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Research Article

Analyzing System Reliability Across Configurational Diversity and Component Variability: Insights From Reliability Block Diagrams and Event Space Method

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Abstract. In this study, we systematically investigated the reliability of a complex system configured in multiple ways with diverse components, employing both Reliability Block Diagram and Event Space Method. Our approach involved evaluating component reliability in series and parallel arrangements, with a subsequent consolidation of these assessments. The primary aim was to set a benchmark for quantifying system reliability across various configurations and conditions. The study outcomes are illustrated using graphs and tables, facilitating a comparative analysis of system reliability across diverse setups.

Keywords. Reliability, Failure rate, Event space method, Reliability block diagram

Mathematics Subject Classification (2020). 90B25, 62N05

1. Introduction

Reliability analysis is a method used to assess and quantify the dependability, consistency, and performance of a system, component, or process. It is an essential concept in various fields, including engineering, manufacturing, economics, and even in everyday life. Reliability analysis aims to understand how well a system or component functions over time, especially in terms of meeting certain performance or quality criteria. Reliability analysis quantifies the effectiveness of a system's performance under specific conditions. To gauge this effectiveness, one must define the system's performance requirements and expected performance outcomes.

Various techniques are available for reliability analysis, including Fault Tree Analysis, Reliability Block Diagrams and Event Tree Method. These methods can help to determine the probability of success and failure. The techniques aim to capture the system's overall reliability by analyzing the failure patterns of its components. Fault Tree Analysis presents a graphical model that aids in analyzing the factors that may lead to system failure. On the other hand, Reliability Block Diagrams help us to create a model of the success relationships present in the complex systems. Event Tree Method provide a detailed overview of all possible operating states including success and failure (Abdelghany *et al*. [2]).

Reliability Block Diagram is a graphical representation of a system's components and connectors. It can be used to determine the overall system reliability based on the reliability of its components. Reliability Block Diagram consists of one or more paths that represent successful system operation. Each path is constructed of blocks/components and lines/connectors. Computational elements are represented by blocks, and lines indicate essential paths to success. If any path results in successful execution, the whole system is considered successful. If all the paths fail, then the overall system also fails (Abd-Allah [1]).

A system is said to be in series, in terms of reliability, if it fails when one or more of its components fail. For a system with multiple elements in series, the overall availability is equal to the product of the availability of each component. A system is said to be in parallel, when one or more of its components fail, the system remains operational. Active redundancy is achieved when parallel elements perform the same functions and work together (Bourouni [7]).

Reliability Block Diagram construction can follow any of three basic component connection patterns: *Series, Active Redundancy or Standby Redundancy*. In a series connection, the entire system depends on all components functioning properly. While, in active redundancy, at least one of the redundant stages must be fully operational. The components in an active redundancy can be connected in a parallel or series-parallel structure, while standby redundancy does not require all components to be active. In order to build the Reliability Block Diagram of a system, three types of information are necessary: the functional interaction of the system's components needs to be understood, the reliability of each component must be determined and the mission times at which reliability is desired need to be identified. Then design engineers utilize this information to determine the appropriate Reliability Block Diagram configuration (series, parallel or series-parallel) to determine the overall reliability of the system (Ahmed [4]).

Reliability is known as the science of failures. By definition, "*Reliability is the ability of a device to perform a required function under specified conditions during a given period*" (Pagès and Gondran [12]), or, "*Reliability is defined as the probability of a component performing its desired task over certain interval of time t*" (Joshi *et al*. [11]). Mathematically it is represented as

$$\mathbb{R}(t) = 1 - \mathbb{P}(x \le t) = \mathbb{P}(x > t). \tag{1.1}$$

Reliability analysis by using the method of Event Space Method is the most well-known analytic method studying the failure modes of complex systems. Event Space Method is a concept based on mutually exclusive events axiom. In this method, only those events that are mutually exclusive and lead to the system success are considered. The reliability of the system can be determined by calculating the probability of the union of all such mutually exclusive events (Joshi *et al*. [11]). The event space method involves exhaustively enumerating all conceivable logical system states. To put it differently, this method assumes that all components initially operate correctly, and then systematically evaluates their potential failures, starting with individual failures, followed by pairs of components failing, and so forth. The system's reliability is subsequently calculated by combining the outcomes of all successful scenarios.

Event Space Method is used for calculating the reliability of the complex system. In this method, we calculate the unreliability of a system by determining the probability of all the mutually exclusive events that can lead to system failure. We construct Reliability Block Diagram for system which has various configurations of components such as series, parallel and mixed structure. The blocks in the diagram represent the components and show how they are arranged and related in terms of reliability. Units that are arranged in parallel are commonly referred to as redundant units. Redundancy is very precious aspect of system design and system reliability because it is one of the methods used to improve the reliability of the system (Joshi *et al*. [11]).

2. Related Work

Joshi *et al*. [11] discussed the reliability of the complex system with different configurations including those with two, three and four components, and compared their reliability.

Abd-Allah [1] has expanded the use of the Reliability Block Diagram to software architectures. He demonstrated that various conceptual features that cut across architectural styles can have either positive or negative results on the effective architectural components modelled failure rates in Reliability Block Diagram. The assumptions of Reliability Block Diagram can also be breached by the preferences made on the conceptual features and different architectural connectors. As software systems become more complex through the use of distribution, coexistence, dynamism, and packaged middleware solutions, estimating the reliability using Reliability Block Diagram becomes more complex as well.

Bourouni [7] presented a model for a reverse osmosis plant availability based on the Reliability Block Diagram method used for failure analysis of repairable systems. Also, executed a comparison with a Fault Tree Analysis. To validate and compare different models, the data from a functioning RO plant in Kuwait was utilised. The results showed that both the methods had good agreement, however, the Reliability Block Diagram method resulted in the lowest unavailability. This difference was explained by the fact that Fault Tree Analysis does not take into consideration on account redundancy and standby configurations. The results indicate the plant experiences a very minimal overall unavailability, reflecting very good performance. The RO modules and high-pressure pumps have higher unavailability rates in the plant, so it is necessary to pay close attention to these components to improve the availability of the entire system. After comparing the Fault Tree Analysis and the Reliability Block Diagram method is more suitable for availability assessment as it can handle complex configurations such as redundancy and standby.

3. Objective

The process of evaluating reliability holds significant importance in the realm of system design and analysis, especially as systems become increasingly complex. As complexity rises, whether due to system expansion or inherent intricacies, a host of factors come into play that can substantially impact reliability and availability. When dealing with highly complex systems or those undergoing expansion or modification, several critical considerations become prominent. First, common-cause failures may emerge, where multiple components fail due to a shared root cause. Component interactions and interdependencies can become more intricate, potentially leading to a cascade of failures if one component malfunctions. Additionally, as systems grow in complexity, interdependence between components, or the specific sequence in which they operate, can become a critical factor in assessing reliability. In this paper, our goal is to create systems with different configurations and evaluate their overall reliability using Reliability Block Diagram and Event Space Method. This method assesses the reliability of individual components within each system configuration to determine the overall reliability of the systems.

4. Reliability of the System by Using Reliability Block Diagram and Event Space Method

4.1 Calculation of Reliability of the System Having *Four* Components in Series-Parallel Mixed Configuration

Configuration 4.1.1. In this configuration, Component 1 and Component 2 are connected in parallel and Component 3 and Component 4 are also connected in parallel, then these two parallel sub-systems of components are again connected in parallel configuration as shown in Figure 1.

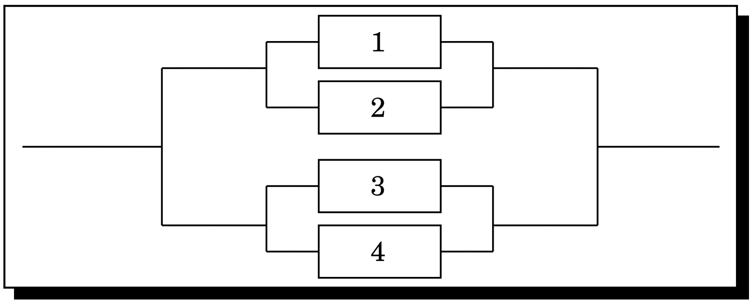


Figure 1. Reliability block diagram of the system having four components.

Let $\boldsymbol{A}$ be the Success of Component 1 event and $\boldsymbol{a}$ be the Failure of Component 1 event.
Let $\boldsymbol{B}$ be the Success of Component 2 event and $\boldsymbol{b}$ be the Failure of Component 2 event.

Let $\boldsymbol{C}$ be the Success of Component 3 event and $\boldsymbol{c}$ be the Failure of Component 3 event.
Let $\boldsymbol{D}$ be the Success of Component 4 event and $\boldsymbol{d}$ be the Failure of Component 4 event.
Also, here

x_i : Success or Failure of Component i's event

$\mathcal{P}(x_i)$: Probability of the Failure of Component i

$\mathbb{R}_s$: Reliability of the system

The mutually exclusive events of given system are:

$x_1 = ABCD$ – All components success, $x_2 = ABCd$ – Component 4 fail,

$x_3 = ABcD$ – Component 3 fail, $x_4 = ABcd$ – Component 3 and 4 fail,

$x_5 = AbCD$ – Component 2 fail, $x_6 = AbCd$ – Component 2 and 4 fail,

$x_7 = AbcD$ – Component 2 and 3 fail, $x_8 = Abcd$ – Component 2, 3 and 4 fail,

$x_9 = aBCD$ – Component 1 fail, $x_{10} = aBCd$ – Component 1 and 4 fail,

$x_{11} = aBcD$ – Component 1 and 3 fail, $x_{12} = aBcd$ – Component 1, 3 and 4 fail,

$x_{13} = abCD$ – Component 1 and 2 fail, $x_{14} = abCd$ – Component 1, 2 and 4 fail,

$x_{15} = abcD$ – Component 1, 2 and 3 fail, $x_{16} = abcd$ – All components fail.

Event of the system x_{16} only results in the system failure. Thus, the probability of the failure of the system is:

$$\mathbb{P}_f = \mathcal{P}(x_{16}).$$

Calculation of the probability of event that leads to the system failure:

$$\mathcal{P}(x_{16}) = (1-\mathcal{R}_1)\cdot(1-\mathcal{R}_2)\cdot(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4).$$

Now, equation (1.1) becomes

$$\mathbb{P}_f = (1-\mathcal{R}_1)\cdot(1-\mathcal{R}_2)\cdot(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4).$$

Reliability of the system = 1− Probability of the failure of the system,
i.e.,

$$\mathbb{R}_s = 1-\mathbb{P}_f = 1-[(1-\mathcal{R}_1)\cdot(1-\mathcal{R}_2)\cdot(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4)].$$

If we consider, Reliability of Component 1 is 96%, Reliability of Component 2 is 99%, Reliability of Component 3 is 95% and Reliability of Component 4 is 97%, i.e.,

$$\mathcal{R}_1 = 96\%,\ \ \mathcal{R}_2 = 99\%,\ \ \mathcal{R}_3 = 95\%,\ \ \mathcal{R}_4 = 97\%.$$

As the reliabilities of the components are specified for 7 days, i.e., 168 hours (assumption), then we will find the value of the Reliability of the system for 7 days as

$$\begin{aligned}\mathbb{R}_s &= 1-[(1-0.96)(1-0.99)(1-0.95)(1-0.97)]\\ &= 1-0.0000006 = 0.9999994 \cong 0.9999.\end{aligned}$$

Configuration 4.1.2. In this configuration, Component 1 and Component 2 are connected in parallel and Component 3 and Component 4 are also connected in parallel, then these two parallel sub-systems of components are connected in series as shown in Figure 2.

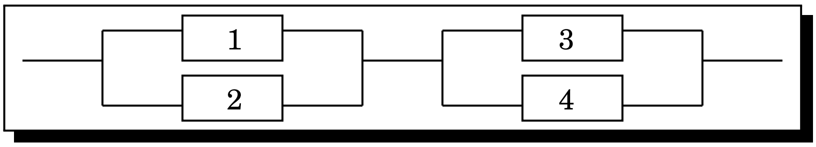


Figure 2. Reliability block diagram of the system having four components

Event of the system x_4, x_8, x_{12}, x_{13}, x_{14}, x_{15}, x_{16} only results in the system failure. Thus, the probability of the failure of the system is:

$$\mathbb{P}_f = \mathcal{P}(x_4 \cup x_8 \cup x_{12} \cup x_{13} \cup x_{14} \cup x_{15} \cup x_{16}).$$

Calculation of the probability of event that leads to the system failure:

$$\begin{aligned}
\mathcal{P}(x_4) &= \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4),\\
\mathcal{P}(x_8) &= \mathcal{R}_1 \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4),\\
\mathcal{P}(x_{12}) &= (1-\mathcal{R}_1) \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4),\\
\mathcal{P}(x_{13}) &= (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot \mathcal{R}_3 \cdot \mathcal{R}_4,\\
\mathcal{P}(x_{14}) &= (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot \mathcal{R}_3 \cdot (1-\mathcal{R}_4),\\
\mathcal{P}(x_{15}) &= (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot \mathcal{R}_4,\\
\mathcal{P}(x_{16}) &= (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4).
\end{aligned}$$

Now, after combining, equation (1.1) becomes

$$\begin{aligned}
\mathbb{P}_f &= (1-\mathcal{R}_3) \cdot [\mathcal{R}_1 \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_4) + \mathcal{R}_1 \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_4) + (1-\mathcal{R}_1) \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_4)]\\
&\quad + (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot [\mathcal{R}_3 \cdot \mathcal{R}_4 + \mathcal{R}_3 \cdot (1-\mathcal{R}_4) + (1-\mathcal{R}_3) \cdot \mathcal{R}_4 + (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4)]\\
&= (1-\mathcal{R}_3) \cdot [\mathcal{R}_1 \cdot \mathcal{R}_2 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 - \mathcal{R}_1 \cdot \mathcal{R}_2 - \mathcal{R}_1 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_2 - \mathcal{R}_1 \cdot \mathcal{R}_2\\
&\quad - \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4] + (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot [\mathcal{R}_3 \cdot \mathcal{R}_4 + \mathcal{R}_3 - \mathcal{R}_3 \cdot \mathcal{R}_4 + \mathcal{R}_4 - \mathcal{R}_3 \cdot \mathcal{R}_4\\
&\quad + 1 - \mathcal{R}_3 - \mathcal{R}_4 + \mathcal{R}_3 \cdot \mathcal{R}_4]\\
&= 1 - \mathcal{R}_1 \cdot \mathcal{R}_3 - \mathcal{R}_1 \cdot \mathcal{R}_4 - \mathcal{R}_2 \cdot \mathcal{R}_3 - \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4\\
&\quad + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4.
\end{aligned}$$

Reliability of the system = 1− Probability of the failure of the system,
i.e.,

$$\begin{aligned}
\mathbb{R}_s &= 1 - \mathbb{P}_f\\
&= 1 - [1 - \mathcal{R}_1 \cdot \mathcal{R}_3 - \mathcal{R}_1 \cdot \mathcal{R}_4 - \mathcal{R}_2 \cdot \mathcal{R}_3 - \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4\\
&\quad + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4]\\
&= \mathcal{R}_1 \cdot \mathcal{R}_3 + \mathcal{R}_1 \cdot \mathcal{R}_4 + \mathcal{R}_2 \cdot \mathcal{R}_3 + \mathcal{R}_2 \cdot \mathcal{R}_4 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 - \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 - \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4\\
&\quad + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4.
\end{aligned}$$

On more simplifying, we get

$$\mathbb{R}_s = \mathcal{R}_1\cdot\mathcal{R}_3\cdot(1-\mathcal{R}_4)+\mathcal{R}_1\cdot\mathcal{R}_4\cdot(1-\mathcal{R}_2)+\mathcal{R}_2\cdot\mathcal{R}_3\cdot(1-\mathcal{R}_1)+\mathcal{R}_2\cdot\mathcal{R}_4\cdot(1-\mathcal{R}_3)+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4.$$

If we consider, Reliability of Component 1 is 96%, Reliability of Component 2 is 99%, Reliability of Component 3 is 95% and Reliability of Component 4 is 97%, i.e.,

$$\mathcal{R}_1 = 96\%,\quad \mathcal{R}_2 = 99\%,\quad \mathcal{R}_3 = 95\%,\quad \mathcal{R}_4 = 97\%.$$

As the reliabilities of the components are specified for 7 days, i.e., 168 hours (assumption), then we will find the value of the Reliability of the system for 7 days as

$$\mathbb{R}_s = 0.02736+0.009312+0.03762+0.048015+0.8757936 = 0.9981006 \cong 0.9981.$$

Configuration 4.1.3. In this configuration, Component 1, Component 2 and Component 3 are connected in parallel and then this sub-system is connected to Component 4 in series as shown in Figure 3.

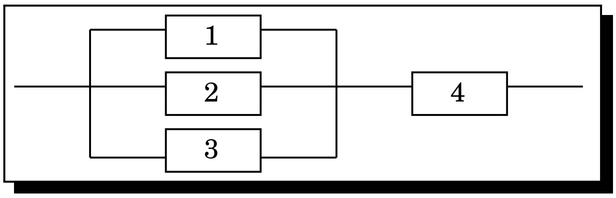


Figure 3. Reliability block diagram of the system having four components

Event of the system x_2, x_4, x_6, x_8, x_{10}, x_{12}, x_{14}, x_{15}, x_{16} only results in the system failure. Thus, the probability of the failure of the system is:

$$\mathbb{P}_f = \mathcal{P}(x_2\cup x_4\cup x_6\cup x_8\cup x_{10}\cup x_{12}\cup x_{14}\cup x_{15}\cup x_{16}).$$

Calculation of the probability of event that leads to the system failure:

$$\mathcal{P}(x_2) = \mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot(1-\mathcal{R}_4),$$
$$\mathcal{P}(x_4) = \mathcal{R}_1\cdot\mathcal{R}_2\cdot(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4),$$
$$\mathcal{P}(x_6) = \mathcal{R}_1\cdot(1-\mathcal{R}_2)\cdot\mathcal{R}_3\cdot(1-\mathcal{R}_4),$$
$$\mathcal{P}(x_8) = \mathcal{R}_1\cdot(1-\mathcal{R}_2)\cdot(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4),$$
$$\mathcal{P}(x_{10}) = (1-\mathcal{R}_1)\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot(1-\mathcal{R}_4),$$
$$\mathcal{P}(x_{12}) = (1-\mathcal{R}_1)\cdot\mathcal{R}_2\cdot(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4),$$
$$\mathcal{P}(x_{14}) = (1-\mathcal{R}_1)\cdot(1-\mathcal{R}_2)\cdot\mathcal{R}_3\cdot(1-\mathcal{R}_4),$$
$$\mathcal{P}(x_{15}) = (1-\mathcal{R}_1)\cdot(1-\mathcal{R}_2)\cdot(1-\mathcal{R}_3)\cdot\mathcal{R}_4,$$
$$\mathcal{P}(x_{16}) = (1-\mathcal{R}_1)\cdot(1-\mathcal{R}_2)\cdot(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4).$$

Now, after combining, equation (1.1) becomes

$$\begin{aligned}\mathbb{P}_f = {} & \mathcal{R}_1\cdot\mathcal{R}_2\cdot[\mathcal{R}_3\cdot(1-\mathcal{R}_4)+(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4)]+\mathcal{R}_1\cdot(1-\mathcal{R}_2)\cdot[\mathcal{R}_3\cdot(1-\mathcal{R}_4)+(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4)]\\ & +(1-\mathcal{R}_1)\cdot\mathcal{R}_2\cdot(1-\mathcal{R}_4)\cdot[\mathcal{R}_3+(1-\mathcal{R}_3)]+(1-\mathcal{R}_1)\cdot(1-\mathcal{R}_2)\cdot[\mathcal{R}_3\cdot(1-\mathcal{R}_4)+(1-\mathcal{R}_3)\cdot\mathcal{R}_4\\ & +(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4)]\end{aligned}$$

$$= \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_4) + \mathcal{R}_1 \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_4) + (1-\mathcal{R}_1) \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_4) + (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3\mathcal{R}_4)$$
$$= (1-\mathcal{R}_4) \cdot [\mathcal{R}_1\mathcal{R}_2 + \mathcal{R}_1 \cdot (1-\mathcal{R}_2) + (1-\mathcal{R}_1) \cdot \mathcal{R}_2] + (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3\mathcal{R}_4)$$
$$= 1 - \mathcal{R}_1 \cdot \mathcal{R}_4 - \mathcal{R}_2 \cdot \mathcal{R}_4 - \mathcal{R}_3 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 .$$

Reliability of the system = 1− Probability of the failure of the system,
i.e.,

$$\mathbb{R}_s = 1 - \mathbb{P}_f$$
$$= 1 - [1 - \mathcal{R}_1 \cdot \mathcal{R}_4 - \mathcal{R}_2 \cdot \mathcal{R}_4 - \mathcal{R}_3 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4]$$
$$= \mathcal{R}_1 \cdot \mathcal{R}_4 + \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_3 \cdot \mathcal{R}_4 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 - \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 - \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 .$$

On more simplifying, we get

$$\mathbb{R}_s = \mathcal{R}_1 \cdot \mathcal{R}_4 \cdot (1-\mathcal{R}_3) + \mathcal{R}_2 \cdot \mathcal{R}_4 \cdot (1-\mathcal{R}_1) + \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot (1-\mathcal{R}_2) + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4.$$

If we consider, Reliability of Component 1 is 96%, Reliability of Component 2 is 99%, Reliability of Component 3 is 95% and Reliability of Component 4 is 97%, i.e.

$$\mathcal{R}_1 = 96\%, \quad \mathcal{R}_2 = 99\%, \quad \mathcal{R}_3 = 95\%, \quad \mathcal{R}_4 = 97\%.$$

As the reliabilities of the components are specified for 7 days, i.e., 168 hours (assumption), then we will find the value of the Reliability of the system for 7 days as

$$\mathbb{R}_s = 0.04656 + 0.038412 + 0.009215 + 0.8757936 = 0.9699806 \cong 0.9700.$$

4.2 Calculation of Reliability of the System Having *Five* Components in Series-Parallel Mixed Configuration

Configuration 4.2.1. In this configuration, Component 1 and Component 2 are connected in parallel and then this parallel sub-system is connected to Component 5 in series. Also, Component 3 and Component 4 are connected in parallel, and then these two sub-systems of components are again connected in parallel as shown in Figure 4.

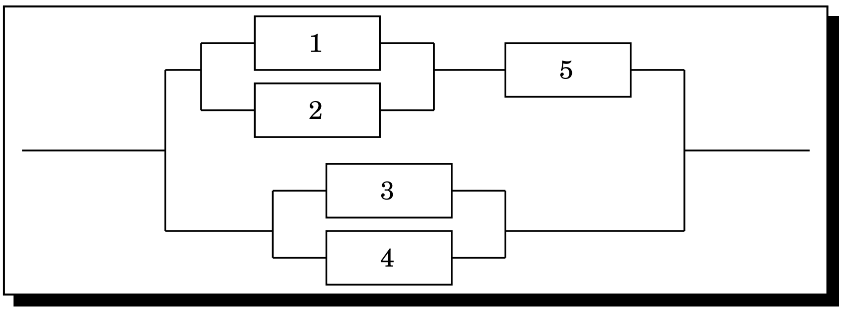


Figure 4. Reliability block diagram of the system having five components

Let $\boldsymbol{A}$ be the Success of Component 1 event and $\boldsymbol{a}$ be the Failure of Component 1 event.
Let $\boldsymbol{B}$ be the Success of Component 2 event and $\boldsymbol{b}$ be the Failure of Component 2 event.
Let $\boldsymbol{C}$ be the Success of Component 3 event and $\boldsymbol{c}$ be the Failure of Component 3 event.

Let $\boldsymbol{D}$ be the Success of Component 4 event and $\boldsymbol{d}$ be the Failure of Component 4 event.
Let $\boldsymbol{E}$ be the Success of Component 5 event and $\boldsymbol{e}$ be the Failure of Component 5 event.
Also, here

x_i : Success or Failure of Component i's event

$\mathcal{P}(x_i)$: Probability of Failure of Component i

$\mathbb{R}_s$: Reliability of the system

The mutually exclusive events of given system are:

$x_1 = ABCDE$ – All components success, $x_2 = ABCDe$ – Component 5 fail,
$x_3 = ABCdE$ – Component 4 fail, $x_4 = ABCde$ – Component 4 and 5 fail,
$x_5 = ABcDE$ – Component 3 fail, $x_6 = ABcDe$ – Component 3 and 5 fail,
$x_7 = ABcdE$ – Component 3 and 4 fail, $x_8 = ABcde$ – Component 3, 4 and 5 fail,
$x_9 = AbCDE$ – Component 2 fail, $x_{10} = AbCDe$ – Component 2 and 5 fail,
$x_{11} = AbCdE$ – Component 2 and 4 fail, $x_{12} = AbCde$ – Component 2, 4 and 5 fail,
$x_{13} = AbcDE$ – Component 2 and 3 fail, $x_{14} = AbcDe$ – Component 2, 3 and 5 fail,
$x_{15} = AbcdE$ – Component 2, 3 and 4 fail, $x_{16} = Abcde$ – Component 2, 3, 4 and 5 fail,
$x_{17} = aBCDE$ – Component 1 fail, $x_{18} = aBCDe$ – Component 1 and 5 fail,
$x_{19} = aBCdE$ – Component 1 and 4 fail, $x_{20} = aBCde$ – Component 1, 4 and 5 fail,
$x_{21} = aBcDE$ – Component 1 and 3 fail, $x_{22} = aBcDe$ – Component 1, 3 and 5 fail,
$x_{23} = aBcdE$ – Component 1, 3 and 4 fail, $x_{24} = aBcde$ – Component 1, 3, 4 and 5 fail,
$x_{25} = abCDE$ – Component 1 and 2 fail, $x_{26} = abCDe$ – Component 1, 2 and 5 fail,
$x_{27} = abCdE$ – Component 1, 2 and 4 fail, $x_{28} = abCde$ – Component 1,2,4 and 5 fail,
$x_{29} = abcDE$ – Component 1, 2 and 3 fail, $x_{30} = abcDe$ – Component 1, 2, 3 and 5 fail,
$x_{31} = abcdE$ – Component 1,2,3 and 4 fail, $x_{32} = abcde$ – All components fail.

Event of the system x_8, x_{16}, x_{24}, x_{31}, x_{32} results in the system failure. Thus, the probability of the failure of the system is:

$$\mathbb{P}_f = \mathcal{P}(x_8 \cup x_{16} \cup x_{24} \cup x_{31} \cup x_{32}).$$

Calculation of the probability of event that leads to the system failure:

$$\mathcal{P}(x_8) = \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5),$$
$$\mathcal{P}(x_{16}) = \mathcal{R}_1 \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5),$$
$$\mathcal{P}(x_{24}) = (1-\mathcal{R}_1) \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5),$$
$$\mathcal{P}(x_{31}) = (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot \mathcal{R}_5,$$
$$\mathcal{P}(x_{32}) = (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5).$$

Now, after combing, equation (1.1) becomes

$$\begin{aligned}
\mathbb{P}_f &= (1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4)\cdot(1-\mathcal{R}_5)\cdot[\mathcal{R}_1\cdot\mathcal{R}_2+\mathcal{R}_1\cdot(1-\mathcal{R}_2)+(1-\mathcal{R}_1)\cdot\mathcal{R}_2] \\
&\quad +(1-\mathcal{R}_1)\cdot(1-\mathcal{R}_2)\cdot(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4)\cdot[\mathcal{R}_5+(1-\mathcal{R}_5)] \\
&= (1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4)\cdot(1-\mathcal{R}_5)\cdot[\mathcal{R}_1+\mathcal{R}_2-\mathcal{R}_1\cdot\mathcal{R}_2]+(1-\mathcal{R}_1)\cdot(1-\mathcal{R}_2)\cdot(1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4) \\
&= (1-\mathcal{R}_3)\cdot(1-\mathcal{R}_4)\cdot[(1-\mathcal{R}_5)\cdot(\mathcal{R}_1+\mathcal{R}_2-\mathcal{R}_1\cdot\mathcal{R}_2)+(1-\mathcal{R}_1)\cdot(1-\mathcal{R}_2)] \\
&= (1-\mathcal{R}_3-\mathcal{R}_4+\mathcal{R}_3\cdot\mathcal{R}_4)[1-\mathcal{R}_1\cdot\mathcal{R}_5-\mathcal{R}_2\cdot\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_5] \\
&= 1-\mathcal{R}_1\cdot\mathcal{R}_5-\mathcal{R}_2\cdot\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_5-\mathcal{R}_3+\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_5+\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_5-\mathcal{R}_4 \\
&\quad +\mathcal{R}_1\cdot\mathcal{R}_4\cdot\mathcal{R}_5+\mathcal{R}_2\cdot\mathcal{R}_4\cdot\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_4\cdot\mathcal{R}_5+\mathcal{R}_3\cdot\mathcal{R}_4-\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5-\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5 \\
&\quad +\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5.
\end{aligned}$$

Reliability of the system = 1− Probability of the failure of the system,
i.e.,

$$\begin{aligned}
\mathbb{R}_s &= 1-\mathbb{P}_f \\
&= 1-[1-\mathcal{R}_1\cdot\mathcal{R}_5-\mathcal{R}_2\cdot\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_5-\mathcal{R}_3+\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_5+\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_5 \\
&\quad -\mathcal{R}_4+\mathcal{R}_1\cdot\mathcal{R}_4\cdot\mathcal{R}_5+\mathcal{R}_2\cdot\mathcal{R}_4\cdot\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_4\cdot\mathcal{R}_5+\mathcal{R}_3\cdot\mathcal{R}_4-\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5 \\
&\quad -\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5] \\
&= \mathcal{R}_1\cdot\mathcal{R}_5+\mathcal{R}_2\cdot\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_5+\mathcal{R}_3-\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_5-\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_5 \\
&\quad +\mathcal{R}_4-\mathcal{R}_1\cdot\mathcal{R}_4\cdot\mathcal{R}_5-\mathcal{R}_2\cdot\mathcal{R}_4\cdot\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_4\cdot\mathcal{R}_5-\mathcal{R}_3\cdot\mathcal{R}_4+\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5 \\
&\quad +\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5.
\end{aligned}$$

On more simplifying, we get

$$\begin{aligned}
\mathbb{R}_s &= \mathcal{R}_3+\mathcal{R}_4-\mathcal{R}_3\cdot\mathcal{R}_4\cdot(1+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_5)+\mathcal{R}_5\cdot(\mathcal{R}_1+\mathcal{R}_2)-\mathcal{R}_1\cdot\mathcal{R}_5\cdot(\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4) \\
&\quad -\mathcal{R}_2\cdot\mathcal{R}_5\cdot(\mathcal{R}_3+\mathcal{R}_4)+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_5\cdot(\mathcal{R}_3+\mathcal{R}_4)+\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5\cdot(\mathcal{R}_1+\mathcal{R}_2).
\end{aligned}$$

If we consider, Reliability of Component 1 is 96%, Reliability of Component 2 is 99%, Reliability of Component 3 is 95%, Reliability of Component 4 is 97% and Reliability of Component 5 is 98%, i.e.,

$$\mathcal{R}_1 = 96\%,\ \ \mathcal{R}_2 = 99\%,\ \ \mathcal{R}_3 = 95\%,\ \ \mathcal{R}_4 = 97\%,\ \ \mathcal{R}_5 = 98\%.$$

As the reliabilities of the components are specified for 7 days, i.e., 168 hours (assumption), then we will find the value of the Reliability of the system for 7 days as

$$\begin{aligned}
\mathbb{R}_s &= 0.95+0.97-1.779777728+1.911-2.737728-1.862784+1.78827264+1.7609865 \\
&= 0.999969412 \cong 0.9999.
\end{aligned}$$

Configuration 4.2.2. In this configuration, Component 1 and Component 2 are connected in parallel and Component 3 and Component 4 are also connected in parallel, then these two parallel sub-systems are connected in series. Then this sub-system is again connected to Component 5 in parallel as shown in Figure 5.

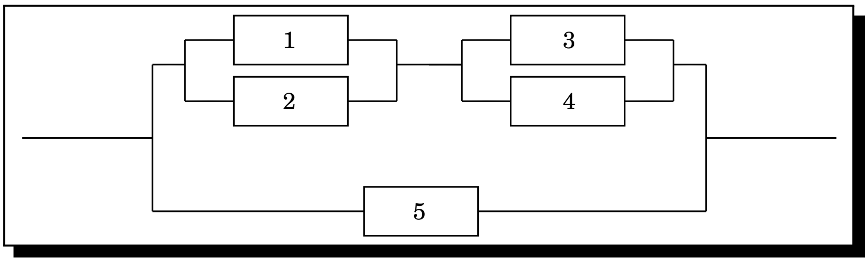


Figure 5. Reliability block diagram of the system having five components

Event of the system x_8, x_{16}, x_{24}, x_{26}, x_{28}, x_{30}, x_{32} results in the system failure. Thus, the probability of the failure of the system is:

$$\mathbb{P}_f = \mathcal{P}(x_8 \cup x_{16} \cup x_{24} \cup x_{26} \cup x_{28} \cup x_{30} \cup x_{32}).$$

Calculation of the probability of event that leads to the system failure:

$$\mathcal{P}(x_8) = \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5),$$
$$\mathcal{P}(x_{16}) = \mathcal{R}_1 \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5),$$
$$\mathcal{P}(x_{24}) = (1-\mathcal{R}_1) \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5),$$
$$\mathcal{P}(x_{26}) = (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot (1-\mathcal{R}_5),$$
$$\mathcal{P}(x_{28}) = (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot \mathcal{R}_3 \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5),$$
$$\mathcal{P}(x_{30}) = (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot \mathcal{R}_4 \cdot (1-\mathcal{R}_5),$$
$$\mathcal{P}(x_{32}) = (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5).$$

Now, after combing, equation (1.1) becomes

$$\begin{aligned}
\mathbb{P}_f &= (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5) \cdot [\mathcal{R}_1 \cdot \mathcal{R}_2 + \mathcal{R}_1 \cdot (1-\mathcal{R}_2) + (1-\mathcal{R}_1) \cdot \mathcal{R}_2] \\
&\quad + (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_5) \cdot [\mathcal{R}_3 \cdot \mathcal{R}_4 + \mathcal{R}_3 \cdot (1-\mathcal{R}_4) + (1-\mathcal{R}_3) \cdot \mathcal{R}_4 + (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4)] \\
&= (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5) \cdot [\mathcal{R}_1 + \mathcal{R}_2 - \mathcal{R}_1 \cdot \mathcal{R}_2] + (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_5) \\
&= (1-\mathcal{R}_5) \cdot [(1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (\mathcal{R}_1 + \mathcal{R}_2 - \mathcal{R}_1 \cdot \mathcal{R}_2) + (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2)] \\
&= (1-\mathcal{R}_5) \cdot [(1-\mathcal{R}_3 - \mathcal{R}_4 + \mathcal{R}_3 \cdot \mathcal{R}_4)(\mathcal{R}_1 + \mathcal{R}_2 - \mathcal{R}_1 \cdot \mathcal{R}_2) + (1-\mathcal{R}_1 - \mathcal{R}_2 + \mathcal{R}_1 \cdot \mathcal{R}_2)] \\
&= (1-\mathcal{R}_5) \cdot [1 - \mathcal{R}_1 \cdot \mathcal{R}_3 - \mathcal{R}_1 \cdot \mathcal{R}_4 - \mathcal{R}_2 \cdot \mathcal{R}_3 - \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \\
&\quad + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4] \\
&= 1 - \mathcal{R}_1 \cdot \mathcal{R}_3 - \mathcal{R}_1 \cdot \mathcal{R}_4 - \mathcal{R}_2 \cdot \mathcal{R}_3 - \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 + \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \\
&\quad + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 - \mathcal{R}_5 + \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 + \mathcal{R}_1 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 \\
&\quad + \mathcal{R}_2 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 - \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 - \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 \\
&\quad + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 .
\end{aligned}$$

Reliability of the system = 1− Probability of the failure of the system,
i.e.,

$$\mathbb{R}_s = 1 - \mathbb{P}_f$$

$$= 1-[1-\mathcal{R}_1\cdot\mathcal{R}_3-\mathcal{R}_1\cdot\mathcal{R}_4-\mathcal{R}_2\cdot\mathcal{R}_3-\mathcal{R}_2\cdot\mathcal{R}_4+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_4+\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_4$$
$$+\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4-\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_4\cdot\mathcal{R}_5+\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_5$$
$$+\mathcal{R}_2\cdot\mathcal{R}_4\cdot\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_4\cdot\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5-\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5$$
$$+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5]$$
$$=\mathcal{R}_1\cdot\mathcal{R}_3+\mathcal{R}_1\cdot\mathcal{R}_4+\mathcal{R}_2\cdot\mathcal{R}_3+\mathcal{R}_2\cdot\mathcal{R}_4-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_4-\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_4-\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4$$
$$+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4+\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_5-\mathcal{R}_1\cdot\mathcal{R}_4\cdot\mathcal{R}_5-\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_5-\mathcal{R}_2\cdot\mathcal{R}_4\cdot\mathcal{R}_5$$
$$+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_4\cdot\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5+\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5$$
$$-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot\mathcal{R}_5\,.$$

On more simplifying, we get

$$\mathbb{R}_s=\mathcal{R}_1\cdot\mathcal{R}_3\cdot(1-\mathcal{R}_2-\mathcal{R}_5+\mathcal{R}_2\cdot\mathcal{R}_4)+\mathcal{R}_1\cdot\mathcal{R}_4(1-\mathcal{R}_3-\mathcal{R}_5+\mathcal{R}_2\cdot\mathcal{R}_5)$$
$$+\mathcal{R}_2\cdot\mathcal{R}_3(1-\mathcal{R}_4-\mathcal{R}_5+\mathcal{R}_1\cdot\mathcal{R}_5)+\mathcal{R}_2\cdot\mathcal{R}_4(1-\mathcal{R}_1-\mathcal{R}_5+\mathcal{R}_3\cdot\mathcal{R}_5)$$
$$+\mathcal{R}_5\cdot(1+\mathcal{R}_1\cdot\mathcal{R}_3\cdot\mathcal{R}_4-\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4)\,.$$

If we consider, Reliability of Component 1 is 96%, Reliability of Component 2 is 99%, Reliability of Component 3 is 95%, Reliability of Component 4 is 97% and Reliability of Component 5 is 98%, i.e.,

$$\mathcal{R}_1=96\%,\quad \mathcal{R}_2=99\%,\quad \mathcal{R}_3=95\%,\quad \mathcal{R}_4=97\%,\quad \mathcal{R}_5=98\%\,.$$

As the reliabilities of the components are specified for 7 days, i.e., 168 hours (assumption), then we will find the value of the Reliability of the system for 7 days as

$$\mathbb{R}_s=0.999962012\cong 0.9999.$$

Configuration 4.2.3. In this configuration, Component 1 and Component 2 are connected in parallel and Component 3 and Component 4 are also connected in parallel, then these two parallel sub-systems with Component 5 are connected in series as shown in Figure 6.

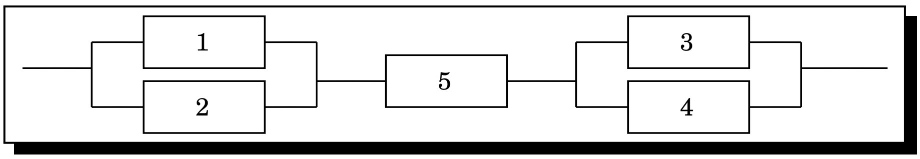


Figure 6. Reliability block diagram of the system having five components

Event of the system x_2, x_4, x_6, x_7, x_8, x_{10}, x_{12}, x_{14}, x_{15}, x_{16}, x_{18}, x_{20}, x_{22}, x_{23}, x_{24}, x_{25}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{31}, x_{32} results in the system failure. Thus, the probability of the failure of the system is:

$$\mathbb{P}_f=\mathcal{P}(x_2\cup x_4\cup x_6\cup x_7\cup x_8\cup x_{10}\cup x_{12}\cup x_{14}\cup x_{15}\cup x_{16}\cup x_{18}\cup x_{20}\cup x_{22}\cup x_{23}\cup x_{24}\cup x_{25}$$
$$\cup x_{26}\cup x_{27}\cup x_{28}\cup x_{29}\cup x_{30}\cup x_{31}\cup x_{32}).$$

Calculation of the probability of event that leads to the system failure:

$$\mathcal{P}(x_2)=\mathcal{R}_1\cdot\mathcal{R}_2\cdot\mathcal{R}_3\cdot\mathcal{R}_4\cdot(1-\mathcal{R}_5)$$

$$\mathcal{P}(x_4) = \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5)$$
$$\mathcal{P}(x_6) = \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_3) \cdot \mathcal{R}_4 \cdot (1-\mathcal{R}_5)$$
$$\vdots$$
$$\mathcal{P}(x_{31}) = (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot \mathcal{R}_4$$
$$\mathcal{P}(x_{32}) = (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4) \cdot (1-\mathcal{R}_5).$$

Now, after combing, equation (1.1) becomes

$$\begin{aligned}
\mathbb{P}_f &= \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot (1-\mathcal{R}_5) + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4 \cdot \mathcal{R}_5) + \mathcal{R}_1 \cdot (1-\mathcal{R}_2) \cdot \mathcal{R}_3 \cdot (1-\mathcal{R}_5) \\
&\quad + \mathcal{R}_1 \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4 \cdot \mathcal{R}_5) + (1-\mathcal{R}_1) \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot (1-\mathcal{R}_5) \\
&\quad + (1-\mathcal{R}_1) \cdot \mathcal{R}_2 \cdot (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4 \cdot \mathcal{R}_5) + (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot \mathcal{R}_3 + (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \cdot (1-\mathcal{R}_3) \\
&= \mathcal{R}_3 \cdot (1-\mathcal{R}_5) \cdot [\mathcal{R}_1 + \mathcal{R}_2 - \mathcal{R}_1 \cdot \mathcal{R}_2] + (1-\mathcal{R}_3) \cdot (1-\mathcal{R}_4 \cdot \mathcal{R}_5) \cdot [\mathcal{R}_1 + \mathcal{R}_2 - \mathcal{R}_1 \cdot \mathcal{R}_2] \\
&\quad + (1-\mathcal{R}_1) \cdot (1-\mathcal{R}_2) \\
&= (1-\mathcal{R}_3 \cdot \mathcal{R}_5 - \mathcal{R}_4 \cdot \mathcal{R}_5 + \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5)(\mathcal{R}_1 + \mathcal{R}_2 - \mathcal{R}_1 \cdot \mathcal{R}_2) + (1-\mathcal{R}_1 - \mathcal{R}_2 + \mathcal{R}_1 \cdot \mathcal{R}_2) \\
&= 1 - \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 - \mathcal{R}_1 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 - \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 - \mathcal{R}_2 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 \\
&\quad + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 + \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 .
\end{aligned}$$

Reliability of the system = 1− Probability of the failure of the system,
i.e.,

$$\begin{aligned}
\mathbb{R}_s &= 1 - \mathbb{P}_f \\
&= 1 - [1 - \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 - \mathcal{R}_1 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 - \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 - \mathcal{R}_2 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 \\
&\quad + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 + \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5] \\
&= \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 + \mathcal{R}_1 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 + \mathcal{R}_2 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 \\
&\quad - \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 - \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 - \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 .
\end{aligned}$$

On more simplifying, we get

$$\begin{aligned}
\mathbb{R}_s &= \mathcal{R}_1 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 \cdot (1-\mathcal{R}_4) + \mathcal{R}_1 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 \cdot (1-\mathcal{R}_2) + \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_5 \cdot (1-\mathcal{R}_1) + \mathcal{R}_2 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5 \cdot (1-\mathcal{R}_3) \\
&\quad + \mathcal{R}_1 \cdot \mathcal{R}_2 \cdot \mathcal{R}_3 \cdot \mathcal{R}_4 \cdot \mathcal{R}_5.
\end{aligned}$$

If we consider, Reliability of Component 1 is 96%, Reliability of Component 2 is 99%, Reliability of Component 3 is 95%, Reliability of Component 4 is 97% and Reliability of Component 5 is 98%, i.e.,

$$\mathcal{R}_1 = 96\%, \ \ \mathcal{R}_2 = 99\%, \ \ \mathcal{R}_3 = 95\%, \ \ \mathcal{R}_4 = 97\%, \ \ \mathcal{R}_5 = 98\%.$$

As the reliabilities of the components are specified for 7 days, i.e., 168 hours (assumption), then we will find the value of the Reliability of the system for 7 days as

$$\mathbb{R}_s = 0.858277728 \cong 0.8583.$$

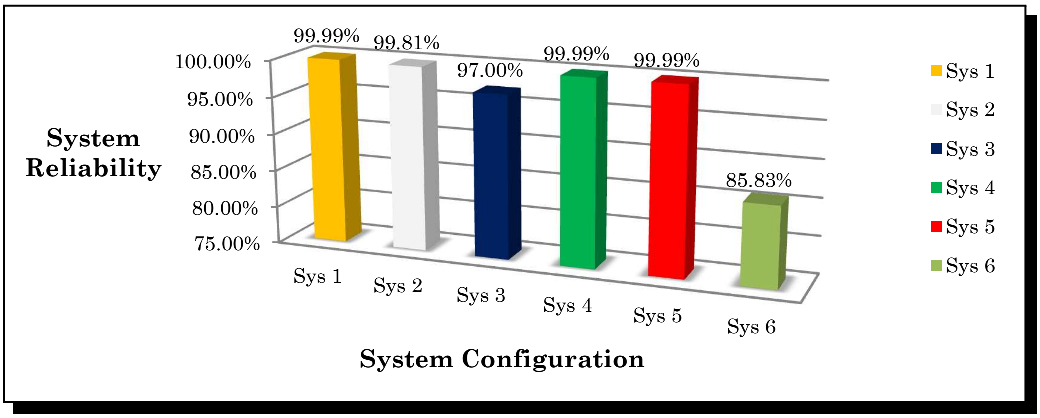
5. Results

The configuration of the system and the Reliability of the system obtained from the study are shown in the following table:

Table 1. Reliability of the system with their respective configuration

Sr. No	Configuration of the system	Reliability of the system
1	Figure 1	99.99%
2	Figure 2	99.81%
3	Figure 3	97.00%
4	Figure 4	99.99%
5	Figure 5	99.99%
6	Figure 6	85.83%

Graphical representation of system configuration and system reliability is shown in Graph 1. Here we consider four and five components arranged in various configuration, where $\mathcal{R}_1 = 96\%$, $\mathcal{R}_2 = 99\%$, $\mathcal{R}_3 = 95\%$, $\mathcal{R}_4 = 97\%$, $\mathcal{R}_5 = 98\%$ for given period. In Table 1, we consider six different configurations and calculated their respective system reliability and. The result obtained in illustrated graphically in Graph 1.



Graph 1. System configuration vs System reliability

6. Conclusion

In this paper, we focus on the analytical study of system reliability with diverse configurations. Our investigation reveals a key finding: when we augment the redundancy within a system, the overall system reliability increases. Redundancy here refers to the inclusion of backup components or pathways, which can take over in case the primary ones fail. It's worth noting that while adding redundancy can substantially enhance system reliability, it often comes at a cost. This expense can manifest in the form of additional components, equipment, or increased maintenance. We also provide a detailed examination of the graphical representations of these various system configurations, helping to visualize how redundancy impact's reliability.

Additionally, we emphasize the advantage of employing parallel redundancy, a configuration where multiple components or pathways work simultaneously to achieve the same task. This strategy is particularly effective in bolstering overall system reliability because it ensures that the system can continue to operate even if one or more components fail.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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