on $\check{\mathfrak{A}}$ satisfying the followings:*

(1) *$\{\zeta_1,\zeta_4\}$, $\{\zeta_2,\zeta_5\}$ and $\{\zeta_3,\zeta_6\}$ are weakly compatible,*

(2) *$\zeta_4(\check{\mathfrak{A}})\subset\zeta_2(\check{\mathfrak{A}})$, $\zeta_5(\check{\mathfrak{A}})\subset\zeta_3(\check{\mathfrak{A}})$, $\zeta_6(\check{\mathfrak{A}})\subset\zeta_1(\check{\mathfrak{A}})$,*

(3) *$\zeta_1(\check{\mathfrak{A}})$ is complete.*

If there exists a constant $k \in (0,1)$ and a real number $b \geq 1$ such that for all $\varpi, w, \xi \in \check{\mathfrak{A}}$:

$$\acute{\mathcal{E}}(\zeta_4\varpi,\zeta_5 w,\zeta_6\xi,\frac{kt}{b}) \geq \max\left\{\acute{\mathcal{E}}\left(\zeta_1\varpi,\zeta_2 w,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\varpi,\zeta_1\varpi,\zeta_2\xi,\frac{t}{b}\right)\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_5\varpi,\zeta_2\varpi,\zeta_2\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\varpi,\zeta_3\varpi,\zeta_3\xi,\frac{t}{b}\right)\right\}. \tag{2.1}$$

Then the six self-maps $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ and ζ_6 have a unique common fixed point.

Proof. Suppose $\mathfrak{p}_0 \in \check{\mathfrak{A}}$ and $\zeta_4(\mathfrak{p}_0) = \mathfrak{q}_1$. Since $\zeta_4(\check{\mathfrak{A}}) \subset \zeta_2(\check{\mathfrak{A}})$, $\zeta_5(\check{\mathfrak{A}}) \subset \zeta_3(\check{\mathfrak{A}})$, $\zeta_6(\check{\mathfrak{A}}) \subset \zeta_1(\check{\mathfrak{A}})$, by iteration, we can define two sequences $\{\mathfrak{p}_m\}$ and $\{\mathfrak{q}_m\}$ in $\check{\mathfrak{A}}$ such that:

$$\mathfrak{q}_{3m+1} = \zeta_4\mathfrak{p}_{3m} = \zeta_2\mathfrak{p}_{3m+1},\quad \mathfrak{q}_{3m+2} = \zeta_5\mathfrak{p}_{3m+1} = \zeta_3\mathfrak{p}_{3m+2},\quad \mathfrak{q}_{3m+3} = \zeta_6\mathfrak{p}_{3m+2} = \zeta_1\mathfrak{p}_{3m+3}.$$

Then from eq. (2.1), we get

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_5\mathfrak{p}_{3m+1},\zeta_6\mathfrak{p}_{3m+2},\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+1},\zeta_3\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+2},\frac{t}{b}\right)\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_5\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\mathfrak{p}_{3m},\zeta_3\mathfrak{p}_{3m},\zeta_3\mathfrak{p}_{3m+2},\frac{t}{b}\right)\right\},$$

$$\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\mathfrak{q}_{3m+3},\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\mathfrak{q}_{3m},\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\mathfrak{q}_{3m+2},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\mathfrak{q}_{3m+2},\frac{t}{b}\right)\right\}.$$

Therefore,

$$\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\mathfrak{q}_{3m+3},\frac{kt}{b}\right) \geq \acute{\mathcal{E}}\left(\mathfrak{q}_{3m},\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\frac{t}{b}\right).$$

Similarly, if we replace $3m+1$ by m, we get

$$\acute{\mathcal{E}}\left(\mathfrak{q}_{m},\mathfrak{q}_{m+1},\mathfrak{q}_{m+2},\frac{kt}{b}\right) \geq \acute{\mathcal{E}}\left(\mathfrak{q}_{m-1},\mathfrak{q}_{m},\mathfrak{q}_{m+1},\frac{t}{b}\right).$$

Hence, $\{\mathfrak{q}_m\}$ is a Cauchy sequence and since $\check{\mathfrak{A}}$ is complete then there exists $\xi \in \check{\mathfrak{A}}$ s.t. $\mathfrak{q}_m \to \xi$. Thus, the subsequences $\{\mathfrak{q}_{3m}\}$, $\{\mathfrak{q}_{3m+1}\}$, $\{\mathfrak{q}_{3m+2}\}$ are also convergent, i.e.,

$$\lim \zeta_2\mathfrak{p}_{3m+1} = \lim \zeta_4\mathfrak{p}_{3m} = \lim \zeta_3\mathfrak{p}_{3m+2} = \lim \zeta_5\mathfrak{p}_{3m+1} = \lim \zeta_1\mathfrak{p}_{3m+3} = \lim \zeta_6\mathfrak{p}_{3m+2} = \xi.$$

Since $\zeta_1(\check{\mathfrak{A}})$ is complete, so there must exist $\mathfrak{T}_4 \in \check{\mathfrak{A}}$ such that $\zeta_1\mathfrak{T}_4 = \xi$.
Now, we claim that $\zeta_4\mathfrak{T}_4 = \xi$,

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\zeta_5\mathfrak{p}_{3m+1},\zeta_6\mathfrak{p}_{3m+2},\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\zeta_1\mathfrak{T}_4,\zeta_2\mathfrak{p}_{3m+1},\zeta_2\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\zeta_1\mathfrak{T}_4,\zeta_2\mathfrak{p}_{3m+2},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_5\mathfrak{T}_4,\zeta_2\mathfrak{T}_4,\zeta_2\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\mathfrak{T}_4,\zeta_3\mathfrak{T}_4,\zeta_3\mathfrak{p}_{3m+2},\frac{t}{b}\right)\right\},$$

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\xi,\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\mathfrak{q}_{3m+2},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\zeta_3\mathfrak{T}_4,\mathfrak{q}_{3m+2},\frac{t}{b}\right)\right\},$$

assuming limit as $m \to \infty$, we get

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\xi,\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\xi,\frac{t}{b}\right),\ \acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\zeta_3\mathfrak{T}_4,\xi,\frac{t}{b}\right)\right\}$$

$$\geq \acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\xi,\frac{t}{b}\right).$$

Therefore,

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\xi,\frac{kt}{b}\right)=1.$$

Thus,

$$\zeta_4\mathfrak{T}_4=\xi=\zeta_1\mathfrak{T}_4.$$

Hence, $\mathfrak{T}_4$ is the coincidence point of ζ_4 and ζ_1.
Now, $\zeta_4(\breve{\mathfrak{A}})\subset\zeta_2(\breve{\mathfrak{A}})$, i.e., $\xi=\zeta_4\mathfrak{T}_4\in\zeta_4(\breve{\mathfrak{A}})\subset\zeta_2(\breve{\mathfrak{A}})$, then there must exist $\mathfrak{T}_5\in\breve{\mathfrak{A}}$ such that $\zeta_2\mathfrak{T}_5=\xi$,

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_5\mathfrak{T}_5,\zeta_6\mathfrak{p}_{3m+2},\frac{kt}{b}\right)\geq\max\left\{\acute{\mathcal{E}}\left(\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{T}_5,\zeta_3\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+2},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_5\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\mathfrak{p}_{3m},\zeta_3\mathfrak{p}_{3m},\zeta_3\mathfrak{p}_{3m+2},\frac{t}{b}\right)\right\},$$

$$\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\zeta_5\mathfrak{T}_5,\mathfrak{q}_{3m+3},\frac{kt}{b}\right)\geq\max\left\{\acute{\mathcal{E}}\left(\mathfrak{q}_{3m},\xi,\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\mathfrak{q}_{3m+2},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\mathfrak{q}_{3m+2},\frac{t}{b}\right)\right\}.$$

Taking limit as $m\to\infty$, we have

$$\acute{\mathcal{E}}\left(\xi,\zeta_5\mathfrak{T}_5,\xi,\frac{kt}{b}\right)\geq\max\left\{\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right)\right\}.$$

Therefore,

$$\acute{\mathcal{E}}\left(\xi,\zeta_5\mathfrak{T}_5,\xi,\frac{kt}{b}\right)=1.$$

Thus,

$$\zeta_5\mathfrak{T}_5=\xi=\zeta_2\mathfrak{T}_5.$$

Hence, $\mathfrak{T}_5$ is a coincidence point of ζ_2 and ζ_5.
Now, $\zeta_5(\breve{\mathfrak{A}})\subset\zeta_3(\breve{\mathfrak{A}})$, i.e., $\xi=\zeta_5\mathfrak{T}_5\in\zeta_5(\breve{\mathfrak{A}})\subset\zeta_3(\breve{\mathfrak{A}})$, then there exists $o_6\in\breve{\mathfrak{A}}$ s.t. $\zeta_3o_6=\xi$,

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_5\mathfrak{p}_{3m+1},\zeta_6o_6,\frac{kt}{b}\right)\geq\max\left\{\acute{\mathcal{E}}\left(\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+1},\zeta_3o_6,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_1\mathfrak{p}_{3m},\zeta_2o_6,\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_5\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m},\zeta_2o_6,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\mathfrak{p}_{3m},\zeta_3\mathfrak{p}_{3m},\zeta_3o_6,\frac{t}{b}\right)\right\},$$

$$\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\zeta_6o_6,\frac{kt}{b}\right)\geq\max\left\{\acute{\mathcal{E}}\left(\mathfrak{q}_{3m},\mathfrak{q}_{3m+1},\zeta_3o_6,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\zeta_2o_6,\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\zeta_2o_6,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\zeta_3o_6,\frac{t}{b}\right)\right\}.$$

Considering the limit as $m\to\infty$, we get

$$\acute{\mathcal{E}}\left(\xi,\xi,\zeta_6o_6,\frac{kt}{b}\right)\geq\max\left\{\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\zeta_2o_6,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\zeta_2o_6,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right)\right\},$$

$$\acute{\mathcal{E}}\left(\xi,\xi,\zeta_6o_6,\frac{kt}{b}\right)\geq 1.$$

Therefore, $\zeta_6o_6=\xi$.

Hence, $\zeta_6 o_6 = \zeta_3 o_6 = \xi$.
Since three pairs of $\{\zeta_1,\zeta_4\}$, $\{\zeta_2,\zeta_5\}$ and $\{\zeta_3,\zeta_6\}$ be weakly compatible mappings they commute at coincidence points.
As, $\zeta_4\mathfrak{I}_4 = \xi = \zeta_1\mathfrak{I}_4$, therefore, $\zeta_1\zeta_4\mathfrak{I}_4 = \zeta_4\zeta_1\mathfrak{I}_4$.
Thus, $\zeta_1\xi = \zeta_4\xi$.
Also, $\zeta_5\mathfrak{T}_5 = \xi = \zeta_2\mathfrak{T}_5$, therefore, $\zeta_2\zeta_5\mathfrak{T}_5 = \zeta_5\zeta_2\mathfrak{T}_5$.
Hence, $\zeta_2\xi = \zeta_5\xi$.
In the similar way, we get $\zeta_3\xi = \zeta_6\xi$, since $\{\zeta_3,\zeta_6\}$ is weakly compatible,

$$\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_5\mathfrak{p}_{3m+1},\zeta_6\xi,\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\zeta_1\xi,\zeta_2\mathfrak{p}_{3m+1},\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_1\xi,\zeta_2\xi,\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_5\xi,\zeta_2\xi,\zeta_2\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\xi,\zeta_3\xi,\zeta_3\xi,\frac{t}{b}\right)\right\}.$$

Proceeding the limit as $m\to\infty$, we get

$$\acute{\mathcal{E}}\left(\zeta_4\xi,\xi,\zeta_6\xi,\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\zeta_1\xi,\xi,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_1\xi,\zeta_2\xi,\frac{t}{b}\right).1,1\right\}.$$

So, $\zeta_4\xi = \xi = \zeta_6\xi$.
Hence, $\zeta_1\xi = \zeta_4\xi = \zeta_3\xi = \zeta_6\xi = \xi$.
Now,

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_5\xi,\zeta_6\xi,\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\zeta_1\mathfrak{p}_{3m},\zeta_2\xi,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_1\mathfrak{p}_{3m},\zeta_2\xi,\frac{t}{b}\right)\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_5\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m},\zeta_2\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\mathfrak{p}_{3m},\zeta_3\mathfrak{p}_{3m},\zeta_3\xi,\frac{t}{b}\right)\right\}.$$

Considering limit as $m\to\infty$,

$$\acute{\mathcal{E}}\left(\xi,\zeta_5\xi,\zeta_6\xi,\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\xi,\zeta_2\xi,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\zeta_2\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\zeta_2\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right)\right\}$$
$$\geq \max\left\{\acute{\mathcal{E}}\left(\xi,\zeta_2\xi,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\zeta_2\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\zeta_2\xi,\frac{t}{b}\right),1\right\}.$$

Thus, $\zeta_5\xi = \xi = \zeta_6\xi$.
Therefore, $\zeta_1\xi = \zeta_4\xi = \zeta_2\xi = \zeta_5\xi = \zeta_3\xi = \zeta_6\xi = \xi$.
Hence, ξ is a common fixed point of the six self-mappings $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ and ζ_6.

Uniqueness: To prove uniqueness of fixed point, let $\mathfrak{u}_o$ be another fixed point of the six self-mappings $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ and ζ_6. Then from equation (2.1), we have

$$\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_5\mathfrak{u}_o,\zeta_6\xi,\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\zeta_1\xi,\zeta_2\mathfrak{u}_o,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_1\xi,\zeta_2\xi,\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_5\xi,\zeta_2\xi,\zeta_2\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\xi,\zeta_3\xi,\zeta_3\xi,\frac{t}{b}\right)\right\},$$
$$\acute{\mathcal{E}}\left(\xi,\mathfrak{u}_o,\xi,\frac{kt}{b}\right) \geq \max\left\{\acute{\mathcal{E}}\left(\xi,\mathfrak{u}_o,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right)\right\}.$$

Therefore, $\acute{\mathcal{E}}\left(\xi,\mathfrak{u}_o,\xi,\frac{kt}{b}\right) \geq 1$.
Hence, $\mathfrak{u}_o = \xi$. □

Theorem 2.14. *Let $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ be a complete generalized E-fuzzy b-metric space and $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ and ζ_6 are six self-maps on $\check{\mathfrak{A}}$ satisfying the followings:*

(i) $\{\zeta_1,\zeta_4\}$, $\{\zeta_2,\zeta_5\}$ *and* $\{\zeta_3,\zeta_6\}$ *are weakly compatible,*

(ii) $\zeta_4(\check{\mathfrak{A}})\subset\zeta_2(\check{\mathfrak{A}})$, $\zeta_5(\check{\mathfrak{A}})\subset\zeta_3(\check{\mathfrak{A}})$, $\zeta_6(\check{\mathfrak{A}})\subset\zeta_1(\check{\mathfrak{A}})$,

(iii) $\zeta_1(\check{\mathfrak{A}})$ *is complete.*

If there exists a constant $k\in(0,1)$ and a real number $b\geq 1$ such that for all $\varpi,w,\xi\in\check{\mathfrak{A}}$:

$$\acute{\mathcal{E}}\left(\zeta_4\varpi,\zeta_5w,\zeta_6\xi,\frac{kt}{b}\right)\geq\min\left\{\acute{\mathcal{E}}\left(\zeta_1\varpi,\zeta_2w,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\varpi,\zeta_1\varpi,\zeta_2\xi,\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_4\varpi,\zeta_2\varpi,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\varpi,\zeta_3\varpi,\zeta_1\xi,\frac{t}{b}\right)\right\}. \tag{2.2}$$

Then the six self-maps $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ and ζ_6 have a unique fixed point.

Proof. Suppose $\mathfrak{p}_0\in\check{\mathfrak{A}}$ and $\zeta_4(\mathfrak{p}_0)=\mathfrak{q}_1$. Since $\zeta_4(\check{\mathfrak{A}})\subset\zeta_2(\check{\mathfrak{A}}),\zeta_5(\check{\mathfrak{A}})\subset\zeta_3(\check{\mathfrak{A}}),\zeta_6(\check{\mathfrak{A}})\subset\zeta_1(\check{\mathfrak{A}})$, by iteration, we can define two sequences $\{\mathfrak{p}_m\}$ and $\{\mathfrak{q}_m\}$ in $\check{\mathfrak{A}}$ such that:

$$\mathfrak{q}_{3m+1}=\zeta_4\mathfrak{p}_{3m}=\zeta_2\mathfrak{p}_{3m+1},\quad \mathfrak{q}_{3m+2}=\zeta_5\mathfrak{p}_{3m+1}=\zeta_3\mathfrak{p}_{3m+2},\quad \mathfrak{q}_{3m+3}=\zeta_6\mathfrak{p}_{3m+2}=\zeta_1\mathfrak{p}_{3m+3}.$$

Then from eq. (2.2), we get

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_5\mathfrak{p}_{3m+1},\zeta_6\mathfrak{p}_{3m+2},\frac{kt}{b}\right)\geq\min\left\{\acute{\mathcal{E}}\left(\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+1},\zeta_3\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+1},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+1},\zeta_3\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\mathfrak{p}_{3m+2},\zeta_3\mathfrak{p}_{3m+2},\zeta_1\mathfrak{p}_{3m},\frac{t}{b}\right)\right\},$$
$$\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\mathfrak{q}_{3m+3},\frac{kt}{b}\right)\geq\min\left\{\acute{\mathcal{E}}\left(\mathfrak{q}_{3m},\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\mathfrak{q}_{3m+1},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+3},\mathfrak{q}_{3m+2},\mathfrak{q}_{3m},\frac{t}{b}\right)\right\}.$$

In the similar way, if we replace $3m+1$ by m, we get

$$\acute{\mathcal{E}}\left(\mathfrak{q}_m,\mathfrak{q}_{m+1},\mathfrak{q}_{m+2},\frac{kt}{b}\right)\geq\acute{\mathcal{E}}\left(\mathfrak{q}_{m-1},\mathfrak{q}_m,\mathfrak{q}_{m+1},\frac{t}{b}\right).$$

Since $\{\mathfrak{q}_m\}$ is a Cauchy sequence and since $\check{\mathfrak{A}}$ is complete, then there exists $\xi\in\check{\mathfrak{A}}$ s.t. $\mathfrak{q}_m\to\xi$. Therefore, the subsequences $\{\mathfrak{q}_{3m}\}$, $\{\mathfrak{q}_{3m+1}\}$, $\{\mathfrak{q}_{3m+2}\}$ are also convergent, i.e.,

$$\lim\zeta_2\mathfrak{p}_{3m+1}=\lim\zeta_4\mathfrak{p}_{3m}=\lim\zeta_3\mathfrak{p}_{3m+2}=\lim\zeta_5\mathfrak{p}_{3m+1}=\lim\zeta_1\mathfrak{p}_{3m+3}=\lim\zeta_6\mathfrak{p}_{3m+2}=\xi.$$

Since $\zeta_1(\check{\mathfrak{A}})$ is complete, so there exists $\mathcal{T}_4\in\check{\mathfrak{A}}$ such that $\zeta_1\mathcal{T}_4=\xi$.
Now, we claim that $\zeta_4\mathcal{T}_4=\xi$,

$$\acute{\mathcal{E}}\left(\zeta_4\mathcal{T}_4,\zeta_5\mathfrak{p}_{3m+1},\zeta_6\mathfrak{p}_{3m+2},\frac{kt}{b}\right)\geq\min\left\{\acute{\mathcal{E}}\left(\zeta_1\mathcal{T}_4,\zeta_2\mathfrak{p}_{3m+1},\zeta_2\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathcal{T}_4,\zeta_1\mathcal{T}_4,\zeta_2\mathfrak{p}_{3m+1},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_4\mathcal{T}_4,\zeta_2\mathfrak{p}_{3m+1},\zeta_3\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\mathfrak{p}_{3m+2},\zeta_3\mathfrak{p}_{3m+2},\zeta_1\mathcal{T}_4,\frac{t}{b}\right)\right\},$$
$$\acute{\mathcal{E}}\left(\zeta_4\mathcal{T}_4,\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\frac{kt}{b}\right)\geq\min\left\{\acute{\mathcal{E}}\left(\xi,\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathcal{T}_4,\xi,\mathfrak{q}_{3m+1},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_4\mathcal{T}_4,\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+3},\mathfrak{q}_{3m+2},\xi,\frac{t}{b}\right)\right\}.$$

Proceeding the limit as $m \to \infty$, we get

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\xi,\frac{kt}{b}\right) \geq \min\left\{\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right)\right\}$$
$$\geq \acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\xi,\frac{t}{b}\right).$$

Therefore,

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{T}_4,\xi,\xi,\frac{kt}{b}\right) = 1.$$

Thus,

$$\zeta_4\mathfrak{T}_4 = \xi = \zeta_1\mathfrak{T}_4.$$

Hence, $\mathfrak{T}_4$ is the coincidence point of ζ_4 and ζ_1.
Now, $\zeta_4(\check{\mathfrak{A}}) \subset \zeta_2(\check{\mathfrak{A}})$, i.e., $\xi \in \zeta_4(\check{\mathfrak{A}}) \subset \zeta_2(\check{\mathfrak{A}})$, then there must exist $\mathfrak{T}_5 \in \check{\mathfrak{A}}$ s.t. $\zeta_2\mathfrak{T}_5 = \xi$,

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_5\mathfrak{T}_5,\zeta_6\mathfrak{p}_{3m+2},\frac{kt}{b}\right) \geq \min\left\{\acute{\mathcal{E}}\left(\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{T}_5,\zeta_3\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{T}_5,\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_2\mathfrak{T}_5,\zeta_3\mathfrak{p}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\mathfrak{p}_{3m+2},\zeta_3\mathfrak{p}_{3m+2},\zeta_1\mathfrak{p}_{3m},\frac{t}{b}\right)\right\},$$

$$\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\zeta_5\mathfrak{T}_5,\mathfrak{q}_{3m+3},\frac{kt}{b}\right) \geq \min\left\{\acute{\mathcal{E}}\left(\mathfrak{q}_{3m},\xi,\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\xi,\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\xi,\mathfrak{q}_{3m+2},\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\mathfrak{q}_{3m},\frac{t}{b}\right)\right\}.$$

Taking limit $m \to \infty$, we get

$$\acute{\mathcal{E}}\left(\xi,\zeta_5\mathfrak{T}_5,\xi,\frac{kt}{b}\right) \geq \min\left\{\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,,\frac{t}{b}\right)\right\},$$
$$\acute{\mathcal{E}}\left(\xi,\zeta_5\mathfrak{T}_5,\xi,\frac{kt}{b}\right) = 1.$$

Thus, $\zeta_5\mathfrak{T}_5 = \xi = \zeta_2\mathfrak{T}_5$.
Hence, $\mathfrak{T}_5$ is a coincidence point of ζ_2 and ζ_5.
Now, $\zeta_5(\check{\mathfrak{A}}) \subset \zeta_3(\check{\mathfrak{A}})$ i.e. $\xi \in \zeta_5(\check{\mathfrak{A}}) \subset \zeta_3(\check{\mathfrak{A}})$, then there must exists $o_6 \in \check{\mathfrak{A}}$ s.t. $\zeta_3 o_6 = \xi$,

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_5\mathfrak{p}_{3m+1},\zeta_6 o_6,\frac{kt}{b}\right) \geq \min\left\{\acute{\mathcal{E}}\left(\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+1},\zeta_3 o_6,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_1\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+1},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m+1},\zeta_3 o_6,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6 o_6,\zeta_3 o_6,\zeta_1\mathfrak{p}_{3m},\frac{t}{b}\right)\right\},$$

$$\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m+2},\zeta_6 o_6,\frac{kt}{b}\right) \geq \min\left\{\acute{\mathcal{E}}\left(\mathfrak{q}_{3m},\mathfrak{q}_{3m+1},\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\mathfrak{q}_{3m+1},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\mathfrak{q}_{3m+1},\mathfrak{q}_{3m},\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6 o_6,\xi,\mathfrak{q}_{3m},\frac{t}{b}\right)\right\}.$$

Assuming the limit as $m \to \infty$,

$$\acute{\mathcal{E}}\left(\xi,\xi,\zeta_6 o_6,\frac{kt}{b}\right) \geq \min\left\{\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6 o_6,\xi,\xi,\frac{t}{b}\right)\right\},$$
$$\acute{\mathcal{E}}\left(\xi,\xi,\zeta_6 o_6,\frac{kt}{b}\right) \geq \acute{\mathcal{E}}\left(\zeta_6 o_6,\xi,\xi,\frac{kt}{b}\right).$$

Therefore, $\zeta_6 o_6 = \xi$.

Thus, $\zeta_6 o_6 = \zeta_3 o_6 = \xi$.
Since three pairs of $\{\zeta_1,\zeta_4\}$, $\{\zeta_2,\zeta_5\}$ and $\{\zeta_3,\zeta_6\}$ be weakly compatible mappings they commute at coincidence points.
As, $\zeta_4\mathfrak{I}_4 = \xi = \zeta_1\mathfrak{I}_4$, therefore $\zeta_1\zeta_4\mathfrak{I}_4 = \zeta_4\zeta_1\mathfrak{I}_4$.
Thus, $\zeta_1\xi = \zeta_4\xi$.
Also, $\zeta_5\mathfrak{T}_5 = \xi = \zeta_2\mathfrak{T}_5$, therefore $\zeta_2\zeta_5\mathfrak{T}_5 = \zeta_5\zeta_2\mathfrak{T}_5$.
Hence, $\zeta_2\xi = \zeta_5\xi$.
In the similar way, we get $\zeta_3\xi = \zeta_6\xi$, since $\{\zeta_3,\zeta_6\}$ is weakly compatible.
Now,

$$\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_5\mathfrak{p}_{3m+1},\zeta_6\xi,\frac{kt}{b}\right) \geq \min\left\{\acute{\mathcal{E}}\left(\zeta_1\xi,\zeta_2\mathfrak{p}_{3m+1},\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_1\xi,\zeta_2\mathfrak{p}_{3m+1},\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_2\mathfrak{p}_{3m+1},\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\xi,\zeta_3\xi,\zeta_1\xi,\frac{t}{b}\right)\right\}.$$

Considering the limit as $m\to\infty$,

$$\acute{\mathcal{E}}\left(\zeta_4\xi,\xi,\zeta_6\xi,\frac{kt}{b}\right) \geq \min\left\{\acute{\mathcal{E}}\left(\zeta_4\xi,\xi,\zeta_6\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_4\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\xi,\xi,\zeta_6\xi,\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_6\xi,\zeta_6\xi,\zeta_4\xi,\frac{t}{b}\right)\right\}.$$

So,

$$\acute{\mathcal{E}}\left(\zeta_4\xi,\xi,\zeta_6\xi,\frac{kt}{b}\right) \geq \acute{\mathcal{E}}\left(\zeta_4\xi,\xi,\zeta_6\xi,\frac{t}{b}\right).$$

Hence, $\zeta_4\xi = \xi = \zeta_6\xi$.
Thus, $\zeta_1\xi = \zeta_4\xi = \zeta_3\xi = \zeta_6\xi = \xi$.
Now,

$$\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_5\xi,\zeta_6\xi,\frac{kt}{b}\right) \geq \min\left\{\acute{\mathcal{E}}\left(\zeta_1\mathfrak{p}_{3m},\zeta_2\xi,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\mathfrak{p}_{3m},\zeta_1\mathfrak{p}_{3m},\zeta_2\xi,\frac{t}{b}\right),\right.$$
$$\left.\acute{\mathcal{E}}\left(\zeta_5\mathfrak{p}_{3m},\zeta_2\mathfrak{p}_{3m},\zeta_2\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\mathfrak{p}_{3m},\zeta_3\mathfrak{p}_{3m},\zeta_3\xi,\frac{t}{b}\right)\right\}.$$

Proceeding the limit as $m\to\infty$, we get

$$\begin{aligned}\acute{\mathcal{E}}\left(\xi,\zeta_5\xi,\xi,\frac{kt}{b}\right) &= \acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_5\xi,\zeta_6\xi,\frac{kt}{b}\right)\\ &\geq \min\left\{\acute{\mathcal{E}}\left(\zeta_1\xi,\zeta_2\xi,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_1\xi,\zeta_2\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_2\xi,\zeta_3\xi,\frac{t}{b}\right),\right.\\ &\qquad\left.\acute{\mathcal{E}}\left(\zeta_6\xi,\zeta_3\xi,\zeta_1\xi,\frac{t}{b}\right)\right\}\\ &\geq \min\left\{\acute{\mathcal{E}}\left(\xi,\zeta_5\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\zeta_5\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\zeta_5\xi,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right)\right\}.\end{aligned}$$

Thus, we get

$$\acute{\mathcal{E}}\left(\xi,\zeta_5\xi,\xi,\frac{kt}{b}\right) \geq \acute{\mathcal{E}}\left(\xi,\zeta_5\xi,\xi,\frac{kt}{b}\right).$$

So, $\zeta_5\xi = \xi = \zeta_6\xi$.
Therefore, $\zeta_1\xi = \zeta_4\xi = \zeta_2\xi = \zeta_5\xi = \zeta_3\xi = \zeta_6\xi = \xi$.

Hence, ξ is a common fixed point of the six self-mappings $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ and ζ_6.

Uniqueness: To prove uniqueness of fixed point, let u_o be another fixed point of the six self-mappings $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ and ζ_6. Then from eq. (2.2), we have

$$\acute{\mathcal{E}}\left(\xi,u_o,\xi,\frac{kt}{b}\right)=\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_5u_o,\zeta_6\xi,\frac{kt}{b}\right)$$
$$\geq \min\left\{\acute{\mathcal{E}}\left(\zeta_1\xi,\zeta_2u_o,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_1\xi,\zeta_2u_o,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_4\xi,\zeta_2u_o,\zeta_3\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\zeta_6\xi,\zeta_3\xi,\zeta_1\xi,\frac{t}{b}\right)\right\}.$$

Therefore,

$$\acute{\mathcal{E}}\left(\xi,u_o,\xi,\frac{kt}{b}\right)\geq \min\left\{\acute{\mathcal{E}}\left(\xi,u_o,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,u_o,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,u_o,\xi,\frac{t}{b}\right),\acute{\mathcal{E}}\left(\xi,\xi,\xi,\frac{t}{b}\right)\right\}.$$

Thus,

$$\acute{\mathcal{E}}\left(\xi,u_o,\xi,\frac{kt}{b}\right)\geq \acute{\mathcal{E}}\left(\xi,u_o,\xi,\frac{t}{b}\right).$$

Hence, $u_o = \xi$. □

3. Consequences

The following results can be directly proved by making use of our main results.In 2018, Sukanya and Jose [16, 17] proved the following results:

Corollary 3.1. *Let* $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ *be a complete E-fuzzy metric space and* $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ *and* ζ_6 *are six self-maps on* $\check{\mathfrak{A}}$ *satisfying the followings:*

(i) $\{\zeta_1,\zeta_4\}$, $\{\zeta_2,\zeta_5\}$ *and* $\{\zeta_3,\zeta_6\}$ *are weakly compatible,*
(ii) $\zeta_4(\check{\mathfrak{A}})\subset\zeta_2(\check{\mathfrak{A}})$, $\zeta_5(\check{\mathfrak{A}})\subset\zeta_3(\check{\mathfrak{A}})$, $\zeta_6(\check{\mathfrak{A}})\subset\zeta_1(\check{\mathfrak{A}})$,
(iii) $\zeta_1(\check{\mathfrak{A}})$ *is complete.*

If there exists a constant $0<k<1$ *and for all* $\varpi,w,\xi\in\check{\mathfrak{A}}$:

$$\acute{\mathcal{E}}(\zeta_4\varpi,\zeta_5w,\zeta_6\xi,kt)$$
$$\geq \max\{\acute{\mathcal{E}}(\zeta_1\varpi,\zeta_2w,\zeta_3\xi,t),\acute{\mathcal{E}}(\zeta_4\varpi,\zeta_1\varpi,\zeta_2\xi,t),\acute{\mathcal{E}}(\zeta_5\varpi,\zeta_2\varpi,\zeta_2\xi,t),\acute{\mathcal{E}}(\zeta_6\varpi,\zeta_3\varpi,\zeta_3\xi,t)\}. \tag{3.1}$$

Then the six self-maps $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ *and* ζ_6 *hava a unique common fixed point.*

Proof. By taking $b=1$ in Theorem 2.13, one can prove the result easily. □

Corollary 3.2. *Let* $(\check{\mathfrak{A}},\acute{\mathcal{E}},\widehat{\mathfrak{S}})$ *be a complete E-fuzzy metric space and* $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ *and* ζ_6 *are six self-maps on* $\check{\mathfrak{A}}$ *satisfying the followings:*

(i) $\{\zeta_1,\zeta_4\}$, $\{\zeta_2,\zeta_5\}$ *and* $\{\zeta_3,\zeta_6\}$ *are weakly compatible,*
(ii) $\zeta_4(\check{\mathfrak{A}})\subset\zeta_2(\check{\mathfrak{A}})$, $\zeta_5(\check{\mathfrak{A}})\subset\zeta_3(\check{\mathfrak{A}})$, $\zeta_6(\check{\mathfrak{A}})\subset\zeta_1(\check{\mathfrak{A}})$,
(iii) $\zeta_1(\check{\mathfrak{A}})$ *is complete.*

If there exists a constant $0<k<1$ *and for all* $\varpi,w,\xi\in\check{\mathfrak{A}}$:

$$\acute{\mathcal{E}}(\zeta_4\varpi,\zeta_5w,\zeta_6\xi,kt)$$
$$\geq \min\{\acute{\mathcal{E}}(\zeta_1\varpi,\zeta_2w,\zeta_3\xi,t),\acute{\mathcal{E}}(\zeta_4\varpi,\zeta_1\varpi,\zeta_2\xi,t),\acute{\mathcal{E}}(\zeta_4\varpi,\zeta_2\varpi,\zeta_3\xi,t),\acute{\mathcal{E}}(\zeta_6\varpi,\zeta_3\varpi,\zeta_1\xi,t)\}. \tag{3.2}$$

Then the six self-maps $\zeta_1,\zeta_2,\zeta_3,\zeta_4,\zeta_5$ *and* ζ_6 *hava a unique common fixed point.*

Proof. By taking $b = 1$ in Theorem 2.14, one can prove the result easily. □

4. Conclusion

Fixed point theory has many applications in several branches of science such as game theory, nonlinear programming, economics, theory of differential equations, etc. In this paper, we introduce a new notion of generalized fuzzy metric space as generalized E-fuzzy b-metric space and prove some common and unique fixed point theorems for three pairs of self-maps under weakly compatible condition. Our result presented in this paper generalize and improve some known result in various generalized fuzzy metric space.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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