



**Special Issue:**

**Recent Trends in Applied and Computational Mathematics**

**Proceedings of the Third International Conference on Recent Trends in Applied and Computational Mathematics (ICRTACM-2022)**

School of Applied Sciences, Department of Mathematics,  
Reva University, Bengaluru, India, 10th & 11th October, 2022

**Editors:** M. Vishu Kumar, A. Salma, B. N. Hanumagowda and U. Vijaya Chandra Kumar

Research Article

# M-Polynomials and Degree-Based Topological Indices of Mycielskian of Paths and Cycles

H. C. Shilpa<sup>\*1</sup> , K. Gayathri<sup>1</sup> , H. M. Nagesh<sup>2</sup>  and N. Narahari<sup>3</sup> 

<sup>1</sup>Department of Mathematics, School of Applied Sciences, REVA University, Kattigenahalli, Bangalore, India

<sup>2</sup>Department of Science & Humanities, PES University, Bangalore, India

<sup>3</sup>Department of Mathematics, University College of Science, Tumkuru University, Tumakuru, India

\*Corresponding author: shilpahc539@gmail.com

**Received:** January 31, 2023

**Accepted:** June 14, 2023

**Abstract.** For a graph  $G$ , the M-polynomial is defined as  $M(G; x, y) = \sum_{\delta \leq \alpha \leq \beta \leq \Delta} m_{\alpha\beta}(G) x^\alpha y^\beta$ , where  $m_{\alpha\beta}(\alpha, \beta \geq 1)$ , is the number of edges  $ab$  of  $G$  such that  $\deg_G(a) = \alpha$  and  $\deg_G(b) = \beta$ , and  $\delta$  is the minimum degree and  $\Delta$  is the maximum degree of  $G$ . The physiochemical properties of chemical graphs are found by topological indices, in particular, the degree-based topological indices, which can be determined from an algebraic formula called M-polynomial. We compute the closest forms of M-polynomial for Mycielskian of paths and cycles. Further, we plot the 3-D graphical representation of M-polynomial. Finally, we derive some degree-based topological indices with the help of M-polynomial.

**Keywords.** Topological indices, M-polynomial, Mycielskian of a graph, Path, Cycle

**Mathematics Subject Classification (2020).** 05C07, 05C31

Copyright © 2023 H. C. Shilpa, K. Gayathri, H. M. Nagesh and N. Narahari. *This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.*

## 1. Introduction

For all terms and definitions we refer to Harary [5]. Let the vertex set of a simple connected graph  $G$  be  $V(G)$  and let  $E(G)$  be its edge set, and let  $n$  and  $m$ , respectively, denotes the order and size of  $G$ . A *chemical graph* is a labeled graph where the atoms correspond to the vertices and the chemical bonds of the compound corresponds to the edges. A numerical quantity which is used to analyse both the physical and chemical properties of compounds is termed as a topological index. A topological index is also called a *graph invariant*. In general, the physiochemical properties and boiling activities of a chemical graph are investigated using topological indices.

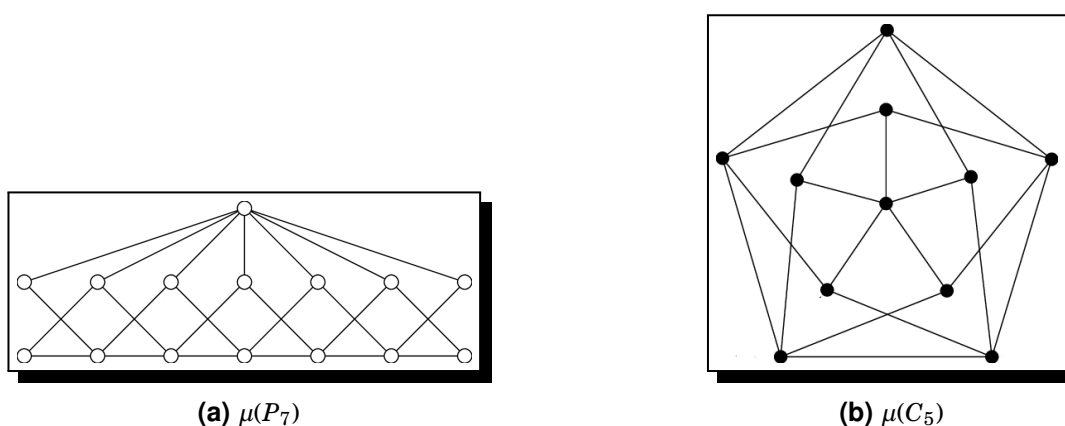
H. Wiener [13] initiated the study of topological indices in the year 1947. The concept of Wiener index was first introduced by Wiener [13] mainly to know the correspondence of the attributes of molecules in a compound along with structural property. Hosoya [6] explained the Wiener index using the concept of distance between vertices in a graph in the year 1972. For more work done on topological indices, we refer the reader to Brückler *et al.* [1], Das *et al.* [2], and Fath-Tabar *et al.* [4].

There are many algebraic polynomials available in the literature. One such a polynomial is Hosoya polynomial, which is used to determine the distance-based topological indices. An important class of algebraic polynomial introduced by Deutsch and Klavžar [3] called *M*-polynomial is used to determine the closest form of various topological indices based on degree. For more work done on finding topological indices using *M*-polynomial, we refer the reader to Khalaf *et al.* [7], Kwun *et al.* [8], Munir *et al.* [10, 11], and Swamy *et al.* [12].

The notion of Mycielskian graph of a given graph  $G$  by Lin *et al.* [9] is defined as follows:

**Definition 1.1.** For a graph  $G$ , the *Mycielskian* of  $G$ , denoted by  $\mu(G)$ , is the graph with  $V(\mu(G)) = X \cup Y \cup \{b\}$  such that  $x_i x_j \in E(\mu(G)) \iff x_i x_j \in E(G)$ , with  $x_i y_j \in E(\mu(G)) \iff x_i x_j \in E(G)$ , with  $y_i b \in E(\mu(G))$ ,  $i \in [1, n]$ ; and no more edges in  $E(\mu(G))$ , where  $x_i \in V(G)$  and  $y_i \in Y$ .

Figure 1(a) and Figure 1(b) shows an example of Mycielskian of a path  $P_7$  and a cycle  $C_5$ , respectively.



**Figure 1**

Inspired by the studies as mentioned above, we aim to calculate the *M*-polynomial and degree-based topological indices of  $\mu(P_7)$  and  $\mu(C_5)$ .

## 2. Methodology

We procedure that we follow is as follows:

- Step 1:* Initially,  $E(\mu(P_7))$  and  $E(\mu(C_5))$  are divided into separate classes depends on the end vertices degree.
- Step 2:* With the help of this edge division (*Step 1*), we compute the M-polynomial of Mycielskian of paths and cycles.
- Step 3:* The degree-based topological indices as listed in Section 3 are computed using M-polynomial.
- Step 4:* Using MATLAB, the 3-D graph corresponding to M-polynomials are plotted.

## 3. Preliminaries

**Definition 3.1.** The M-polynomial of a graph  $G$  is defined as  $M(G; x, y) = \sum_{\delta \leq \alpha \leq \beta \leq \Delta} m_{\alpha\beta}(G)x^\alpha y^\beta$ , where  $m_{\alpha\beta}(\alpha, \beta \geq 1)$ , is the number of edges  $ab$  of  $G$  such that  $\deg_G(a) = \alpha$  and  $\deg_G(b) = \beta$ , and  $\delta$  is the minimum degree and  $\Delta$  is the maximum degree of  $G$ .

Using M-polynomial, degree-based topological indices are derived with the help of operations as mentioned in following table. Let  $M(G; x, y) = f(x, y)$ . Then using M-polynomial, degree-based topological indices are derived with the help of operations as mentioned below.

Notation	Topological Index	Derivation from $M(G; x, y)$
$M_1(G)$	First Zagreb index	$(D_x + D_y)(f(x, y)) _{x=1; y=1}$
$M_2(G)$	Second Zagreb index	$(D_x D_y)(f(x, y)) _{x=1; y=1}$
${}^m M_2(G)$	Second modified Zagreb index	$(S_x S_y)(f(x, y)) _{x=1; y=1}$
$SSD(G)$	Symmetric division index	$(D_x S_y + D_y S_x)(f(x, y)) _{x=1; y=1}$
$H(G)$	Harmonic index	$2S_x J(f(x, y)) _{x=1}$
$I(G)$	Inverse sum index	$S_x J D_x D_y(f(x, y)) _{x=1}$

Here,  $D_x(f(x, y)) = x \left( \frac{\partial f(x, y)}{\partial x} \right)$ ,  $D_y(f(x, y)) = y \left( \frac{\partial f(x, y)}{\partial y} \right)$ ,  $S_x(f(x, y)) = \int_0^x \left( \frac{f(t, y)}{t} \right) dt$ ,  $S_y(f(x, y)) = \int_0^y \left( \frac{f(x, t)}{t} \right) dt$ , and  $J[f(x, y)] = f(x, x)$  are the operators.

## 4. M-polynomial of Mycielskian of Paths

In this section, we find the M-polynomial of Mycielskian of paths.

**Theorem 4.1.** Let  $P_n$  be a path of order  $n \geq 2$ . Then  $M(\mu(G))$  is

$$M(\mu(G); x, y) = 2x^2y^3 + 4x^2y^4 + 2x^2y^n + 2(n - 3)x^3y^4 + (n - 2)x^3y^n + (n - 3)x^4y^4.$$

*Proof.* Let  $G = P_n$  be a path of order  $n, n \geq 2$ . It is easy to observe from Figure 1(a) that

$$|V(\mu(G))| = 2n + 1 \quad \text{and} \quad |E(\mu(G))| = 4n - 3.$$

Since each vertex of  $G$  is of degree either 2 or 3 or 4 or  $n$ , the partitions of  $V(\mu(G))$  be:

$$V_1(\mu(G)) := \{v \in V(\mu(G)) : \deg_{\mu(G)}(v) = 2\},$$

$$\begin{aligned}
 V_2(\mu(G)) &:= \{v \in V(\mu(G)) : \deg_{\mu(G)}(v) = 3\}, \\
 V_3(\mu(G)) &:= \{v \in V(\mu(G)) : \deg_{\mu(G)}(v) = 4\}, \\
 V_4(\mu(G)) &:= \{v \in V(\mu(G)) : \deg_{\mu(G)}(v) = n\}.
 \end{aligned}$$

Clearly,

$$|V_1(\mu(G))| = 4, \quad |V_2(\mu(G))| = n - 2, \quad |V_3(\mu(G))| = n - 2, \quad |V_4(\mu(G))| = 1.$$

Furthermore, the partitions of edge set  $E(\mu(G))$  are:

$$\begin{aligned}
 E_1(\mu(G)) &:= \{e = ab \in E(\mu(G)) : \deg_{\mu(G)}(a) = 2, \deg_{\mu(G)}(b) = 3\}, \\
 E_2(\mu(G)) &:= \{e = ab \in E(\mu(G)) : \deg_{\mu(G)}(a) = 2, \deg_{\mu(G)}(b) = 4\}, \\
 E_3(\mu(G)) &:= \{e = ab \in E(\mu(G)) : \deg_{\mu(G)}(a) = 2, \deg_{\mu(G)}(b) = n\}, \\
 E_4(\mu(G)) &:= \{e = ab \in E(\mu(G)) : \deg_{\mu(G)}(a) = 3, \deg_{\mu(G)}(b) = 4\}, \\
 E_5(\mu(G)) &:= \{e = ab \in E(\mu(G)) : \deg_{\mu(G)}(a) = 3, \deg_{\mu(G)}(b) = n\}, \\
 E_6(\mu(G)) &:= \{e = ab \in E(\mu(G)) : \deg_{\mu(G)}(a) = 4, \deg_{\mu(G)}(b) = 4\}.
 \end{aligned}$$

Clearly,

$$\begin{aligned}
 |E_1(\mu(G))| &= 2, \quad |E_2(\mu(G))| = 4, \quad |E_3(\mu(G))| = 2, \quad |E_4(\mu(G))| = 2(n - 3), \\
 |E_5(\mu(G))| &= n - 2, \quad |E_6(\mu(G))| = n - 3.
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 M(G; x, y) &= \sum_{\delta \leq \alpha \leq \beta \leq \Delta} m_{\alpha\beta}(\mu(G)) x^\alpha y^\beta \\
 &= m_{23}(\mu(G)) x^2 y^3 + m_{24}(\mu(G)) x^2 y^4 + m_{2n}(\mu(G)) x^2 y^n + m_{34}(\mu(G)) x^3 y^4 \\
 &\quad + m_{3n}(\mu(G)) x^3 y^n + m_{44}(\mu(G)) x^4 y^4 \\
 &= 2x^2 y^3 + 4x^2 y^4 + 2x^2 y^n + 2(n - 3)x^3 y^4 + (n - 2)x^3 y^n + (n - 3)x^4 y^4. \quad \square
 \end{aligned}$$

For Mycielskian of paths, degree-based topological indices are computed using this *M*-polynomial in the next theorem.

**Theorem 4.2.** *Let  $G = P_n$  be a path of order  $n \geq 2$ . Then,*

$$\begin{aligned}
 M_1(\mu(G)) &= n^2 + 25n - 34, \\
 M_2(\mu(G)) &= 3n^2 + 38n - 76, \\
 {}^m M_2(\mu(G)) &= \frac{11n^2 + 23n + 16}{48n}, \\
 SSD(\mu(G)) &= \frac{2n^3 + 39n^2 - 7n - 12}{6n}, \\
 H(\mu(G)) &= \frac{345n^3 + 2426n^2 + 3055n + 846}{420n^2 + 2100n + 2520}, \\
 I(\mu(G)) &= \frac{885n^3 + 2372n^2 - 1070n - 5388}{105n^2 + 525n + 630}.
 \end{aligned}$$

*Proof.* From Theorem 4.1, we have

$$M(\mu(G); x, y) = 2x^2 y^3 + 4x^2 y^4 + 2x^2 y^n + 2(n - 3)x^3 y^4 + (n - 2)x^3 y^n + (n - 3)x^4 y^4.$$

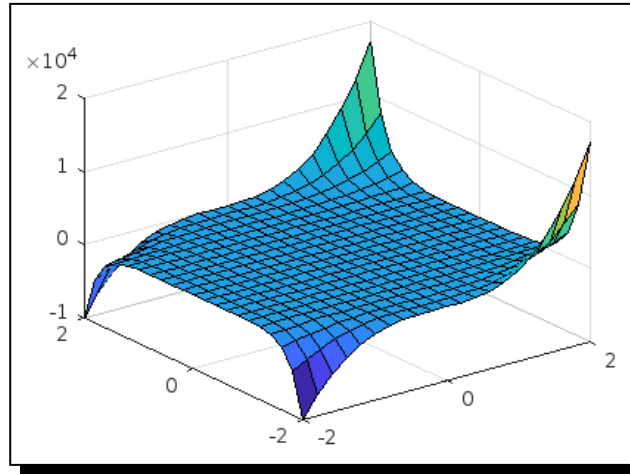
Then, we have

$$\begin{aligned}
 D_x(f(x, y)) &= 3(n-2)x^3y^n + 4x^2y^n + 4(n-3)x^4y^4 + 6(n-3)x^3y^4 + 8x^2y^4 + 4x^2y^3, \\
 D_y(f(x, y)) &= n(n-2)x^3y^n + 2nx^2y^n + 4(n-3)x^4y^4 + 8(n-3)x^3y^4 + 16x^2y^4 + 6x^2y^3, \\
 (D_yD_x)(f(x, y)) &= 3n(n-2)x^3y^n + 4nx^2y^n + 16(n-3)x^4y^4 + 24(n-3)x^3y^4 + 32x^2y^4 + 12x^2y^3, \\
 S_x(f(x, y)) &= \frac{((4n-8)x^3 + 12x^2)y^n + ((3n-9)x^4 + (8n-24)x^3 + 24x^2)y^4 + 12x^2y^3}{12}, \\
 S_y(f(x, y)) &= \frac{((12n-24)x^3 + 24x^2)y^n + ((3n^2-9n)x^4 + (6n^2-18n)x^3 + 12nx^2)y^4 + 8nx^2y^3}{12n}, \\
 S_xS_y(f(x, y)) &= \frac{((16n-32)x^3 + 48x^2)y^n + ((3n^2-9n)x^4 + (8n^2-24n)x^3 + 24nx^2)y^4 + 16nx^2y^3}{48n}, \\
 S_yD_x(f(x, y)) &= \frac{x(((18n-36)x^2 + 24x)y^n + ((6n^2-18n)x^3 + (9n^2-27n)x^2 + 12nx)y^4 + 8nxy^3)}{6n}, \\
 S_xD_y(f(x, y)) &= \frac{((n^2-2n)x^3 + 3nx^2)y^n + ((3n-9)x^4 + (8n-24)x^3 + 24x^2)y^4 + 9x^2y^3}{3}, \\
 2S_xJ(f(x, y)) &= \frac{2(105n^3 + 210n^2 - 945n - 1890)x^8 + 2(240n^3 + 480n^2 - 2160n - 4320)x^7}{840n^2 + 4200n + 5040} \\
 &\quad + \frac{2(560n^2 + 2800n + 3360)x^6 + 2(336n^2 + 1680n + 2016)x^5}{840n^2 + 4200n + 5040} \\
 &\quad + \frac{2x^{n+3}(840n^2 - 3360) + 2x^{n+2}(1680n + 5040)}{840n^2 + 4200n + 5040}, \\
 S_xJD_xD_y(f(x, y)) &= \frac{(210n^3 + 420n^2 - 1890n - 3780)x^8 + (360n^3 + 720n^2 - 3240n - 6480)x^7}{105n^2 + 525n + 630} \\
 &\quad + \frac{(560n^2 + 2800n + 3360)x^6 + (252n^2 + 1260n + 1512)x^5}{105n^2 + 525n + 630} \\
 &\quad + \frac{x^{n+3}(315n^3 - 1260n) + x^{n+2}(420n^2 + 1260n)}{105n^2 + 525n + 630}.
 \end{aligned}$$

Now, we have the following:

- (i)  $M_1(\mu(G)) = (D_x(f(x, y) + D_y(f(x, y)))|_{x=1; y=1} = n^2 + 25n - 34,$
- (ii)  $M_2(\mu(G)) = (D_x(f(x, y))(D_y(f(x, y)))|_{x=1; y=1} = 3n^2 + 38n - 76,$
- (iii)  ${}^mM_2(\mu(G)) = (S_x(f(x, y))(S_y(f(x, y)))|_{x=1; y=1} = \frac{11n^2 + 23n + 16}{48n},$
- (iv)  $SSD(\mu(G)) = (D_xS_y(f(x, y)) + D_yS_x(f(x, y)))|_{x=1; y=1} = \frac{2n^3 + 39n^2 - 7n - 12}{6n},$
- (v)  $H(\mu(G)) = 2S_xJ(f(x, y))|_{x=1} = \frac{345n^3 + 2426n^2 + 3055n + 846}{420n^2 + 2100n + 2520},$
- (vi)  $I(\mu(G)) = S_xJD_xD_y(f(x, y))|_{x=1} = \frac{885n^3 + 2372n^2 - 1070n - 5388}{105n^2 + 525n + 630}.$

□



**Figure 2.** Plot of M-polynomial of Mycielskian of a path  $P_8$

### 5. M-polynomial of Mycielskian of Cycles

We now find the M-polynomial of Mycielskian of cycles.

**Theorem 5.1.** *Let  $C_n$  be a cycle of order  $n \geq 3$ . Then  $m(\mu(G))$  is*

$$M(\mu(G); x, y) = 2nx^3y^4 + nx^3y^n + nx^4y^4.$$

*Proof.* Let  $G = C_n$  be a cycle of order  $n, n \geq 3$ . From Figure 1(b) it is easy to observe that

$$|V(\mu(G))| = 2n + 1 \quad \text{and} \quad |E(\mu(G))| = 4n.$$

Since each vertex of  $G$  is of degree either 2 or 4 or  $n$ , the partitions of  $V(\mu(G))$  be:

$$V_1(\mu(G)) = \{a \in V(\mu(G)) : \deg_{\mu(G)}(a) = 3\},$$

$$V_2(\mu(G)) = \{a \in V(\mu(G)) : \deg_{\mu(G)}(a) = 4\},$$

$$V_3(\mu(G)) = \{a \in V(\mu(G)) : \deg_{\mu(G)}(a) = n\}.$$

Clearly,

$$|V_1(\mu(G))| = n, \quad |V_2(\mu(G))| = n, \quad |V_3(\mu(G))| = 1.$$

Furthermore, the partitions of edge set  $E(\mu(G))$  are:

$$E_1(\mu(G)) = \{e = ab \in E(\mu(G)) : \deg_{\mu(G)}(a) = 3, \deg_{\mu(G)}(b) = 4\},$$

$$E_2(\mu(G)) = \{e = ab \in E(\mu(G)) : \deg_{\mu(G)}(a) = 3, \deg_{\mu(G)}(b) = n\},$$

$$E_3(\mu(G)) = \{e = ab \in E(\mu(G)) : \deg_{\mu(G)}(a) = 4, \deg_{\mu(G)}(b) = 4\}.$$

Clearly,

$$|E_1(\mu(G))| = 2n, \quad |E_2(\mu(G))| = n, \quad |E_3(\mu(G))| = n.$$

Therefore,

$$\begin{aligned} M(G; x, y) &= \sum_{\delta \leq \alpha \leq \beta \leq \Delta} m_{\alpha\beta}(\mu(G))x^\alpha y^\beta \\ &= m_{34}(\mu(G))x^3y^4 + m_{3n}(\mu(G))x^3y^n + m_{44}(\mu(G))x^4y^4 \\ &= 2nx^3y^4 + nx^3y^n + nx^4y^4. \end{aligned}$$

□

**Theorem 5.2.** *Let  $C_n$  be a cycle of order  $n \geq 3$ . Then,*

$$M_1(\mu(G)) = n^2 + 25n,$$

$$M_2(\mu(G)) = 3n^2 + 40n,$$

$${}^m M_2(\mu(G)) = \frac{11n + 16}{48},$$

$$SSD(\mu(G)) = \frac{2n^2 + 37n + 18}{6},$$

$$H(\mu(G)) = \frac{23n^2 + 125n}{28n + 84},$$

$$I(\mu(G)) = \frac{59n^2 + 114n}{7n + 21}.$$

*Proof.* From Theorem 5.1, we have

$$M(\mu(G); x, y) = 2nx^3y^4 + nx^3y^n + nx^4y^4.$$

Then,

$$D_x(f(x, y)) = 3nx^3y^n + 4nx^4y^4 + 6nx^3y^4,$$

$$D_y(f(x, y)) = n^2x^3y^n + 4nx^4y^4 + 8nx^3y^4,$$

$$(D_y D_x)(f(x, y)) = 3n^2x^3y^n + 16nx^4y^4 + 24nx^3y^4,$$

$$S_x(f(x, y)) = \frac{4nx^3y^n + (3nx^4 + 8nx^3)y^4}{12},$$

$$S_y(f(x, y)) = \frac{4x^3y^n + (nx^4 + 2nx^3)y^4}{4},$$

$$S_x S_y(f(x, y)) = \frac{16x^3y^n + (3nx^4 + 8nx^3)y^4}{48},$$

$$S_y D_x(f(x, y)) = \frac{6x^3y^n + (2nx^4 + 3nx^3)y^4}{2},$$

$$S_x D_y(f(x, y)) = \frac{n^2x^3y^n + (3nx^4 + 8nx^3)y^4}{3},$$

$$2S_x J(f(x, y)) = \frac{(56nx^{n+3} + (7n^2 + 21n)x^8 + (16n^2 + 48n)x^7)}{28n + 84},$$

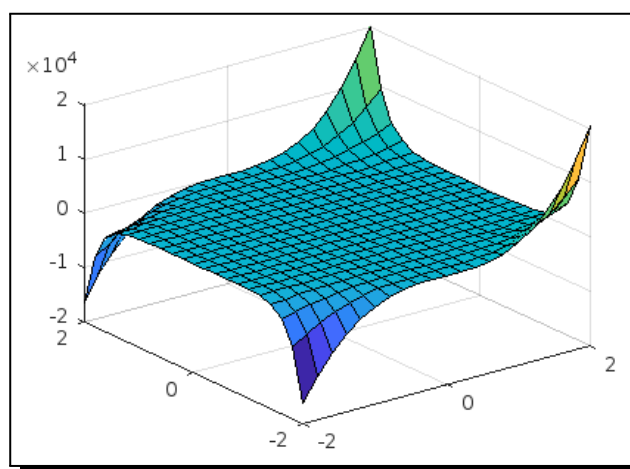
$$S_x J D_x D_y(f(x, y)) = \frac{21n^2x^{n+3} + (14n^2 + 42n)x^8 + (24n^2 + 72n)x^7}{7n + 21}.$$

Now, we have the following:

- (i)  $M_1(\mu(G)) = (D_x(f(x, y)) + D_y(f(x, y)))|_{x=1; y=1} = n^2 + 25n,$
- (ii)  $M_2(\mu(G)) = (D_x(f(x, y))(D_y(f(x, y))))|_{x=1; y=1} = 3n^2 + 40n,$
- (iii)  ${}^m M_2(\mu(G)) = (S_x(f(x, y))(S_y(f(x, y))))|_{x=1; y=1} = \frac{11n + 16}{48},$
- (iv)  $SSD(\mu(G)) = (D_x S_y(f(x, y)) + D_y S_x(f(x, y)))|_{x=1; y=1} = \frac{2n^2 + 37n + 18}{6},$
- (v)  $H(\mu(G)) = 2S_x J(f(x, y))|_{x=1} = \frac{23n^2 + 125n}{28n + 84},$

$$(vi) I(\mu(G)) = S_x J D_x D_y (f(x, y))|_{x=1} = \frac{59n^2 + 114n}{7n + 21}.$$

□



**Figure 3.** Plot of M-polynomial of Mycielskian of a cycle  $C_8$

## 6. Conclusion

We have discussed the closest forms of M-polynomial for Mycielskian of paths and cycles. The graphical representation of M-polynomial is given and derived some degree-based topological indices from M-polynomial. The M-polynomial can be determined for many graph classes, derived graphs, graph products, graph operations, and graph powers.

### Competing Interests

The authors declare that they have no competing interests.

### Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

## References

- [1] F. M. Brückler, T. Došlić, A. Graovac and I. Gutman, On a class of distance-based molecular structure descriptors, *Chemical Physics Letters* **503**(4-6) (2011), 336 – 338, DOI: 10.1016/J.Cplett.2011.01.033.
- [2] K. Ch. Das and N. Trinajstić, Comparison between first geometric-arithmetic index and atom-bond connectivity index, *Chemical Physics Letters* **497**(1-3) (2010), 149 – 151, DOI: 10.1016/j.cplett.2010.07.097.
- [3] E. Deutsch and S. Klavžar, M-polynomial and degree-based topological indices, *Iranian Journal of Mathematical Chemistry* **6**(2) (2015), 93 – 102, DOI: 10.22052/IJMC.2015.10106.
- [4] G. Fath-Tabar, B. Furtula and I. Gutman, A new geometric-arithmetic index, *Journal of Mathematical Chemistry* **47** (2010), 477 – 486, DOI: 10.1007/s10910-009-9584-79.
- [5] F. Harary, *Graph Theory*, Addison-Wesley Publishing Company, New York (1969).



- [6] H. Hosoya, Topological index. A newly proposed quantity characterizing the topological nature of structural isomers of saturated hydrocarbons, *Bulletin of the Chemical Society of Japan* **44**(9) (1971), 2332 – 2339, DOI: 10.1246/bcsj.44.2332.
- [7] A. J. M. Khalaf, S. Hussain, D. Afzal, F. Afzal and A. Maqbool, M-Polynomial and topological indices of book graph, *Journal of Discrete Mathematical Sciences and Cryptography* **23**(6) (2020), 1217 – 1237, DOI: 10.1080/09720529.2020.1809115.
- [8] Y. C. Kwun, M. Munir, W. Nazeer, S. Rafique and S. M. Kang, M-Polynomials and topological indices of V-Phenylenic nanotubes and nanotori, *Scientific Reports* **7** (2017), Article number: 8756, DOI: 10.1038/s41598-017-08309-y.
- [9] W. Lin, J. Wu, P. C. B. Lam and G. Gu, Several parameters of generalized Mycielskians, *Discrete Applied Mathematics* **154**(8) (2006), 1173 – 1182, DOI: 10.1016/j.dam.2005.11.001.
- [10] M. Munir, W. Nazeer, A. R. Nizami, S. Rafique and S. M. Kang, M-polynomials and topological indices of titania nanotubes, *Symmetry* **8**(11) (2016), 117, DOI: 10.3390/sym8110117.
- [11] M. Munir, W. Nazeer, S. Rafique and S. M. Kang, M-Polynomial and degree-based topological indices of polyhex nanotubes, *Symmetry* **8**(12) (2016), 149, DOI: 10.3390/sym8120149.
- [12] N. N. Swamy, C. K. Gangappa, P. Poojary, B. Sooryanarayana and N. H. Mudalagiraiah, Topological indices of the subdivision graphs of the nanostructure  $TUC_4C_8(R)$  using M-polynomials, *Journal of Discrete Mathematical Sciences and Cryptography* **25**(1) (2022), 265 – 282, DOI: 10.1080/09720529.2022.2027604.
- [13] H. Wiener, Structural determination of the paraffin boiling points, *Journal of the American Chemical Society* **69** (1947), 17 – 20, DOI: 10.1021/ja01193a005.

