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Research Article

Independent Domination Degree of Standard Graphs of Adriatic (*a*,*b*)-KA Indices

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Abstract. The dominating set D of the graph $K=(V,E)$ is the independent dominating set (Ids), the independent domination number $i(K)$ of the graph K is the minimum cardinality of id. In this article, we introduce the new independent degree domination (idd) of each vertices $s\in V(K)$, denoted by $d_{id}(s)$ and compute the Adriatic (a,b)-KA index for book graphs, cycle middle graphs and windmill graphs.

Keywords. Topological index, Adriatic (a,b)-KA index, Independent degree domination, Independent minimal dominating number

Mathematics Subject Classification (2020). 05C05, 05C12, 05C35

1. Introduction

Let K be a simple graph with vertex set $V(K)$ and edge set $E(K)$. The degree of independence of $d_{id}(s)$ of vertex s is the number of edges contained in s. The Ids of K are the dominating and independent set in K. The independent domination number of K denoted by $i(K)$ is the minimum *id* size, and $\alpha(K)$ is the maximum *id* size of k.

For any vertex $s \in V(K)$, the *independent domination degree* (*idd*) ([4], [5], [7], [13]) denoted by $d_{id}(s)$ and defined as the number of minimal dominating sets of K which contain s. The degree of an independent domination, both minimum and maximum are denoted $\delta_{id}(K) = \delta_{id}$ and $\Delta_{id}(K) = \Delta_{id}$, respectively, where $\delta_{id} = \min\{d_{id}(s) : s \in V(K)\}$ and $\Delta_{id} = \max\{d_{id}(s) : s \in V(K)\}$.

The misbalance independent degree index [1] of K is defined as

$$\alpha_1(K) = \sum_{rs\in E(K)} |d_{id}(r) - d_{id}(s)|.$$

Minus F-index or nonzero Zagreb index [9] and Jahabani *et al*. in [8], is

$$MF(K) = \sum_{rs\in E(K)} |d_{id}(r)^2 - d_{id}(s)^2|.$$

The σ index [6] of a graph K,

$$\sigma(K) = \sum_{rs\in E(K)} [d_{id}(r) - d_{id}(s)]^2.$$

The misbalance independent indeg index [14] of K defined as

$$\alpha_{-1}(K) = \sum_{rs\in E(K)} \left|\frac{1}{d_{id}(r)} - \frac{1}{d_{id}(s)}\right|.$$

The misbalance independent irdeg index of K is defined as

$$\alpha_{-\frac{1}{2}}(K) = \sum_{rs\in E(K)} \left|\frac{1}{\sqrt{d_{id}(r)}} - \frac{1}{\sqrt{d_{id}(s)}}\right|.$$

The misbalance independent rodeg index of K is

$$\alpha_{\frac{1}{2}}(K) = \sum_{rs\in E(K)} |\sqrt{d_{id}(r)} - \sqrt{d_{id}(s)}|.$$

The general independent minus index [10] of a graph K is defined as

$$M_1^a(K) = \sum_{rs\in E(K)} [|d_{id}(r) - d_{id}(s)|]^a,$$

where a is a real number.

The misbalance independent sdeg index [11] of a graph K is

$$\alpha_{-2}(K) = \sum_{rs\in E(K)} \left|\frac{1}{d_{id}(r)^2} - \frac{1}{d_{id}(s)^2}\right|.$$

The general independent misbalance deg index [12] of a graph K is

$$\alpha_a(K) = \sum_{rs\in E(K)} [|d_{id}(r)^a - d_{id}(s)^a|],$$

where $a = \{-\frac{1}{2}, \frac{1}{2}, -1, 1\}$.

In [3], the Randic index, is

$$IRA(K)=\sum_{rs\in E(K)}\left(\frac{1}{\sqrt{d_{id}(r)}}-\frac{1}{\sqrt{d_{id}(s)}}\right)^2.$$

In [2], the IRB index of graph K is

$$IRB(G)=\sum_{rs\in E(K)}(\sqrt{d_{id}(r)}-\sqrt{d_{id}(s)})^2.$$

The Adriatic (a,b)-KA index and coindex of a graph K as

$$MKA^1_{a,b}(K)=\sum_{rs\in E(K)}[|d_{id}(r)^a-d_{id}(s)^a|]^b,$$

$$\overline{MKA}^1_{a,b}(K)=\sum_{rs\notin E(K)}[|d_{id}(r)^a-d_{id}(s)^a|]^b.$$

We easily see that

(i) $\alpha_1(K)=MKA^1_{1,1}(K)$,

(ii) $MF(K)=MKA^1_{2,1}(K)$,

(iii) $\sigma(K)=MKA^1_{1,2}(K)$,

(iv) $\alpha_{-1}(K)=MKA^1_{-1,1}(K)$,

(v) $\alpha_{-\frac{1}{2}}(K)=MKA^1_{-\frac{1}{2},1}(K)$,

(vi) $\alpha_{\frac{1}{2}}(K)=MKA^1_{\frac{1}{2},1}(K)$,

(vii) $M_1^a(K)=MKA^1_{1,a}(K)$,

(viii) $\alpha_{-2}(K)=MKA^1_{-2,1}(K)$,

(ix) $\alpha_a(K)=MKA^1_{a,1}(K)$,

(x) $IRA(K)=MKA^1_{-\frac{1}{2},2}(K)$,

(xi) $IRB(K)=MKA^2_{\frac{1}{2},2}(K)$.

2. Main Results

We compute the (a,b)-KA index of book graph, cycle middle graph and windmill graph.

2.1 Book Graph

Let $K=B_f$ is a book graph, there are two kinds of edges based on the *idd* of the end vertices of each edges, as shown in Table 1.

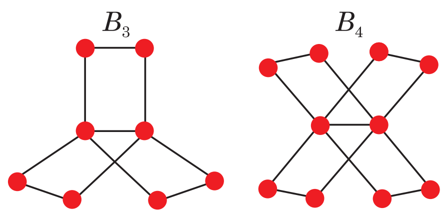

Figure 1. Book graph of B_3, B_4

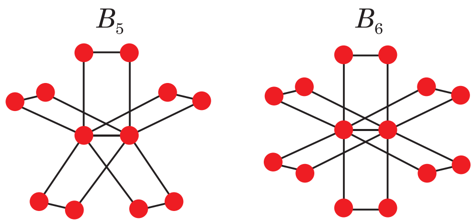

Figure 2. Book graph of B_5, B_6

Table 1. Edge partition of book graph

$d_{id}(r)$, $d_{id}(s)/rs \in E(G)$	$(f+1,1)$	$(f+1,f+1)$
Number of edges	$2f$	f

Theorem 2.1. *Let $K = B_f$ be a book graph, then*

$$MKA^1_{a,b}(K) = (|(f+1)^a - 1^a|^b)2f.$$

Proof. Using definition and Table 1, we deduce

$$\begin{aligned} MKA^1_{a,b}(K) &= \sum_{rs\in E(K)} [|d_{id}(r)^a - d_{id}(s)^a|]^b \\ &= (|(f+1)^a - 1^a|^b)|E_1| + (|(f+1)^a - (f+1)^a|^b)|E_2| \\ &= (|(f+1)^a - 1^a|^b)2f + (|(f+1)^a - (f+1)^a|^b)f \\ &= (|(f+1)^a - 1^a|^b)2f. \end{aligned}$$ □

From Theorem 2.1. Note the following results.

Result 2.1. (i) $\alpha_1(K) = MKA^1_{1,1}(K) = 2f^2$,

(ii) $MF(K) = MKA^1_{2,1}(K) = 2f^3 + 4f^2$,

(iii) $\sigma(K) = MKA^1_{1,2}(K) = 2f^3$,

(iv) $\alpha_{-1}(K) = MKA^1_{-1,1}(K) = \left(\frac{1}{f+1} - 1\right)2f$,

(v) $\alpha_{-\frac{1}{2}}(K) = MKA^1_{-\frac{1}{2},1}(K) = \left(\frac{1}{\sqrt{f+1}} - 1\right)2f$,

(vi) $\alpha_{\frac{1}{2}}(K) = MKA^1_{\frac{1}{2},1}(K) = (\sqrt{f+1} - 1)2f$,

(vii) $M^a_1(K) = MKA^1_{1,a}(K) = f^a\, 2f$,

(viii) $\alpha_{-2}(K) = MKA^1_{-2,1}(K) = \left(\frac{1}{(f+1)^2} - 1\right)2f$,

(ix) $\alpha_a(K) = MKA^1_{a,1}(K) = ((f+1)^a - 1)2f$,

(x) $IRA(K) = MKA^1_{-\frac{1}{2},2}(K) = \left(\frac{1}{\sqrt{f+1}} - 1\right)^2 2f$,

(xi) $IRB(K) = MKA^1_{\frac{1}{2},2}(K) = (\sqrt{f+1} - 1)^2\, 2f$.

2.2 Middle Cycle Graph

Let $K = M(C_f)$ be a middle cycle graph, there are 3 types of edge based on *idd* of end vertices of each edges as given in Table 2.

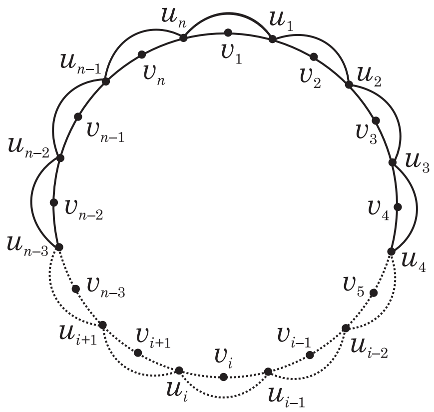

Figure 3. Middle cycle graph

Table 2. Edge partition of middle cycle graph

$d_{id}(r)$, $d_{id}(s)/rs \in E(K)$	$(f-2,f-2)$	$(f-2,f-1)$	$(f-1,f-2)$
Number of edges	$2f$	$2f$	$2f$

Theorem 2.2. *Let $K = M(C_f)$ be a middle cycle graph, then*

$$MKA^1_{a,b}(K) = (|(f-2)^a - (f-1)^a|^b)2f + (|(f-1)^a - (f-2)^a|^b)2f\,.$$

Proof. Using definition and Table 2, we deduce

$$\begin{aligned} MKA^1_{a,b}(K) &= \sum_{rs\in E(K)} [|d_{id}(r)^a - d_{id}(s)^a|]^b \\ &= (|(f-2)^a-(f-2)^a|^b)|E_1| + (|(f-2)^a-(f-1)^a|^b)|E_2| + (|(f-1)^a-(f-2)^a|^b)|E_3| \\ &= (|(f-2)^a-(f-2)^a|^b)2f + (|(f-2)^a-(f-1)^a|^b)2f + (|(f-1)^a-(f-2)^a|^b)2f \\ &= (|(f-2)^a-(f-1)^a|^b)2f + (|(f-1)^a-(f-2)^a|^b)2f\,. \end{aligned}$$ □

From Theorem 2.2. We establish the following results.

Remark 2.1.

(i) $\alpha_1(K) = MKA^1_{1,1}(K) = 4f$,

(ii) $MF(K) = MKA^1_{2,1}(K) = |-2f+3|2f + |2f-3|2f$,

(iii) $\sigma(K) = MKA^1_{1,2}(K) = 4f$,

(iv) $\alpha_{-1}(K) = MKA^1_{-1,1}(K) = \frac{4f}{(f-1)(f-2)}$,

(v) $\alpha_{-\frac{1}{2}}(K) = MKA^1_{-\frac{1}{2},1}(K) = \left|\frac{1}{\sqrt{f-2}} - \frac{1}{\sqrt{f-1}}\right|(fq) + \left|\frac{1}{\sqrt{f-1}} - \frac{1}{\sqrt{f-2}}\right|(2f)$,

(vi) $\alpha_{\frac{1}{2}}(K) = MKA^1_{\frac{1}{2},1}(K) = |\sqrt{f-2} - \sqrt{f-1}|(2f) + |\sqrt{f-1} - \sqrt{f-1}|(2f)$,

(vii) $M_1^a(K) = MKA^1_{1,a}(K) = 4f$,

(viii) $\alpha_{-2}(K) = MKA^1_{-2,1}(K) = \left|\frac{3}{(f-1)^2}(f-2)^2\right|(4f)$,

(ix) $\alpha_a(K) = MKA^1_{a,1}(K) = |(f-2)^a - (f-1)^a|(2f) + |(f-1)^a - (f-2)^a|(2f)$,

(x) $IRA(K) = MKA^1_{-\frac{1}{2},2}(K) = \left(\left|\frac{1}{(f-2)^{\frac{1}{2}}} - \frac{1}{(f-1)^{\frac{1}{2}}}\right|^2\right)(2f) + \left(\left|\frac{1}{(f-1)^{\frac{1}{2}}} - \frac{1}{(f-2)^{\frac{1}{2}}}\right|^2\right)(2f)$,

(xi) $IRB(K) = MKA^2_{\frac{1}{2},2}(K) = (|(f-2)^{\frac{1}{2}} - (f-1)^{\frac{1}{2}}|^2)(2f) + (|(f-1)^{\frac{1}{2}} - (f-2)^{\frac{1}{2}}|^2)(2f)$.

2.3 Windmill Graph

Let $K = W_f^g$ be a windmill graph. By calculation, we find that K has $(g-1)f+1$ vertices and $\frac{fg(g-1)}{2}$ edges. In a windmill graph there are two types of edge based on independent degree (id) of end vertices of each edges as given in Table 3.

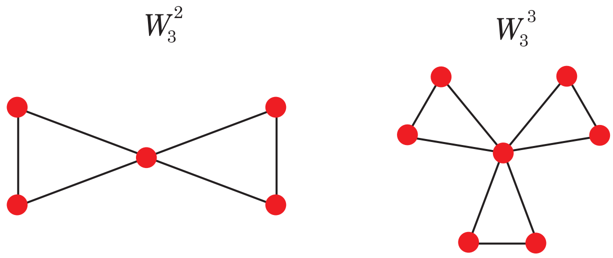

Figure 4. Windmill graph of W_3^2, W_3^3

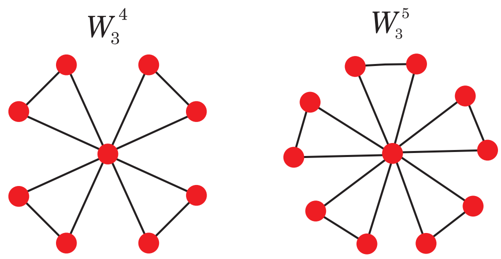

Figure 5. Windmill graph of W_3^4, W_3^5

Table 3. Edge partition of Windmill graph

$d_{id}(r), d_{id}(s)/rs \in E(K)$	$(g-1, g-1)$	$(g-1, (g-1)f)$
Number of edges	$\frac{(g-1)(g-2)f}{2}$	$(g-1)q$

Theorem 2.3. *Let $K = W_f^g$ be a windmill graph, then*

$$MKA^1_{a,b}(K) = (|(g-1)^a - ((g-1)q)^a|^b)(g-1)f.$$

Proof. Using definition and Table 3, we deduce

$$
\begin{aligned}
MKA^1_{a,b}(K) &= \sum_{rs\in E(K)} [|d_{id}(r)^a - d_{id}(s)^a|]^b \\
&= (|(g-1)^a - (g-1)^a|^b\,|E_1| + (|(g-1)^a - ((g-1)f)^a|^b)|E_2| \\
&= (|(g-1)^a - (g-1)^a|^b)\frac{(g-1)(g-2)f}{2} + (|(g-1)^a - ((g-1)f)^a|^b)(g-1)f \\
&= (|(g-1)^a - ((g-1)f)^a|^b)(g-1)f. \qquad \square
\end{aligned}
$$

From Theorem 2.2. We establish the following results.

Result 2.2. (i) $\alpha_1(K) = MKA^1_{1,1}(K) = (|(g-1) - ((g-1)f)|)(g-1)f$,

(ii) $MF(K) = MKA^1_{2,1}(K) = (|(g-1)^2 - ((g-1)q)^2|)(g-1)f$,

(iii) $\sigma(K) = MKA^1_{1,2}(K) = (|(g-1) - ((g-1)f)|^2)(g-1)f$,

(iv) $\alpha_{-1}(K) = MKA^1_{-1,1}(K) = (|(g-1)^{-1} - ((g-1)f)^{-1}|)(g-1)f$,

(v) $\alpha_{-\frac{1}{2}}(K) = MKA^1_{-\frac{1}{2},1}(K) = (|(g-1)^{\frac{-1}{2}} - ((g-1)f)^{\frac{-1}{2}}|)(g-1)f$,

(vi) $\alpha_{\frac{1}{2}}(K) = MKA^1_{\frac{1}{2},1}(K) = (|(g-1)^{\frac{1}{2}} - ((g-1)f)^{\frac{1}{2}}|)(g-1)f$,

(vii) $M^a_1(K) = MKA^1_{1,a}(K) = (|(g-1) - ((g-1)f)|^a)(g-1)f$,

(viii) $\alpha_{-2}(K) = MKA^1_{-2,1}(K) = (|(g-1)^{-2} - ((g-1)f)^{-2}|)(g-1)f$,

(ix) $\alpha_a(K) = MKA^1_{a,1}(K) = (|(g-1)^a - ((g-1)f)^a|)(g-1)f$,

(x) $IRA(K) = MKA^1_{-\frac{1}{2},2}(K) = (|(g-1)^{\frac{-1}{2}} - ((g-1)f)^{\frac{-1}{2}}|^2)(g-1)f$,

(xi) $IRB(K) = MKA^2_{\frac{1}{2},2}(K) = (|(g-1)^{\frac{1}{2}} - ((g-1)f)^{\frac{1}{2}}|^2)(g-1)f$.

3. Conclusion

In this paper, the precise values for the independent degree domination indices number of book graphs, middle graph of cycles and windmill graphs are computed.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] M. O. Albertson, The irregularity of a graph, *Ars Combinatoria* **46** (1997), 219 – 225.

[2] A. Alwardi, A. Alqesmah, R. Rangarajan and I. N. Cangul, Entire Zagreb indices of graphs, *Discrete Mathematics, Algorithms and Applications* **10**(3) (2018), 1850037, DOI: 10.1142/S1793830918500374.

[3] M. Atapour, A. Jahanbani and R. Khoeilar, New bounds for the Randić index of graphs, *Journal of Mathematics* **2021** (2021), Article ID 9938406, 8 pages, DOI: 10.1155/2021/9938406.

[4] F. V. Fomin, F. Grandoni, A. V. Pyatkin and A. A. Stepanov, On maximum number of minimal dominating sets in graphs, *Electronic Notes in Discrete Mathematics* **22** (2005), 157 – 162, DOI: 10.1016/j.endm.2005.06.028.

[5] W. Goddard and M. A. Henning, Independent domination in graphs: A survey and recent results, *Discrete Mathematics* **313**(7) (2013), 839 – 854, DOI: 10.1016/j.disc.2012.11.031.

[6] I. Gutman, M. Togan, A. Yurttas, A. S. Cevik and I. N. Cangul, Inverse problem of sigma index, *MATCH Communications in Mathematical and in Computer Chemistry* **79** (2018), 491 – 508, URL: https://match.pmf.kg.ac.rs/electronic_versions/Match79/n2/match79n2_491-508.pdf.

[7] T. W. Haynes, S. Hedetniemi and P. Slater, *Fundamentals of Domination in Graphs*, 1st edition, CRC Press, Boca Raton, 464 pages (1998), DOI: 10.1201/9781482246582.

[8] A. Jahanbani and H. Shooshtary, Nano-Zagreb index and multiplicative nano-zagreb index of some graph operations, *International Journal of Computing Science and Applied Mathematics* **5**(1) (2019), 15 – 22, DOI: 10.12962/j24775401.v5i1.4659.

[9] V. R. Kulli, Minus F and square F-indices and their polynomials of certain dendrimers, *Earthline Journal of Mathematical Sciences* **1**(2) (2019), 172 – 185, DOI: 10.34198/ejms.1219.171185.

[10] V. R. Kulli, B. Chaluvaraju and T. Vidya, Computation of adriatic (a,b)-KA index of some nanostructues, *International Journal of Mathematics Trends and Technology* **67**(4) (2021), 79 – 87, DOI: 10.14445/22315373/IJMTT-V67I4P511.

[11] V. Lokesha, K. Z. Yasmeen, B. Chaluvaraju and T. Deepika, Computation of misbalance type degree indices of certain classes of derived graph, *International Journal of Mathematical Combinatorics* **3** (2019), 61 – 69, URL: https://fs.unm.edu/IJMC/IJMC-3-2019.pdf.

[12] T. Réti, R. Sharfdini, Á. D. Kiss and H. Hagobin, Graph irregularity indices used as molecular Descriptors in QSPR studies, *MATCH Communications in Mathematical and in Computer Chemistry* **79** (2018), 509 – 524, URL: https://match.pmf.kg.ac.rs/electronic_versions/Match79/n2/match79n2_509-524.pdf.

[13] M. L. Roden and P. J. Slater, Liar's domination in graphs, *Discrete Mathematics* **309**(19) (2009), 5884 – 5890, DOI: 10.1016/j.disc.2008.07.019.

[14] D. Vukičević and M. Gaśperov, Bond additive modeling 1. Adriatic indices, *Croatica Chemica Acta* **83**(3) (2010), 243 – 260, URL: https://hrcak.srce.hr/file/93215.