Communications in Mathematics and Applications
Vol. 15, No. 3, pp. 1223–1230, 2024
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v15i3.2661

http://www.rgnpublications.com

Research Article

Rainbow Coloring of Magic Fuzzy Random Graphs

R. Abirami* and N. Sarala

Department of Mathematics, A.D.M. College for Women (Autonomous) (affiliated to Bharathidasan University), Nagapattinam, Tamil Nadu, India
***Corresponding author:** abiramielanchezhiyan@gmail.com

Received: April 5, 2024 **Accepted:** May 11, 2024

Abstract. This study introduces the concept of rainbow coloring to magic fuzzy random graphs, which combine the properties of fuzzy random graphs and magic graphs. A comprehensive study of rainbow coloring is presented in this study, which explores fundamental properties and computational issues. Also, it explores the role of rainbow coloring in magic fuzzy random graph connectivity and robustness, provides insights into the interaction between graph coloring and structure. Rainbow coloring is inherently theoretical and this finding contributes to our understanding of network design and distributed systems as well.

Keywords. Fuzzy random graph, Labeling of a fuzzy random graph, Magic labeling of a fuzzy random graph, Rainbow coloring

Mathematics Subject Classification (2020). 03E72, 03B52

1. Introduction

Rainbow magic coloring of fuzzy random graphs involves assigning colors to the vertices of a graph where the edges are fuzzy, meaning they have degrees of membership. The goal is to use a rainbow color set (no repeated colors) such that adjacent edges have distinct colors and the fuzzy nature accommodates uncertainty in edge connections. This concept combines graph theory, fuzzy graph and coloring algorithms to address uncertainty in network relationships.

Kauffmann [10] defined a fuzzy graph in 1973, drawing on Zadeh's fuzzy relation [16]. Azriel Rosenfeld [11] wrote one of the seminal works on fuzzy graph theory. Concepts including pathways, connectivity, bridges, cut vertices, forests, trees were introduced and studied by Rosenfeld [11].

The crisp graph's generalization is known as a fuzzy graph. As a result, it makes sense that many of its attributes resemble a crisp graph while simultaneously diverging in various areas. Compared to classical and fuzzy models, fuzzy labeling models provide the system with increased accuracy, adaptability, and compatibility. Fuzzy labeling is a notion that was first presented by Gani *et al.* [7,8].

In 1964, Sunitha and Vijayakumar [14] presented the magic graph notation for the first time. Regardless of the vertex selected, they characterised a graph as magical if its edges are labeled inside the real number range and the total of the labels surrounding any given vertex equals a constant. Fathalian *et al.* [4] introduces the fuzzy magic labeling of simple graphs in 2019. Later, Ajay and Chellamani [1] and Sudha *et al.* [13] discussed the fuzzy magic labeling of Neutrosophic path and star graphs.

Furthermore, the authors Femila and Asha [1] introduced the fuzzy labeling and fuzzy magic labeling for Hamiltonian graphs. Chartrand *et al.* [5] introduced the idea of rainbow coloring. When two edges of a path have different colors, it's referred to as a rainbow. Moreover, rainbow coloring [2,6], rainbow connection numbers [15], rainbow hamiliton path [9] are described in later studies.

2. Preliminaries

Definition 2.1 ([12]). Let $\mathbb{V}_{\mathcal{FR}} = \{v_i, i = 1,2,\dots,n\}$ be a set of n vertices has $n(n-1)/2$ possible edges $\mathbb{E}_{\mathcal{FR}}$ between them. Then, there are two sets of edges,

$$\mathbb{E}_{\mathcal{F}} = \{(v_i,v_j)/1 = v_i < v_j = n; i,j = 1,2,\dots,n \text{ and } (v_i,v_j) \text{ are fuzzy edges}\},$$

$$\mathbb{E}_{\mathcal{R}} = \{(v_i,v_j)/1 = v_i < v_j = n; i,j = 1,2,\dots,n \text{ and } (v_i,v_j) \text{ are random edges}\}$$

that are disjoint. Consider the mapping

$$\varphi_{\mathcal{G}} : \mathbb{V}_{\mathcal{FR}} \to [0,1] v_i \to \varphi_{\mathcal{G}}(v_i)$$

$$\psi_{\mathcal{G}} : \mathbb{V}_{\mathcal{FR}} \times \mathbb{V}_{\mathcal{FR}} \to [0,1]_P \times [0,1]_{f_A}$$

$$(v_i,v_j) \to \psi_{\mathcal{G}}(v_i,v_j) = (P(v_i,v_j), f_A(v_i,v_j))$$

with $P(v_i,v_j) = 0$ if and only if $f_A(v_i,v_j) = 0$. Then, the quartet $\mathbb{G}_{\mathcal{FR}} = (\mathbb{V}_{\mathcal{FR}}, \mathbb{E}_{\mathcal{FR}})$ is called a fuzzy random graph if $\psi_{\mathcal{G}}(v_i,v_j) = \min\{\varphi_{\mathcal{G}}(v_i), \varphi_{\mathcal{G}}(v_j)\}$, where (v_i,v_j) corresponds to the edge between v_i and v_j, $P(v_i,v_j)$ and $f_A(v_i,v_j)$ represents the probability of the edge (v_i,v_j) and the membership function for the edge (v_i,v_j) within the fuzzy set A in X, respectively.

Definition 2.2 ([12]). In a fuzzy random graph $\mathbb{G}_{\mathcal{FR}} = (\mathbb{V}_{\mathcal{FR}}, \mathbb{E}_{\mathcal{FR}})$, a path P is an array of different vertices, $u_0, u_1, \dots u_m$ (potentially excluding u_0 and u_m) such that $\mu(u_{i-1}u_i) > 0$, $i = 1,2,\dots,m$. The path's length is denoted by m in that case. The edges of the path are the consecutive pairings. If $u_0 = u_m$ and $m = 3$ then P is a cycle and two vertices are said to be connected when a path connects them. The length of the shortest path connecting the vertices that are furthest apart is known as the diameter of a fuzzy random graph.

Definition 2.3 ([12]). In a fuzzy random graph $\mathbb{G}_{\mathcal{FR}} = (\mathbb{V}_{\mathcal{FR}}, \mathbb{E}_{\mathcal{FR}})$, the degree of a fuzzy random vertex v_i is defined as

$$d_{\mathbb{G}_{\mathcal{FR}}}(v_i) = \sum_{v_i \neq v_j \in E_F} \psi_{\mathcal{G}}(v_i,v_j) + \sum_{v_i \neq v_j \in E_R} nP$$

for $(v_i,v_j)\in\mathbb{E}_{\mathcal{FR}}$ and $\psi_{\mathcal{G}}(v_i,v_j)=0$ for (v_i,v_j) not in $\mathbb{E}_{\mathcal{FR}}$, where $\psi_{\mathcal{G}}(v_i,v_j)=P$,

$$d_{\mathbb{G}_{\mathcal{FR}}}(v_i)=\sum_{v_i,v_j\in E_F}\psi_{\mathcal{G}}(v_i,v_j)+\sum_{v_i,v_j\in E_R}n\psi_{\mathcal{G}}(v_i,v_j).$$

Definition 2.4 ([12])**.** The minimal degree of vertex, $\delta(\mathbb{G}_{\mathcal{FR}})=\min\{d_{\mathbb{G}_{\mathcal{FR}}}(v_i,v_j);(v_i,v_j)\in\mathbb{E}_{\mathcal{FR}}\}$. The maximal degree of vertex, $\Delta(\mathbb{G}_{\mathcal{FR}})=\max\{d_{\mathbb{G}_{\mathcal{FR}}}(v_i,v_j);(v_i,v_j)\in\mathbb{E}_{\mathcal{FR}}\}$.

Definition 2.5 ([12])**.** The order $\mathcal{O}(\mathbb{G}_{\mathcal{FR}})$ and size $\mathcal{S}(\mathbb{G}_{\mathcal{FR}})$ of fuzzy random graph $\mathbb{G}_{\mathcal{FR}}=(\mathbb{V}_{\mathcal{FR}},\mathbb{E}_{\mathcal{FR}})$ is defined as, $\mathcal{O}(\mathbb{G}_{\mathcal{FR}})=\sum\varphi_{\mathcal{G}}(v_i)$, $v_i\in\mathbb{V}_{\mathcal{FR}}$; $\mathcal{S}(\mathbb{G}_{\mathcal{FR}})=\sum\psi_{\mathcal{G}}(v_i,v_j)$, $(v_i,v_j)\in\mathbb{E}_{\mathcal{FR}}$.

Definition 2.6 ([12])**.** The fuzzy random graph $\mathbb{G}_{\mathcal{FR}}=(\mathbb{V}_{\mathcal{FR}},\mathbb{E}_{\mathcal{FR}})$ is called a complete fuzzy random graph if $\psi_{\mathcal{G}}(v_i,v_j)=\min\{\varphi_{\mathcal{G}}(v_i),\varphi_{\mathcal{G}}(v_j)\}$, for all $(v_i,v_j)\in\mathbb{E}_{\mathcal{FR}}$.

3. Labeling and Coloring of Fuzzy Random Graphs

Definition 3.1. A fuzzy random graph labeling is the process of providing a graph's edges and vertices values. When $\varphi_{\mathcal{G}}:\mathbb{V}_{\mathcal{FR}}\to[0,1]$ and $\psi_{\mathcal{G}}:\mathbb{V}_{\mathcal{FR}}\times\mathbb{V}_{\mathcal{FR}}\to[0,1]$ are bijective, the edges and vertices' membership values are unique, and for every $(v_i,v_j)\in\mathbb{E}_{\mathcal{FR}}$, $\psi_{\mathcal{G}}(v_i,v_j)=\min\{\varphi_{\mathcal{G}}(v_i),\varphi_{\mathcal{G}}(v_j)\}$, then $\mathbb{G}_{\mathcal{FR}}=(\mathbb{V}_{\mathcal{FR}},\mathbb{E}_{\mathcal{FR}})$ is considered a fuzzy random graph labeling.

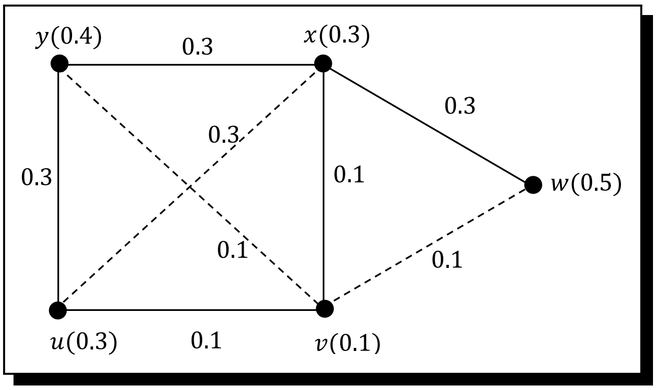

Figure 1. Labeling of fuzzy random graph

Definition 3.2. In a fuzzy random graph $\mathbb{G}_{\mathcal{FR}}=(\mathbb{V}_{\mathcal{FR}},\mathbb{E}_{\mathcal{FR}})$, a mapping $C_{\mathcal{G}}:\mathbb{V}_{\mathcal{FR}}\to\mathbb{N}$ that identifies $C_{\mathcal{G}}(v_i)$ as the color of vertex $v_i\in\mathbb{V}_{\mathcal{FR}}$ and ensures that no two adjacent vertices can have the same color is known as a coloring function, i.e., $C_{\mathcal{G}}(v_i)\neq C_{\mathcal{G}}(v_j)$, for all $(v_i,v_j)\in\mathbb{E}_{\mathcal{FR}}$. Similarly, for all edges in $\mathbb{G}_{\mathcal{FR}}=(\mathbb{V}_{\mathcal{FR}},\mathbb{E}_{\mathcal{FR}})$ with mapping $C_{\mathcal{G}}:\mathbb{E}_{\mathcal{FR}}\to\mathbb{N}$ such that every adjacent edges have distinct color.

Definition 3.3. Let $\mathbb{G}_{\mathcal{FR}}=(\mathbb{V}_{\mathcal{FR}},\mathbb{E}_{\mathcal{FR}})$ be a fuzzy random graph of order n. If any pair of vertices in $\mathbb{G}_{\mathcal{FR}}$ is connected by a rainbow path that is, a path along which no two edges are colored the same, then an edge coloring of $\mathbb{G}_{\mathcal{FR}}$ is referred to as a rainbow coloring as in Figure 2.

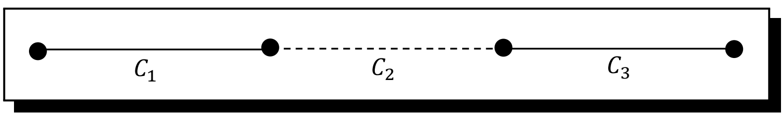

Figure 2. Rainbow coloring of fuzzy random graph

When two edges of a path have different colors, it can be referred to a rainbow. A path is known as a rainbow path in a vertex colored fuzzy random graph if its internal vertices vary in colors. The bare minimum of colors needed to accomplish this is known as the rainbow connection number, and it is represented as $\mathfrak{rc}(\mathbb{G}_{\mathcal{FR}})$.

4. Rainbow Coloring of Magic Fuzzy Random Graph

Definition 4.1. Let $\mathbb{G}_{\mathcal{FR}}$ be a connected fuzzy random graph which contains rainbow path. A labeling of graph $\mathbb{G}_{\mathcal{FR}}$ is a bijection $f:\mathbb{V}_{\mathcal{FR}}\to[0,1]$. The bijection f is called rainbow magic vertex labeling if for every vertex $v_i\in\mathbb{V}_{\mathcal{FR}}$, $\varphi_{\mathcal{G}}(v_i)+\sum\psi_{\mathcal{G}}(v_i,v_j)=\mathcal{M}_{\mathcal{G}}(v_i)$

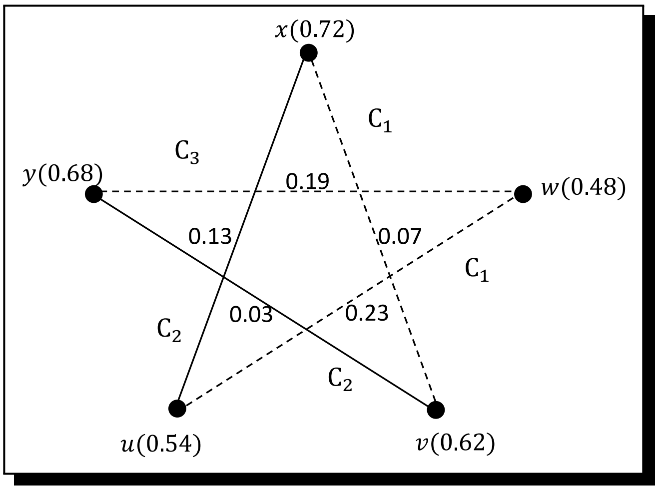

Figure 3. Rainbow magic vertex labeling of fuzzy random graph

In Figure 3, $\mathcal{M}_{\mathcal{G}}(v_i)=0.9$ and $\mathfrak{rc}(\mathbb{G}_{\mathcal{FR}})=3$.

Definition 4.2. Let $\mathbb{G}_{\mathcal{FR}}=(\mathbb{V}_{\mathcal{FR}},\mathbb{E}_{\mathcal{FR}})$ be a connected fuzzy random graph with rainbow path. A bijection $f:\mathbb{E}_{\mathcal{FR}}\to[0,1]$ labels graph $\mathbb{G}_{\mathcal{FR}}$. The bijection f is called rainbow magic edge labeling if for every edge $(v_i,v_j)\in\mathbb{E}_{\mathcal{FR}}$, $\varphi_{\mathcal{G}}(v_i)+\varphi_{\mathcal{G}}(v_j)+\psi_{\mathcal{G}}(v_i,v_j)=\mathcal{M}_{\mathcal{G}}(e)$.

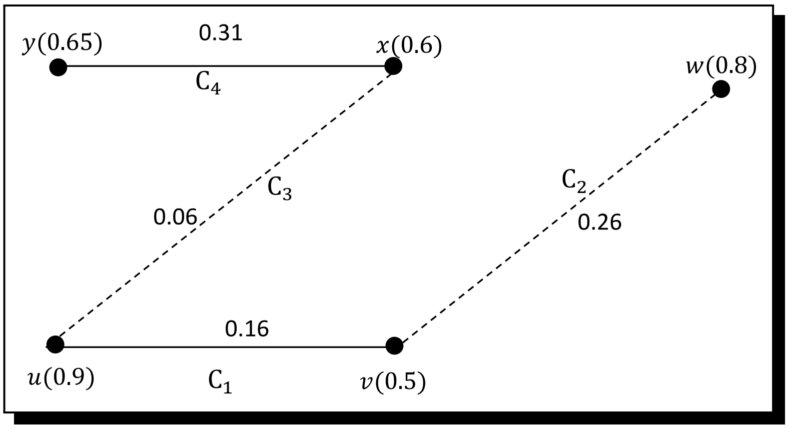

Figure 4. Rainbow magic edge labeling of fuzzy random graph

In Figure 4, $\mathcal{M}_{\mathcal{G}}(e)=1.56$ and $\mathfrak{rc}(\mathbb{G}_{\mathcal{FR}})=4$.

Definition 4.3. A fuzzy random graph which is both rainbow vertex magic $\mathcal{M}_{\mathcal{G}}(v_i)$ and rainbow edge magic $\mathcal{M}_{\mathcal{G}}(e)$ is known as rainbow magic fuzzy random graph with magic constant $\mathcal{M}_{\mathcal{G}}$. (Need not to be $\mathcal{M}_{\mathcal{G}}(v_i) = \mathcal{M}_{\mathcal{G}}(e)$.)

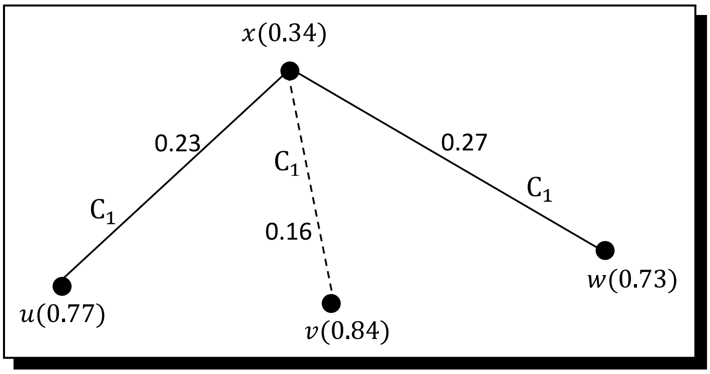

Figure 5. Rainbow magic fuzzy random graph

In Figure 5, $\mathcal{M}_{\mathcal{G}}(u) = 1.0$, $\mathcal{M}_{\mathcal{G}}(e) = 1.34$, $\mathfrak{rc}(\mathbb{G}) = 1$.

5. Properties of Rainbow Coloring of Magic Fuzzy Random Graphs

Theorem 5.1. *For magic fuzzy random graph $\mathbb{G}_{\mathcal{FR}}$, $\delta(\mathbb{G}_{\mathcal{FR}}) = \Delta(\mathbb{G}_{\mathcal{FR}}) = \mathcal{S}(\mathbb{G}_{\mathcal{FR}}) = \mathcal{O}(\mathbb{G}_{\mathcal{FR}})$ holds.*

Proof. Let $\mathbb{G}_{\mathcal{FR}} = (\mathbb{V}_{\mathcal{FR}}, \mathbb{E}_{\mathcal{FR}})$ be a magic fuzzy random graph. As the minimal and maximal degrees of vertices in $\mathbb{G}_{\mathcal{FR}}$ are represented by $\delta(\mathbb{G}_{\mathcal{FR}})$ and $\Delta(\mathbb{G}_{\mathcal{FR}})$ respectively,

$$\delta(\mathbb{G}_{\mathcal{FR}}) = \Delta(\mathbb{G}_{\mathcal{FR}}). \tag{5.1}$$

Since, order of $\mathbb{G}$,

$$\mathcal{O}(\mathbb{G}_{\mathcal{FR}}) = \sum \varphi_{\mathcal{G}}(v_i), \quad v_i \in \mathbb{V}_{\mathcal{FR}}$$

and size of $\mathbb{G}$,

$$\mathcal{S}(\mathbb{G}_{\mathcal{FR}}) = \sum \psi_{\mathcal{G}}((v_i, v_j), \quad (v_i, v_j) \in \mathbb{E}_{\mathcal{FR}}.$$

By,

$$\max\{d_{\mathbb{G}_{\mathcal{FR}}}(v_i, v_j); (v_i, v_j) \in \mathbb{E}_{\mathcal{FR}}\} = \sum \psi_{\mathcal{G}}(v_i, v_j)$$

implies that

$$\Delta(\mathbb{G}_{\mathcal{FR}}) = \mathcal{S}(\mathbb{G}_{\mathcal{FR}}). \tag{5.2}$$

Also,

$$\min\{d_{\mathbb{G}_{\mathcal{FR}}}(v_i, v_j); (v_i, v_j) \in \mathbb{E}_{\mathcal{FR}}\} = \sum \varphi_{\mathcal{G}}(v_i)$$

implies that

$$\mathcal{S}(\mathbb{G}_{\mathcal{FR}}) = \mathcal{O}(\mathbb{G}_{\mathcal{FR}}). \tag{5.3}$$

Hence from (5.1), (5.2) and (5.3), we get

$$\delta(\mathbb{G}_{\mathcal{FR}}) \leq \Delta(\mathbb{G}_{\mathcal{FR}}) \leq \mathcal{S}(\mathbb{G}_{\mathcal{FR}}) \leq \mathcal{O}(\mathbb{G}_{\mathcal{FR}}).$$ □

Theorem 5.2. *Let $\mathbb{G}_{\mathcal{FR}} = (\mathbb{V}_{\mathcal{FR}}, \mathbb{E}_{\mathcal{FR}})$ be a magic fuzzy random graph. Then*

(i) $\sum \varphi_{\mathcal{G}}(v_i) < |\mathbb{V}_{\mathcal{FR}}|$, *where* $v_i \in \mathbb{V}_{\mathcal{FR}}$,

(ii) $\sum \psi_{\mathcal{G}}(v_i, v_j) < |\mathbb{E}_{\mathcal{FR}}|$, *where* $(v_i, v_j) \in \mathbb{E}_{\mathcal{FR}}$,

(iii) $\sum \varphi_{\mathcal{G}}(v_i) < \sum \psi_{\mathcal{G}}(v_i, v_j)$, *where* $v_i \in \mathbb{V}_{\mathcal{FR}}$, $(v_i, v_j) \in \mathbb{E}_{\mathcal{FR}}$.

Proof. For (i) and (ii), since for all $v_i \in \mathbb{V}_{\mathcal{FR}}$ and $(v_i, v_j) \in \mathbb{E}_{\mathcal{FR}}$, $\varphi_{\mathcal{G}}(v_i) < 1$, $\psi_{\mathcal{G}}(v_i, v_j) < 1$, we get

$$\sum \varphi_{\mathcal{G}}(v_i) < |\mathbb{V}_{\mathcal{FR}}|$$

For (iii), for all $v_i \in \mathbb{V}_{\mathcal{FR}}$ and $(v_i, v_j) \in \mathbb{E}_{\mathcal{FR}}$, $\psi_{\mathcal{G}}(v_i, v_j) < \varphi_{\mathcal{G}}(v_i) \wedge \varphi_{\mathcal{G}}(v_j) = \varphi_{\mathcal{G}}(v_i)$,

$$\sum \psi_{\mathcal{G}}(v_i, v_j) < \sum \varphi_{\mathcal{G}}(v_i).$$ □

Result 5.1. (i) *Contrary to general perception, not all magic fuzzy random graphs are fuzzy labeling graphs.*

(ii) *Let $\mathbb{G}_{\mathcal{FR}} = (\mathbb{V}_{\mathcal{FR}}, \mathbb{E}_{\mathcal{FR}})$ be a connected fuzzy random graph with rainbow coloring, then*

(a) $\mathfrak{rc}(\mathbb{G}_{\mathcal{FR}}) = 1$ *if and only if* $\mathbb{G}_{\mathcal{FR}}$ *is a tree.*

(b) $1 = \mathfrak{rc}(\mathbb{G}_{\mathcal{FR}}) = n - 1$, *where n is a number of vertices in* $\mathbb{G}_{\mathcal{FR}}$.

(c) $\mathfrak{rc}(\mathbb{G}_{\mathcal{FR}}) = \mathrm{diam}(\mathbb{G}_{\mathcal{FR}})$.

(iii) *Every properly edge colored complete fuzzy random graph has a rainbow Hamilton path.*

(iv) *Every connected fuzzy random graph with minimal degree atleast two has a rainbow connection number equal to its diameter*

Theorem 5.3. *The complete fuzzy random graph with magic labeling has rainbow connection number one.*

Proof. Let $\mathbb{G}_{\mathcal{FR}} = (\mathbb{V}_{\mathcal{FR}}, \mathbb{E}_{\mathcal{FR}})$ be a complete magic fuzzy random graph with n vertices, nc_2 edges and magic constant $\mathcal{M}_{\mathcal{G}}$. Then, every pair of vertices has an edge between them. It entails that there must be one diameter. A rainbow path of length one therefore exists. That is to say, one color is necessary at most to color the rainbow path, $\mathfrak{rc}(\mathbb{G}) = 1$. Hence the complete fuzzy random graph with magic labeling has rainbow connection number one. □

6. Applications

Rainbow coloring of magic fuzzy random graphs has several applications in various fields.

(i) *Network security*: Rainbow coloring can be applied to secure communication networks by ensuring that different channels or paths in the network have diverse sets of colors, making it harder for attackers to intercept or disrupt communication.

(ii) *Traffic management*: In transportation networks, rainbow coloring of magic fuzzy random graphs can help optimize traffic flow by assigning different routes or lanes distinct colors, aiding in congestion management and route planning.

(iii) *Wireless sensor networks*: Rainbow coloring techniques can be used in wireless sensor networks to assign unique colors to sensor nodes, facilitating efficient data routing, localization and energy management.

(iv) *Social networks*: Rainbow coloring of magic fuzzy random graphs can be utilized in social network analysis to identify and analyze different communities or clusters within a network, based on the color assignments of nodes representing individuals or groups.

(v) *Bioinformatics*: Rainbow coloring of magic fuzzy random graphs can help identify functional modules or pathways in biological networks, such as gene regulatory networks or protein-protein interaction networks. This is achieved by assigning colors to vertices based on their biological features or activities.

(vi) *Optical networks*: Rainbow coloring techniques can be applied in optical networks to allocate wavelengths to different communication channels, ensuring efficient utilization of the optical spectrum and minimizing interference.

(vii) *Graph theory and algorithms*: Rainbow coloring of fuzzy random graphs contributes to the development of novel graph coloring algorithms and combinatorial optimization techniques, advancing research in theoretical computer science and algorithm design.

(viii) *Parallel and distributed computing*: Rainbow coloring of magic fuzzy random graphs can aid in task scheduling and load balancing in parallel and distributed computing systems by assigning distinct colors to computing resources or tasks, optimizing resource utilization and reducing execution time.

These applications demonstrate the versatility and significance of rainbow coloring techniques, particularly in the context of magic fuzzy random graphs across various domains.

7. Conclusion

This study introduced the Magic labeling, including edge and vertex magic labeling was defined for a fuzzy random graph. Also, for magic labeling of fuzzy random graph, the rainbow coloring is defined with rainbow connection number which is denoted by $\mathcal{M}_{\mathcal{G}}$. Furthermore, some theorems and results relating to magic labeling and rainbow coloring of fuzzy random graphs are demonstrated together with their applications.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] D. Ajay and P. Chellamani, Fuzzy magic labeling of Neutrosophic path and star graph, *Advances in Mathematics: Scientific Journal* **9**(8) (2020), 6059 – 6070.

[2] I. A. Arputhamary and M. H. Mercy, Rainbow coloring of some ladder graphs, *Global Journal of Pure and Applied Mathematics* **11**(3) (2015), 1231 – 1240.

[3] G. Chartrand, G. L. Johns, K. A. Mckeon and P. Zhang, Rainbow connection in graphs, *Mathematica Bohemica* **133**(1) (2008), 85 – 98, DOI: 10.21136/mb.2008.133947.

[4] M. Fathalian, R. A. Borzooei and M. Hamidi, Fuzzy magic labeling of simple graphs, *Journal of Applied Mathematics and Computing* **60** (2019), 369 – 385, DOI: 10.1007/s12190-018-01218-x.

[5] L. T. C. M. Femila and S. Asha, Hamiltonian fuzzy labeling and Hamiltonian fuzzy magic labeling of graphs, *Malaya Journal of Matematik* **9**(1) (2021), 782 – 788, DOI: 10.26637/MJM0901/0138.

[6] F. Fujie-Okamoto, K. Kolasinski, J. Lin and P. Zhang, Vertex rainbow colorings of graphs, *Discussiones Mathematicae Graph Theory* **32**(2) (2012), 63 – 80, DOI: 10.7151/dmgt.1586.

[7] A. N. Gani and D. Rajalaxmi, A note on fuzzy labeling, *International Journal of Fuzzy Mathematical Archive* **4**(2) (2014), 88 – 95.

[8] A. N. Gani, M. Akram and D. Rajalaxmi, Novel properties of fuzzy labeling graphs, *Journal of Mathematics* **2014**(1) (2014), Article ID 375135, DOI: 10.1155/2014/375135.

[9] S. Gould, T. Kelly, D. Kühan and D. Osthus, Almost all optimally coloured complete graphs contain a rainbow Hamilton path, *Journal of Combinatorial Theory, Series B* **156** (2022), 57 – 100, DOI: 10.1016/j.jctb.2022.04.003.

[10] A. Kauffman, *Introduction à la théorie des sous-ensembles flous à l'usage des ingénieurs: Éléments théoriques de base*, Volume 1, Masson, 410 pages (1973).

[11] A. Rosenfeld, Fuzzy graphs, in: *Fuzzy Sets and their Applications to Cognitive and Decision Processes*, Proceedings of the US–Japan Seminar on Fuzzy Sets and their Applications, Held at the University of California, Berkeley, California, July 1–4, 1974, Academic Press, (1975), pp. 77 – 95, DOI: 10.1016/B978-0-12-775260-0.50008-6.

[12] N. Sarala and R. Abirami, Fuzzy random graphs, *Bulletin for Technology and History Journal* **23**(10) (2023), 8 – 19, DOI: 10.37326/bthnlv22.8/1312.

[13] S. Sudha, M. Lathamaheswari, S. Broumi and F. Smaradhche, Neutrosophic fuzzy magic labeling graph with its application in academic performance of the students, *Neutrosophic Sets and Systems* **60** (2023), Article 8, URL: https://digitalrepository.unm.edu/nss_journal/vol60/iss1/8.

[14] M. S. Sunitha and A. Vijayakumar, A characterization of fuzzy trees, *Information Sciences* **113**(3-4) (1999), 293 – 300, DOI: 10.1016/S0020-0255(98)10066-X.

[15] B. Suthar, D. Parmar, D. Patel, S. Sharma and Y. Modi, Rainbow connection number of some graphs, *Webology* **18**(6) (2021), 2863 – 2875.

[16] L. A. Zadeh, Fuzzy sets as a basis for a theory of possibility, *Fuzzy Sets and Systems* **1**(1) (1978), 3 – 28, DOI: 10.1016/0165-0114(78)90029-5.