Communications in Mathematics and Applications
Vol. 15, No. 1, pp. 397–405, 2024
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v15i1.2598

http://www.rgnpublications.com

Research Article

Rough Decision Making of Facial Expression Detection

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Received: February 21, 2024 **Accepted:** April 18, 2024

Abstract. This paper introduces a novel framework named rough decision making which extends the principles of rough set theory. The mathematical model proposed consists of a fuzzy interval-valued knowledge-based information system that includes a non-empty finite universe of objects, a derived interval-valued fuzzy set of attributes, and a fuzzy interval-valued set of decisions. To manage complexity, the roughness of decision-making is introduced by defining lower and upper rough decisions for each object. Further, this model is implemented on a real-time affective image database RAF-DB for facial expression detection. This approach is found to provide a comprehensive analysis of facial expressions, demonstrating effective classification even in the presence of uncertainty.

Keywords. Rough set theory, Decision making problems, Facial expression detection

Mathematics Subject Classification (2020). 62C86, 97C20, 94A08, 90C70

1. Introduction

The universal nature of *Decision-Making Problems* emphasizes their influence in wide-ranging contexts from the intricacies of everyday life to the complexities of advanced technologies. As decision making problems evolve in complexity, there is a need for powerful methods to manage uncertain, indeterminate, incomplete and imprecise information especially at a large scale. Several works have researched the usage of rough set theory (Pawlak [8]) for decision making (Slowinski *et al*. [10]). Li and Zhou [4] reviews rough set models and probabilistic rough set models introducing a model that offers optimistic, pessimistic, and equable decisions, considering the cost of misclassification. A rough set approach to intuitionistic fuzzy soft set-based decision making (Zhang [12]) that utilizes soft set theory, intuitionistic fuzzy soft sets, and rough set theory to address uncertainty and decision-making problems. Decision making in fuzzy rough set theory (Chacón-Gómez *et al*. [1]) analyzes classification methods for new objects in fuzzy rough set theory-based decision-making, leveraging relevance indicators like support, certainty, and credibility associated with decision rules.

The main objective of this work is to provide a nuanced approach in handling decision making problems. For any given dataset, along with its attributes, the focus is to make a decision to any one of the variables in the decision set. When the dataset is large and with uncertain information an appropriate mathematical model is required to capture the information and to arrive at the decision. This article uses an extension of rough set theory and introduces the concept of rough decision of the objects of a universe U. Further, the developed concepts are explored with the expression detection of images on RAF-DB, a real time database of effective images.

2. Materials and Methods

In this section, the definitions that are required to study the forthcoming sections are provided followed by the methodology adopted in this approach. The source code for the mathematical model demonstrated for facial expression detection is publicly available at:

- *Rough decision making of facial expression detection*[1]
- *Code rough decision making of facial expression detection*[2]

2.1 Interval Valued Fuzzy Information System With Decision Variables

Let $I = (U, A)$ be an interval valued fuzzy information system where U is a non-empty finite set of objects and A be a set of interval valued fuzzy attributes,

$$A = \{a_1, a_2, \dots, a_k\}$$

is a set of parameters such that for each $x \in U$,

$$\mu_x : A \to P([0,1]),$$

where $P([0,1])$ is the set

$$P([0,1]) = \{(I_1, I_2) \mid (I_1, I_2) \subseteq [0,1]\}.$$

Let D be the set of decision variables $\{d_1, d_2, \dots, d_r\}$. Let

$$\mu_D(a_i) D \to P([0,1])$$

be the interval valued fuzzy set in which each decision variable is associated with a set of interval valued fuzzy set for each attribute a_i. Hence the following interval valued fuzzy information system with decision variables is defined,

[1]URL: https://github.com/charu210703/Rough-Decision-Making-of-Facial-Expression-Detection/blob/main/Rough_Decision_Making_of_Facial_Expression_Detection.ipynb

[2]URL: https://github.com/a-bhavana04/Rough_Decision_Making/blob/main/Code_Rough_Decision_Making_of_Facial_Expression_Detection.ipynb

Table 1. Interval-valued Fuzzy Information System

U/A	a_1	a_2	a_3
x_1	I_{11}	I_{12}	I_{13}
x_2	I_{21}	I_{22}	I_{23}
x_3	I_{31}	I_{32}	I_{33}

and the table of decision variables as,

Table 2. Decision Variables

D/A	a_1	a_2	a_3
d_1	D_{11}	D_{12}	D_{13}
d_2	D_{21}	D_{22}	D_{23}
d_3	D_{31}	D_{32}	D_{33}

where I_{ij} is an interval contained in $[0,1]$ with respect to the attributes a_j for the object x_i. Similarly, $D_{ij} \subseteq [0,1]$ is an interval for the decision d_i with respect to the attribute a_j. This decision table can be obtained using the expert knowledge and can also be based on the trained dataset. Now the objective is to associate each object of U to a decision in D.

Definition 2.1. Given a knowledge based information system

$$I = (U, A, D, \mu_A, \mu_D)$$

an object $x_i \in U$ is said to be related to a decision $d_j \in D$, i.e., $x_i \sim d_j$ if

$$I_{i1} \sim d_{1j}, I_{i2} \sim d_{2j}, \dots, I_{ik} \sim d_{kj},$$

where $I_{i1} \subseteq d_{j1}$ (or) $d_{j1} \subseteq I_{i1}$ and so on.

Definition 2.2. The lower decision approximation of an object is $x_i \in U$ is

$$x_i(D)_- = \{d_j \in D \mid I_{i1} \sim D_{1j}, I_{i2} \sim D_{2j}, \dots, I_{ik} \sim D_{kj}\},$$

for all I_r corresponding to x_i.

Definition 2.3. The upper rough decision of $x_i \in U$ is

$$x_i(D)^- = \{d_j \in D \mid \text{ at least some of } I_{i1}, I_{i2}, \dots, I_{ik} \text{ are related by } I_{i1} \sim D_{1j}, I_{i2} \sim D_{2j}, \dots, I_{ik} \sim D_{kj}\}.$$

Definition 2.4. The rough decision of an object, $x_i \in U$ is

$$RD(x_i) = (x_i(D)_-, x_i(D)^-).$$

The novelty in this proposed model lies in the fact that it takes into account interval-valued fuzzy sets for taking the rough decision rather than single values belonging to the upper and lower subsets. This framework thus gives the provision of making definite and possible decisions for each object. The advantage of this model is that the information about an object is extracted with respect to the attributes and the decision is made with respect to these attributes which will play a major role in classification tasks involving decision-making. Hence this acts as a two way gate to take decisions in uncertain and indeterminate conditions.

2.2 Decision on Facial Expression Using Rough Decision of an Object

Facial expressions are essential for understanding non-verbal communication, offering valuable insights into thoughts and emotions. Analyzing various facial muscle movements requires a cross-parametric approach. Automated facial expression recognition has transformed applications in healthcare and education. Diverse methodologies, from machine learning, deep learning (Ngwe *et al*. [6]) and graph based frameworks (Liao *et al*. [5]) are utilized for facial expression detection. Models incorporating rough set theory, like Chen *et al*. [2] explore geometric features for expression detection. Tuncer *et al*. [11] and Parcham *et al*. [7] address the uncertainty of this task using fuzzy set models. Praba *et al*. [9] proposes a hybrid structure for real-time detection, enhancing robustness. This paper focuses on handling roughness in decision-making for expression classification, using interval-valued fuzzy sets derived from geometric features. Membership in Lower and Upper Rough Decision Sets provides nuanced insights into facial expression detection within decision-making frameworks.

2.3 Dataset Description

The Real-world Affective Faces Database (RAF-DB) (Lim and Deng [3]) is a vast collection of approximately 30,000 diverse facial images spread across a range of age groups. Each image has been annotated by independent evaluators.

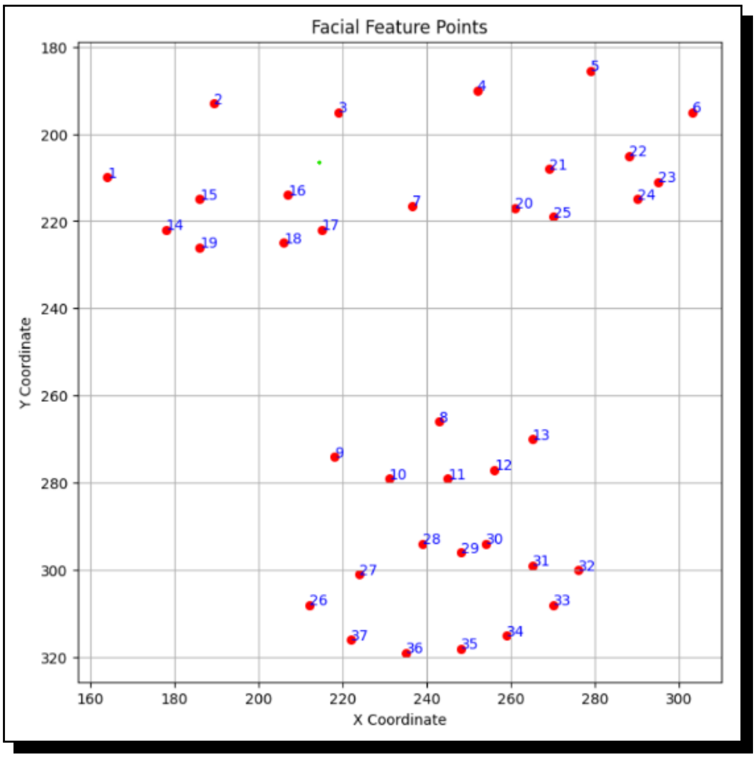


Figure 1. 37 Facial points automatically annotated[3] [4]

[3]URL: https://github.com/charu210703/Rough-Decision-Making-of-Facial-Expression-Detection/blob/main/Rough_Decision_Making_of_Facial_Expression_Detection.ipynb

[4]URL: https://github.com/a-bhavana04/Rough_Decision_Making/blob/main/Code_Rough_Decision_Making_of_Facial_Expression_Detection.ipynb

These images exhibit a wide range of variations, including subjects' characteristics, head poses, lighting conditions, and occlusions. RAF-DB includes annotations such as multi-dimensional expression vectors, landmark locations, bounding boxes, and provides classifiers for basic and compound emotions. There are 5 manually and 37 automatically annotated facial feature points.

These 37 automatically annotated facial landmark points are utilized to select a set of 2300 images with eye center, nose tip, nose center and lip midpoints oriented forming a relatively straight configuration to create a set of aligned images.

2.4 Approach

The geometric parameters based approach focused on five key facial parameters. The coordinates of the facial landmarks are used to calculate the mentioned parameters using the given formulae:

- Area between the eye and eyebrow
$$\text{Area} = \frac{1}{2}\left|\sum_{i=1}^{n}(x_{i-1}y_i - x_i y_{i-1})\right|.$$
- Area between the eyelids
$$\text{Area} = \frac{1}{2}\left|\sum_{i=1}^{n}(x_{i-1}y_i - x_i y_{i-1})\right|.$$
- Lip landmarks
$$\text{Vertical distance} = \|\text{Upper midpoint} - \text{Lower midpoint}\|,$$
where
$$\text{Upper midpoint} = \frac{1}{3}\sum_{i=1}^{3}\text{Upper lip landmark}_i,$$
$$\text{Lower midpoint} = \frac{1}{3}\sum_{i=1}^{3}\text{Lower lip landmark}_i.$$
- Distance between eyebrow center and nose center
$$\text{Distance}_{\text{eb\_cen\_nose\_cen}} = \sqrt{(\text{eb\_cen\_x} - \text{nose\_cen\_x})^2 + (\text{eb\_cen\_y} - \text{nose\_cen\_y})^2}.$$
- Distance between eyebrow center and nose tip
$$\text{Distance}_{\text{eb\_cen\_nose\_tip}} = \sqrt{(\text{eb\_cen\_x} - \text{nose\_tip\_x})^2 + (\text{eb\_cen\_y} - \text{nose\_tip\_y})^2}.$$

Confidence intervals to quantify the variability associated with each emotion across these facial features were derived.

$$\text{Mean}_p = \frac{1}{n}\sum_{i=1}^{n} x_i\,,$$
$$SD_p = \frac{SD_p}{\sqrt{n}}\,,$$
$$CI_p = \text{Mean}_p \pm (\text{Critical value} \times SE_p),$$

Mean_p is the mean of the data for parameter p,

SE_p is the standard error of the mean for parameter p,

SD_p is the standard deviation of the data for parameter p.

In this application the universe, U is defined as the set of all the images used from the database. Utilizing the aligned facial images, confidence intervals for seven distinct emotions were derived.

$$U = \{\text{Set of all aligned facial images}\}.$$

These are split into train and test images. The set A consists of interval-valued fuzzy sets derived for each emotion based on the facial feature points from the set of train images.

$$A = \{(I-\varepsilon, I+\varepsilon) \text{ for each } a_j \in A\}.$$

The set D represents the decisions that can be taken which in this case is the set of emotions.

$$D = \{1:\text{Surprise}, 2:\text{Fear}, 3:\text{Disgust}, 4:\text{Happiness}, 5:\text{Sadness}, 6:\text{Anger}, 7:\text{Neutral}\}.$$

Figure 2. Samples images [3]

The given tables illustrate the intervals derived based on the attributes to make a decision.

Table 3. Emotions and their confidence intervals

Emotions	Param_1	Param_2	Param_3	Param_4	Param_5
Surprise	[0.02, 0.267]	[0.007, 0.11]	[0.009, 0.017]	[0.013, 0.026]	[0.009, 0.018]
Fear	[0, 0.469]	[0, 0.143]	[0.004, 0.031]	[0.009, 0.043]	[0.006, 0.03]
Disgust	[0.016, 0.081]	[0.005, 0.029]	[0.006, 0.012]	[0.011, 0.019]	[0.008, 0.014]
Happiness	[0.031, 0.094]	[0.011, 0.035]	[0.009, 0.011]	[0.013, 0.018]	[0.01, 0.013]
Sadness	[0.034, 0.228]	[0.011, 0.085]	[0.008, 0.018]	[0.015, 0.03]	[0.011, 0.021]
Anger	[0.039, 0.151]	[0.015, 0.057]	[0.01, 0.018]	[0.015, 0.029]	[0.011, 0.022]
Neutral	[0.053, 0.149]	[0.018, 0.063]	[0.009, 0.012]	[0.016, 0.024]	[0.012, 0.017]

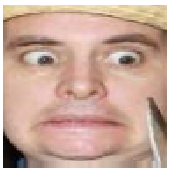
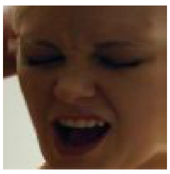
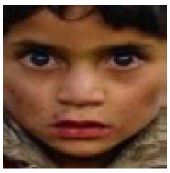
Table 4. Sample images with their parameters

img_name	Param_1	Param_2	Param_3	Param_4	Param_5
train_01220	0.023	0.006	0.008	0.008	0.005
train_03104	0.076	0.027	0.013	0.019	0.015
train_10549	0.044	0.019	0.006	0.013	0.010
train_00339	0.138	0.038	0.019	0.023	0.016
train_00467	0.032	0.011	0.009	0.013	0.009
train_03320	0.014	0.007	0.005	0.009	0.007
train_07499	0.071	0.031	0.010	0.017	0.011
train_03041	0.095	0.033	0.020	0.023	0.018
train_03953	0.016	0.005	0.007	0.008	0.006
train_03849	0.136	0.046	0.015	0.023	0.018

3. Results and Discussion

The lower and upper rough decision sets obtained from the mathematical model implemented are tabulated in Table 5. The results obtained are in accordance with the labels provided by the dataset. The use of rough decision making enables the nuanced labelling of data by ranking the possibility of the emotion using facial parameters. This method yields better results in instances that have greater uncertainty in the emotion classification.

Table 5. Results

Test image	Lower rough decision	Upper rough decision	Label	Intervals contained	Rough decision
	[‘fear’]	[‘ang’, ‘sur’, ‘sad’]	fear	[[‘sur’, 2], [‘fear’, 5], [‘disg’, 0], [‘happ’, 0], [‘sad’, 2], [‘ang’, 1], [‘neu’, 0]]	100% fear
	[‘fear’]	[‘sur’]	fear	[[‘sur’, 1], [‘fear’, 5], [‘disg’, 0], [‘happ’, 0], [‘sad’, 0], [‘ang’, 0], [‘neu’, 0]]	100% fear
	[]	[‘happ’, ‘sur’, ‘disg’, ‘fear’, ‘sad’]	sad	[[‘sur’, 1], [‘fear’, 1], [‘disg’, 1], [‘happ’, 1], [‘sad’, 2], [‘ang’, 0], [‘neu’, 0]]	16.66%-surprise, 16.66%-fear, 16.66%-disgust, 16.66%-happy, 33.33%-sad

4. Conclusion

The novel rough decision making mathematical model was introduced by defining upper and lower rough decision sets. Interval-valued fuzzy sets were utilized to proficiently handle the uncertainty that exists in datasets. Further, this framework was implemented and tested on a real-time database of images RAF-DB for facial expression detection. Future work within the application explored in this paper can extend to classification into complex emotions as well as applications such as detection of attention levels in students using facial expressions. Further, this mathematical model can also be implemented and examined for decision-making models in various fields.

5. Data Availability

The Real-world Affective Faces Database (RAF-DB) is private and available for non-commercial research purposes only. For analyzing the facial landmarks the images were processed and features extracted into a CSV file containing the following parameters:

- Image Name
- Age
- Gender
- x and y coordinates of 5 accurate landmark locations
- x and y coordinates of 37 automatically annotated landmark locations

Acknowledgements

We extend our sincere gratitude to the management and principal of Sri Sivasubramaniya Nadar College of Engineering for their support and encouragement by providing a platform for conducting impactful research. Special thanks are extended to Dr. S. Li and Dr. W. Dong for providing the RAF-DB database, enabling the implementation of our mathematical model.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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