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Research Article

An Integrated Framework for Inventory Management Considering Demand and Supply Uncertainties

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Abstract. Due to uncertainties on both the demand and supply sides, managing inventories is the most critical and cost-effective challenge faced by supply chain managers in the supply chain network. While inventory management problems with uncertain demand and supply have been covered in the supply chain management literature for a long time, it is fair to say that no one approach can be applied universally across all scenarios. This paper presents an integrated inventory management framework considering non-linear optimization and multi-criteria decision-making tools. First, it presents a non-linear model to minimize the total inventory management cost at a centralized location (e.g., distribution center) by considering multiple inventory aggregation scenarios. A series of sensitivity analyses are conducted to examine the impact of demand and supply uncertainties on various key performance indicators related to inventory operations. Lastly, the best inventory aggregation model is selected by using a multi-criterion-based TOPSIS (Technique for order performance by similarity to ideal solution)) analysis.

Keywords. Inventory aggregation, Distribution network, Multi-criteria decision modeling, Supply uncertainties, Demand uncertainties, TOPSIS

Mathematics Subject Classification (2020). 90B05, 90B06, 90B50

1. Introduction

In today's global business environment, one of the most important components involved in the success of any business is inventory management. The fundamental role of inventory in any business is to bridge the gap between demand and supply and facilitate equilibrium. Inventory control and inventory visibility are two very important elements in any business that cannot be ignored as they act as cost drivers and the balance sheet is directly affected by them. Inventory management is a critical issue in multiple stages of the supply chain (Routroy and Kodali [22]). To achieve the maximum need of the customer at the minimum cost is the priority of any business (Vishkaei *et al*. [35]). Inventory represents one of the largest assets ranging from twenty to sixty percent of the total assets in most of the manufacturing firms. Weber *et al*. [36] suggest that purchased materials and services represent up to 80% of the total product cost for high-technology firms. The authors also describe that the raw material purchased for most U.S. firms constitutes 40-60% of the unit cost of a product. Therefor, the stock may be ordered in a optimal way to maximize the profit (Giri *et al*. [14]). Inventory management policies can directly impact the profits of any business firm especially when considering the overall profit of supply chains rather than a single firm (Arnold, 1998). Since the uncertain environment will eventually affect almost all organizations, all business processes under require a more flexible and responsive structure (Agrawal [1]). Supply chain uncertainty refers to a decision-making environment in which the decision maker does not know exactly what to decide due to lack of transparency in the supply chain and the impact of possible actions. It may be caused by a malfunctioning production process at the supplier, late delivery due to unexpected weather conditions, or unacceptable quality of the delivered products. As an example, aggregation can reduce the demand uncertainty by pooling independent demands of two or more regions (Eyan and Fouque [12]). In this regard, Edirisinghe and Atkins [10] discuss the effect of risk pooling when a single supplier or a depot distributes a single commodity to multiple retailers into two-echelon framework. Similarly, Puga and Tancrez [20] analyzed the location inventory problem for the design of a supply chain network with uncertain demand.

Various solution approaches are suggested by the researchers for solving the different inventory control models. A few of them are exact solution approaches, heuristics approaches, meta-heuristic approaches, hybrid approaches and fuzzy approaches, and other computational techniques (Vats *et al*. [33]). Based on the literature review it can be concluded that the inventory required for a particular service level depends on the number of inventory stocking locations if inventory assumptions are satisfied. This does not mean that reducing the number of warehouses will automatically reduce inventory. In previous studies, most authors generally assume that the demand and lead times to be stochastic, deterministic, constant, or negligible. In the present study it is assumed that the demand and the lead time is random variable and the effects of both demand and lead time uncertainty are analyzed seperately. The combined effect of both uncertainty demand and lead time are also analyzed on various performance parameters. It is undeniably challenging to keep up with the suitable client assistance level under unsure states of interest and supply. In this current investigation the numerical model is created so that this model is sufficient vigorous to offer the suitable support level under the cut off states of vulnerability of interest and lead time. There is a wide possibility to handle the multi-criteria based network scenario selection problem with inventory optimization problem using MADM approaches simultaneously.

The objective of this research is to present a comprehensive framework to develop an optimal inventory aggregation strategy that deals with uncertainties in a supply chain system. More specifically, it presents a non-linear model to minimize the total inventory management cost at a centralized location by considering multiple inventory aggregation scenarios. Through sensitivity analysis, multiple distribution networks and aggregation scenarios involving various level of demand and supply uncertainities are analyzed in the paper. Lastly, by using a multi-criteria decision making tool, the best inventory aggregation scenario for a given supply chain is selected. The proposed methodology is illustrated by using a stylized case study representing a two-stage supply chain.

The structure of this paper is organized as follows. Section 2 provides a brief overview of related literature. In Section 3, we present the proposed inventory aggregation approach, mathematical model, solution approach, and numerical illustration along with the results and discussion. A TOPSIS approach which is implemented for finding the best possible scenario of inventory aggregation in a distribution network is discussed in Section 3.6. Analysis of results and the effects of different types of uncertainty are examined. Section 4 wraps up the paper by providing concluding remarks and a few options for future research.

2. An Overview of Related Literature

The adoption of inventory aggregation as a strategy to reduce the inventory safety stock dates back to several decades ago (Eppen [11]). Several studies have been conducted since then in multiple domains of inventory management dealing with uncertain demand and supply. Tagaras [30] demonstrated that inventory pooling also improves the customer service level while reducing total inventory management cost through lateral transshipment. Chang and Lin [7] also validate the statement of (Eppen [11]) and also prove that the centralized system is more effective than decentralized system. Weng [37] studied the effect of inventory aggregation over demand uncertainty of multiple products sharing product modularity in a two-echelon distribution system. Xu and Evers [38] found that sometimes partial aggregation of inventory is favored over complete aggregation.

Inventory aggregation in supply chain can reduce the safety stock. The amount of safety stock depends on the level of pooling between the wholesalers and retailers. The safety stocks have an adverse effect on the operational efficiency of the supply chain as they increase the inventory carrying costs in supply chains (Gaur and Ravindran [13]). This approach is more effective when inventory carrying costs are major components of the overall supply chain costs. It is quite hard to determine the appropriate level of safety stocks in a multistage production and distribution system (Inderfurth [17]). Synder *et al*. [29] presented an inventory location and allocation model using a risk pooling approach to minimize the total cost. Bernstein *et al*. [5] found that inventory aggregation could support the unbalanced capacities thereby improving the overall sales. Schmitt *et al*. [24] found that a decentralized inventory system produces optimum results due to risk diversification effect when demand is deterministic and supply is stochastic. On the other hand, when the converse was true with respect to demand and supply characteristics, a centralized inventory system yielded optimum results due to inventory aggregation effect.

Hu *et al*. [15] used a network analysis method to operationalize risk pooling and supply chain hierarchy and find that risk pooling significantly reduces analysts' forecast errors and increases (decreases) their use of public (private) information. Brunaud *et al*. [6] worked on traditional supply chain planning models and extended these model to optimize the inventory policies. Salimi and Vahdani [23] presented a mathematical model to design an efficient bio-fuel supply chain network at pre-disaster stage that considering failure in the connecting links between the facilities. In which the probability of failure of the links is forecasted by a spatial statistic approach and also due to the fact that disasters can cause disruptions in bio-refineries, leads to use the risk-pooling effect in order to reduce total costs. Cosma *et al*. [9] provided a mathematical model for this problem and as well a solution approach based on genetic algorithms for solving the problem. Oeser and Romano [19] has completed survey with 223 German hospitals and their study explores how ten risk pooling methods can be adapted and applied in the healthcare context to reduce economic losses while maintaining a given service level. Vats *et al*. [34] represented a mathematical approach for inventory optimization using risk-pooling concept for different types of the product, and it provides optimum inventory policy (reorder point, ordering quantity, and total cost) along with *expected shortages per cycle* (ESC), *fill rate* (FR), *safety stock* (SS), and *average inventory* (AI).

The most popular inventory aggregation modeling approaches are news vendor model (Eppen [11]), multi-location news vendor model (Chang and Lin [7]), lateral transshipment inventory distribution model (Tagaras [30]), bi-criteria non-linear stochastic integer programming model (Gaur and Ravindran [13]), multivariate dependent demand distribution model (Charles and Rajaram [8]), the stochastic location model (Synder *et al*. [29]), MINLP model for multi-echelon inventory model (You and Grossman [40]), unidirectional transship model for two locations (Arikan and Sibermayr [2]), bi-objective mathematical model for inventory distribution routing model (Momenikiyai *et al*. [18]), non-linear mathematical model containing various costs (Vats *et al*. [34]), robust mixed integer linear programming model (Trilokee *et al*. [32]), dydadic regression model (Putman [21]), three product newsvendor model (Zhang *et al*. [42]), robust multi-location news vendor model (Singh *et al*. [28]). Singh *et al*. [26] designed an inventory model of a supply chain for a deteriorating item under the selling price. This study optimized retailer's replenishment rate when demand rate is declining with time. The most widely used approaches dealing with uncertainty are mathematical modeling and other techniques, METRIC modeling technique, Markov decision process technique, simulation, Stackelberg game and fuzzy set theory. Singh *et al*. [27] and Sharma *et al*. [25] have analyzed a supply chain model with multiple market demands suggested by selling price.

In summary, prior research has demonstrated that inventory aggregation is one of the widely accepted approaches to deal with uncertainties in supply chain management. This approach has been found effective in reducing safety stock and improving customer service level. From modeling perspective, inventory aggregation model involves an inventory location and allocation problem which involves non-linear and stochastic constraints. Our research reveals that the majority of prior works have assumed both demand and supply as deterministic. This paper presents an integrated framework to manage inventory involving uncertain demand and supply conditions. The proposed methodology consists of a non-linear optimization model and TOPSIS multi-criteria decision making tool to deal with uncertainties in demand and supply.

3. Methodology and Modelling or Problem Statement and Optimization Process

Multiple wholesalers and retail stores have been considered in a two-stage supply chain network to perform this study. Different scenarios of inventory aggregation are considered in this two-stage supply chain network under the uncertain environment of demand and supply. A Mathematical model has been developed incorporating the inventory aggregation approach. And an illustrative example has been provided for obtaining optimized results for all scenarios.

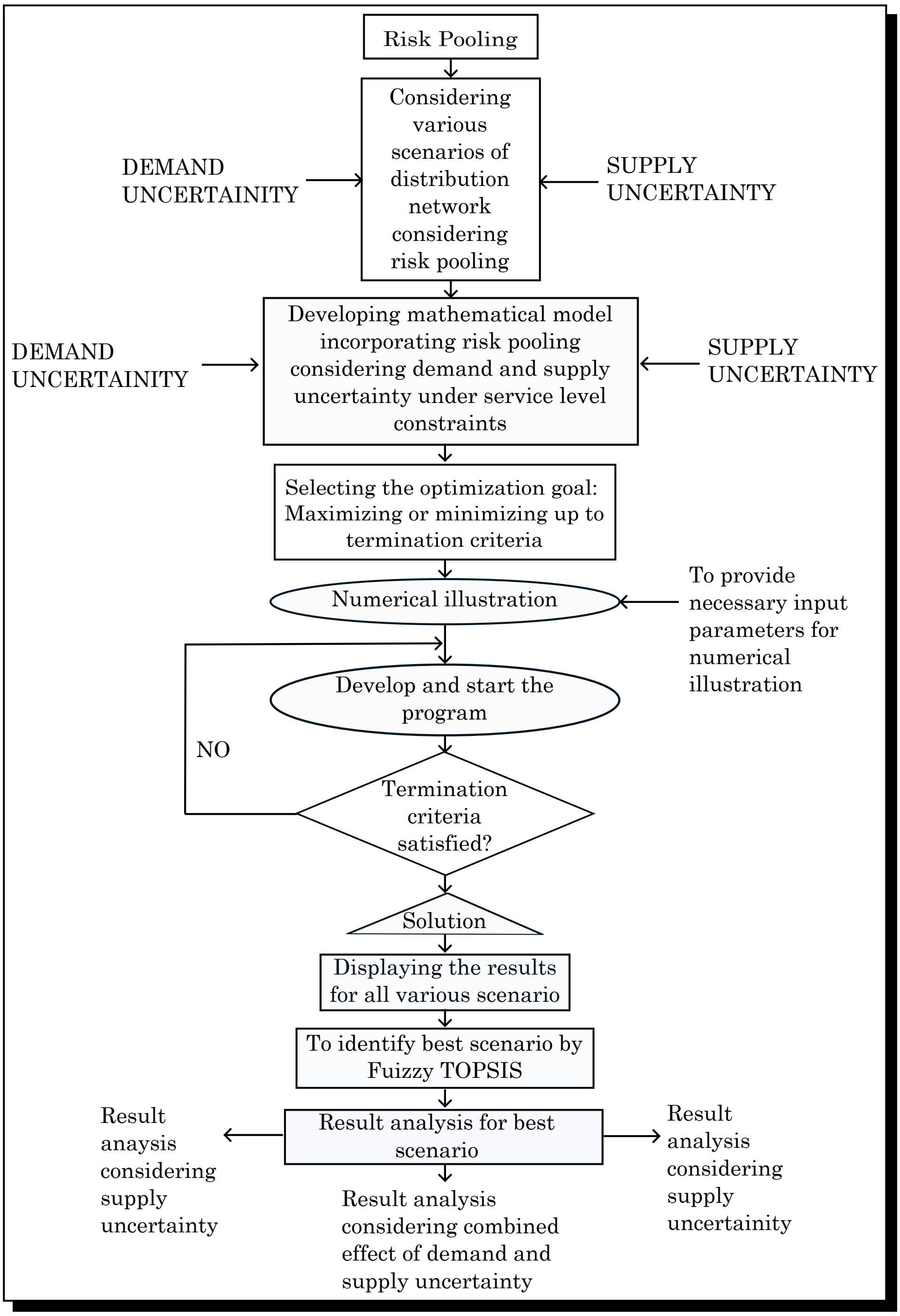

Figure 1. Proposed integrated framework for analysis of uncertainties in inventory management

A non-linear programming approach has been used to solve the above problem This approach provides the optimization results for all scenarios and the best scenario is identified by TOPSIS method. An integrated framework for the analysis of uncertainties in inventory management has been also proposed.

Notations and Model Background

Let, L = average lead time from central wholesaler to retail store,

D = average demand of retail store,

μ_{ltd} = average demand during lead time,

σ_{ltd} = standard deviation of demand during lead time,

σ = standard deviation of demand,

σ_L = standard deviation of lead time.

The average of demand and the standard deviation of demand during lead time may be calculated as follows:

$$\mu_{ltd} = D \times L,$$

$$\sigma_{ltd} = (\sigma^2 \times L + \sigma_L^2 \times D^2)^{\frac{1}{2}}.$$

From Figure 2, a wholesaler D1 is able to satisfy the demands of the retail stores R1, R2 and R3 so these three offices can be collected for wholesaler D1. Additionally, a wholesaler D2 is able to satisfy the demands of the retail stores R2, R3 and R4 so these three retail stores can be collected for D2. Similarly, wholesaler D3 is fit for to satisfy the demands of retail stores R3 and R4.

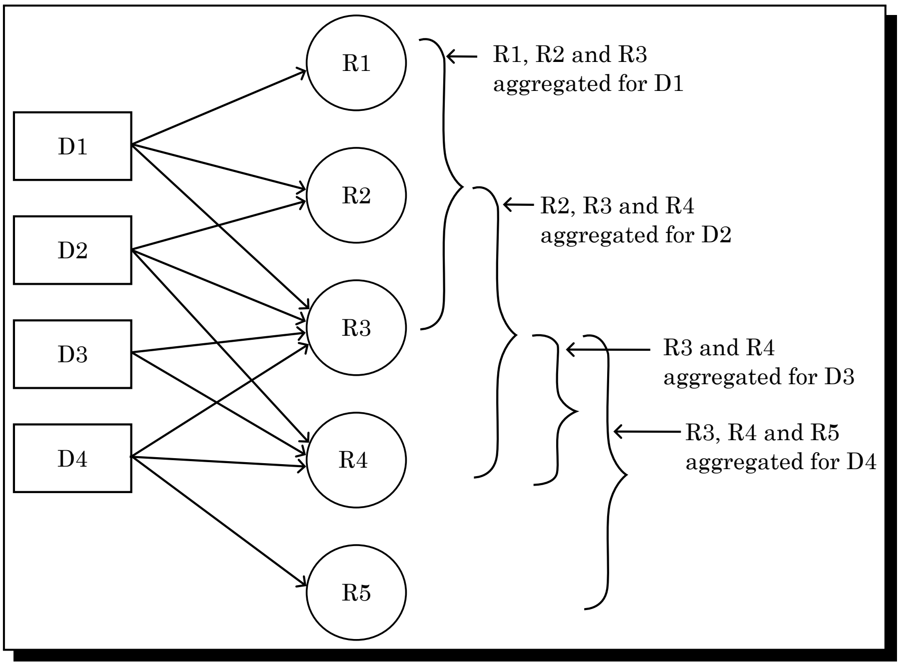

Figure 2. Example of inventory aggregation

Therefore, R3 and R4 can be aggregated for D3. The average of aggregated demand and the standard deviation of aggregated demand for a wholesaler can be given by are given as follows:

Let the wholesalers are denoted by set j and the retail stores are denoted by set k.

D_k = demands of retail stores' k served by wholesaler,

σ_1 = standard deviation of demand of retail store 1,

σ_2 = standard deviation of demand of retail store 2,

ρ_{1-2} = coefficient of correlation between the demands of retail store 1 and retail store 2,

$\bar{D}_j$ = average daily demands for a wholesaler j,

$\bar{D}_j$ = summation of all demands of retail stores = $\sum_{J\in B_i} D_k$,

σ_j = Standard deviation of aggregated demand,

$\sigma_j = \sum_{(r,s)} (\sigma_1^2+\sigma_2^2+2\rho_{1-2}\sigma_1\sigma_2)^{\frac{1}{2}}$.

Here $(\bar{D}_j,\sigma_j)$ are the demand and standard deviation of demand for pooling. If a wholesaler is shipping more than one retail store and the demand of all retail stores are aggregated, then the lead time demand may be represented as

$$\mu_{xj} = \bar{D}_j \times L,$$

where $\bar{D}_j$ is average aggregated demand for wholesaler j,

$$\sigma_{xj} = (\sigma_j^2 \times L + \sigma_L^2 \times \bar{D}_j^2)^{\frac{1}{2}},$$

where σ_{xj} is the aggregated demand's standard deviation for wholesaler j.

3.1 Mathematical Modelling Using Inventory Aggregation Approach in Distribution Network

Inventory aggregation may reduce the demand (demand) uncertainty and this approach may be utilized in the effective inventory management (Eynan and Fouque [12]). This approach plays a vital role when stock conveying costs are the major component of supply chain costs (Gaur and Ravindran [13]) and it can provide required customer service level under uncertain conditions of lead time and demand. Due to these unique reasons, this approach may be applied for developing a mathematical model for inventory optimization and network selection by addressing the both type of uncertainty. For developing this model, a two-stage distribution network is considered between the wholesalers and the retail stores. It is assumed that there are some certain numbers of wholesalers and retail stores in the network. A single wholesaler can have the potential to fulfill the demands of more than one retail store but a single retail store cannot fulfill its demand by more than one wholesaler.

By implementing this condition so many scenarios of aggregation may be generated in the modelling. Here the main and the primary objectives are to reduce the overall cost of the system and to get optimized inventory policy for all possible scenarios of aggregation and the second objective is to select the best scenario of aggregation in the network. Therefore, a two-stage problem is developed to perform this study. In the first stage problem, a non-linear mathematical model is developed which provides the optimum inventory policy along with minimized total cost after numerical illustration and the second stage solution will provide the required level of inventory aggregation in the given two-stage network which can be achieved by the TOPSIS approach. For developing this non-linear mathematical model following major assumptions are considered:

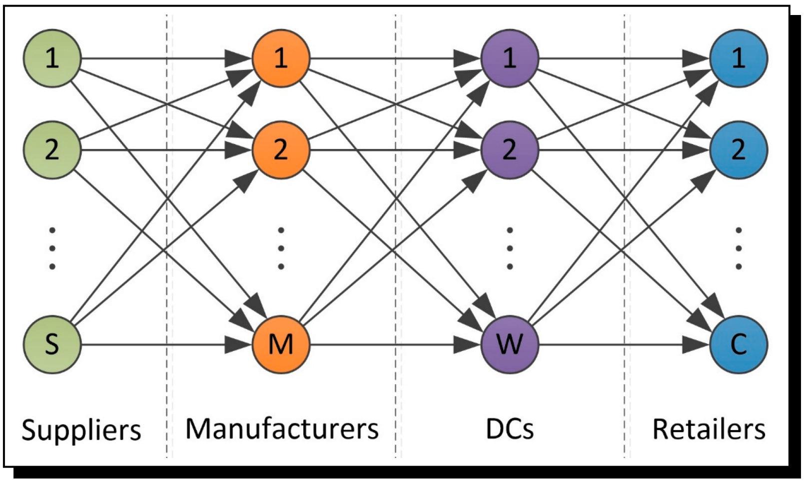

Figure 3. A representative two-stage supply chain network consisting of multiple wholesalers and retail stores

Assumptions

(i) A continuous review policy is adopted to review of inventory system.

(ii) Demand and lead time are independent random variables and are assumed to be normally distributed.

(iii) The fill rate for all the wholesalers is assumed to be the same.

(iv) Partial fulfillment is not allowed for retail stores.

(v) A fixed wholesaler can fulfill the demand of some specific retail stores only.

(vi) Reverse flow of inventory is not considered.

(vii) Shortages and backorder are not considered.

(viii) Pipe line inventories are not considered.

For developing a non-linear problem, we are considering two-stage distribution networks between wholesalers and retail stores. The sets, symbols, notation, parameters, objective function and constraints are discussed in the following subsections:

Sets of the Proposed Model

Wholesalers: j,
Retail store: k.

Parameters

(i) C_j: Cost of foundation of wholesalers at specific locations.

(ii) O_{PD_j}: Operating cost of wholesaler.

(iii) C_{SH}: Shipment cost.

(iv) C_{CI}: Stock conveying cost.

(v) F_{OC_j}: Fixed ordering cost at wholesaler j.

(vi) $\bar{D}_j$: Normal accumulated demands for a wholesaler j.

(vii) D_k: Demand of retail store k that has to be fulfilled by the wholesaler in supply chain.

(viii) X_{j-k}: It is the distance between the wholesaler j and retail store k.

Decision Variables

(i) Q_j: It shows the economic order quantity. It is the minimum order quantity, that is ordered by the retail store to the wholesaler at every order.

(ii) r_j: It represents the reorder point of the wholesaler j.

Objective Function

The objective function of non-linear stochastic problem may be formulated as:

$$\begin{aligned}\text{Total anticipated cost} = {} & \text{Cost of foundation} + \text{Fixed ordering cost} \\ & + \text{Operating cost of wholesaler} + \text{Shipment cost} \\ & + \text{Stock conveying cost.}\end{aligned}$$

(i) *Cost of foundation of wholesaler:* It is the summation of all expenses of foundation of the relative multitude of wholesalers arranged at various locations:

$$\text{Total cost of foundation of wholesaler} = \sum_j C_j .$$

(ii) *Operating cost of wholesaler:* It is the cost involved for running a wholesaler:

$$\text{Total operating cost of wholesaler} = \sum_j \sum_k O_{PD_j} * \bar{D}_j .$$

(iii) *Shipment cost from wholesaler to retail store:* It is the cost of transportation involved for shifting the units from wholesaler j to retail store k:

$$\text{Total shipment cost} = \sum_j \sum_k C_{SH} * X_{j-k} * D_k .$$

(iv) *Ordering Cost:* This is the cost involved when a wholesaler receives an order from retail stores. Number of order received by a wholesaler:

$$\text{Total ordering cost} = \sum_j F_{OC_j} \left(\frac{\bar{D}_j}{Q_j} \right).$$

(v) *Stock conveying cost:* It is the cost involved for maintain inventory in supply chain:

$$\text{Average cycle inventory} = \frac{Q_j}{2},$$

$$\text{Safety stock} = r - \mu_{xj},$$

$$\text{Total Cost of carrying the inventory} = \sum_j \left\{ \frac{Q_j}{2} + (r_j - \mu_{xj}) \right\} * C_{CI} = \sum_j \left\{ \frac{Q_j}{2} + SS \right\} * C_{CI},$$

where μ_{xj} is the mean demand during lead time for a wholesaler j.

Constraints

(i) *Constraints of ordering quantity*: The order quantity is always a nonnegative quantity:

$$Q_j \geq 0, \quad \text{for all } j,$$

(ii) *Safety stock constraints*: The safety stock cannot be negative:

$$SS \geq 0, \quad \text{for all } j,$$

(iii) *CSL Constraint*: Cycle service level (CSL), which is characterized as the likelihood of not loading out in a cycle and it is the negligible portion of renewal cycles that end with all the client demand being met:

$$CSL \geq 0.95.$$

Product Miles (PM)

The product-miles may be defined as:

$$\text{Product miles} = \left(\sum_k \sum_j X_{j-k} * D_k \right).$$

The mathematical model can be represented as:

$$\text{Minimization of } TC = \sum_j C_j + \sum_j \sum_k O_{PD_j} * \bar{D}_j + \sum_j \sum_k C_{SH} * X_{j-k} * D_k + \sum_j F_{OC_j} \left(\frac{\bar{D}_j}{Q_j} \right) + \sum_j \left\{ \frac{Q_j}{2} + SS \right\} * C_{CI}$$

subjected to constraints:

$$Q_j \geq 1$$

$$r_j \geq \mu_{xj}$$

$$CSL \geq 0.95$$

3.2 Illustrative Example

For numerical illustration of the above problem, 5 wholesalers and 7 retail stores are considered in a network (Figure 4) as discussed in Vats *et al*. [34]. It is assumed that these wholesalers are capable of fulfilling the demand of all 7 retail stores. The condition is imposed that a fixed wholesaler can fulfill the demand of some specific retail stores only and a retail store cannot fulfill its demand by more than one wholesaler. The 5 wholesalers in the system are denoted by D1, D2, D3, D4 and D5, and 7 the retail stores are depicted by R1, R2, R3, R4, R5, R6 and R7 as shown in Figure 4.

For solving the numerical problem, the parameters are taken from Vats *et al*. [34]. Table 1 represents important parameters for wholesalers.

Table 2 provides the cost of setting a facility and the cost of operation for all the 5 wholesalers.

Table 3 represents the distance (KM) from wholesaler i to retail store j.

Table 4 shows the monthly average demand and the standard deviations of demand for all 7 retail stores.

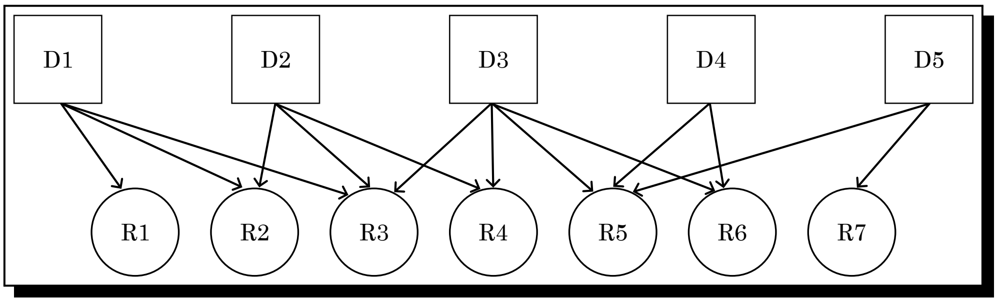

Figure 4. Distribution network considered for numerical illustration

Table 1. Wholesaler parameters

Sr. No.	Important parameters for wholesaler	Values for each item
1	Fixed ordering cost (Fo_j)	650.00 INR per unit
2	Shipment cost (T)	0.45 INR per unit per kilometer
3	Holding cost of inventory items (C)	0.25 INR per unit
4	Cost of each product (v)	55.00 INR per unit
5	Minimum delivery time (L)	0.15 day

Table 2. Operating cost and facility cost

Wholesaler	Operating cost of wholesaler (OP_j) (INR)	Cost of establishment (F_j) (INR)
1	6.00	50000.00
2	7.00	35000.00
3	5.00	85000.00
4	5.00	35000.00
5	6.00	40000.00

Table 3. Distance from wholesaler to retail store

Wholesaler	Retail stores						
	1	2	3	4	5	6	7
1	45.00	55.00	65.00				
2		35.00	40.00				
3			65.00	35.00	35.00	65.00	
4					55.00	35.00	
5					85.00		70.00

Table 4. Daily mean demand of retail store and its standard deviation

Retail store's Number	Daily mean demand	Standard deviation of daily mean demand
1	2500	75
2	3500	50
3	2500	50
4	3000	75
5	2500	100
6	4000	100
7	3500	75

3.3 Scenarios of Aggregation in the Network and the Solution Procedure

As discussed in the previous section that, for solving the above problem, 5 wholesalers and 7 retail stores are considered in the given network (refer Figure 3). In most general way, the retail stores R1, R2 and R3 may be served by wholesaler D1. Retail stores R2 and R4 may be served by wholesaler D2. Retail stores R5 and R6 can be served by wholesaler D4. Retail stores R5 and R7 can be served by wholesaler D5. The main condition for scenario generation is that all the retail stores are served simultaneously by some specific wholesalers but no retail store can fulfill its demand by more than one wholesaler. After forcing the above condition in between the wholesaler and the retail store, the Scenario-1 shows the aggregation of retail stores R1 and R2 for wholesaler D1, similarly retail stores R4, R5 and R6 are aggregated for wholesaler D3. D2 has the potential to complete the demand of R3 and D5 has the potential to complete the demand of R7. In this case, all the retail stores are served by all wholesalers. Similarly, a series of all possible 20 scenarios can be generated for the demand fulfillment of all retail stores by wholesalers in the given network (Table 5).

Table 5. Possible scenarios of aggregation in network

Scenarios	Inventories aggregated at Wholesaler I for Retail stores j ($j = 1, 2, \dots$)				
	D1	D2	D3	D4	D5
Scenario-1	R1, R2	R3	R4, R5, R6	-	R7
Scenario-2	R1, R2	R3	R4, R5	R6	R7
Scenario-3	R1, R2	R3	R4	R5, R6	R7
Scenario-4	R1, R2	-	R3, R4, R5, R6	-	R7
Scenario-5	R1, R2	-	R3, R4, R5	R6	R7
Scenario-6	R1, R2	-	R3, R4	R5, R6	R7
Scenario-7	R1, R2	-	R3, R4	R6	R5, R7
Scenario-8	R1, R3	R2	R4, R5, R6	-	R7
Scenario-9	R1, R3	R2	R4, R5	R6	R7
Scenario-10	R1, R3	R2	R4	R5, R6	R7
Scenario-11	R1, R3	R2	R4	R6	R5, R7
Scenario-12	R1, R2, R3	-	R4, R5, R6	-	R7
Scenario-13	R1, R2, R3	-	R4, R5	R6	R7
Scenario-14	R1, R2, R3	-	R4	R5, R6	R7
Scenario-15	R1, R2, R3	-	R4	R6	R5, R7
Scenario-16	R1	R2, R3	R4, R5, R6	-	R7
Scenario-17	R1	R2	R3, R4, R5, R6	-	R7
Scenario-18	R1	R2	R3, R4, R5	R6	R7
Scenario-19	R1	R2	R3, R4	R5, R6	R7
Scenario-20	R1	R2	R3, R4	R6	R5, R7

The given problem is solved in an Intel (R) Core (TM)2 Duo CPU T 6570 system having RAM 4GB by using AIMMS software in which CONPOT solver is inbuilt. After obtaining the solution, one important performance parameters fill rate is also evaluated,

$$FR = \left(1 - \frac{ESC}{Q}\right),$$

where $ESC = -SS\left[1 - NORMDIST\left(\frac{SS}{\sigma_L}, 0, 1, 1\right)\right] + \sigma_L NORMDIST\left(\frac{SS}{\sigma_L}, 0, 1, 0\right)$.

3.4 Results of Optimization

As the given problem is designed for cost minimization and a non-linear programming is developed in AIMMS software to solve the above problem. The optimization results for all 20 possible scenarios are shown in Figure 5.

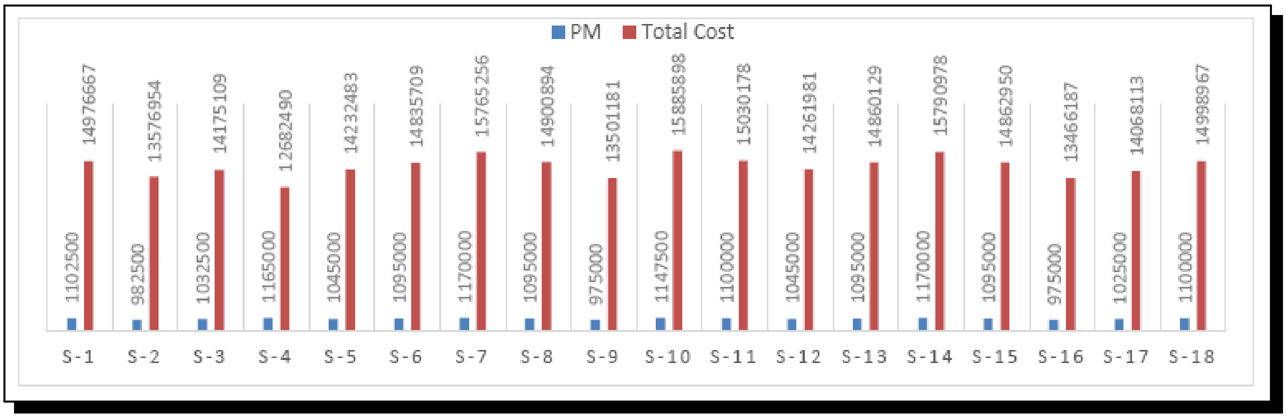

Figure 5. Total cost vs product miles

Figure 5 shows the optimization results for all twenty scenarios and the scenario 4 is providing the minimum cost among all scenarios. Therefore, this scenario should be selected for the inventory analysis in the network. In this study the importance of cycle service level (CSL), fill rates (FR), product-miles (PM) cannot be ignored. That's why for finding more appropriate results, some weightage of cycle service level (CSL), fill rates (FR), product-miles (PM) has to be provided. In the second stage of this study, best scenario has to be selected against the given performance parameters such as cycle service level (CSL), fill rates (FR), product-miles (PM) and the total cost. In the second stage of this study a multi-criteria decision-making problem arises.

3.5 TOPSIS Approach for Selecting the Best Scenario of Aggregation

There are four performance parameters via fill rate (FR), cycle service level (CSL), product-miles (PM) and the total cost for all twenty scenarios of aggregation in the given distribution network. Now it is implicit to select the best inventory aggregation scenario among all scenarios and to select the best inventory policy against the given performance parameters of cycle service level (CSL), fill rates (FR), product-miles (PM). The results obtained through the first stage are very close and very similar. Due to the similarity of solutions in the first stage results, the TOPSIS approach is endorsed here for selecting the best scenario of aggregation. TOPSIS is the acronym for the Technique for Order Preference by Similarity to Ideal Solution and this approach is suggested by Hwang *et al*. [16]. This approach is widely accepted in various multi-attribute decision-making problems. It stems from the concept of an ideal point of displacement, i.e., the shortest distance between compromise solutions (Belenson and Kapur [4], and Zeleny [41]). Yoon and Hwang [39] further suggested that the ranking of alternatives would be based on the shortest distance from (positive) ideal solution (PIS) and the furthest distance from negative ideal solution (NIS). The standard methodology involved in TOPSIS approach is discussed in the following section:

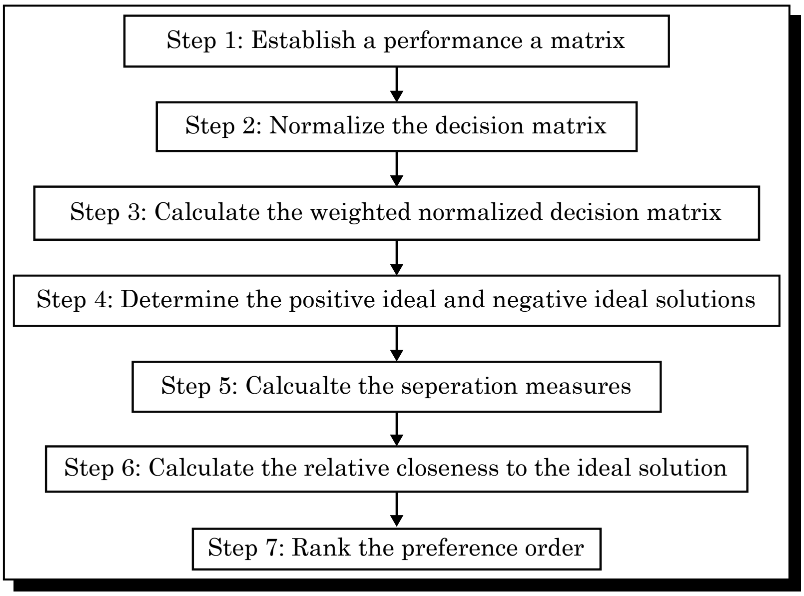

Figure 6. Methodology of TOPSIS approach

Step 1: *Establish the decision matrix*:

First of all weights of criteria weights are evaluated and a decision matrix is constructed between the criteria's and scenarios. The 4 criteria's are denoted by C_1, C_2, C_3 and C_4 and the 20 scenarios are denoted by $S_1, S_2, S_3, \dots, S_{20}$ and the decision matrix is denoted by DM.

$$DM = \begin{array}{c} \\ S_1 \\ S_2 \\ \vdots \\ S_{20} \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} X_{11} & X_{12} & \dots & X_{14} \\ X_{21} & X_{22} & \dots & X_{24} \\ \vdots & \vdots & \ddots & \vdots \\ X_{201} & X_{202} & \dots & X_{204} \end{pmatrix} \end{array}.$$

Step 2: *Calculate a normalized decision matrix*:

In this step, normalized decision matrix (NDM) is found out. The normalized decision matrix may be found as

$$r_{ij} = \frac{X_{ij}}{\sqrt{\left(\sum\limits_{i=1}^{m} X_{ij}\right)^{\frac{1}{2}}}}.$$

Step 3: *Determine the weighted normalized decision matrix*:

The weighted normalized decision matrix is found by multiplying the weights of criteria and the normalized decision matrix,

$$V_{ij} = w_{ij} \times r_i,$$

where $\sum w_{ij} = 1$.

Step 4: *Determine the positive and negative ideal solution*:
The positive ideal (A^+) and the negative ideal (A^-) solutions are defined according to the weighted decision matrix:

$$PIS = A^+ = \{V_1^+, V_2^+, \dots, V_n^+\}, \text{ where } V_j^+ = \{(\max(V_{ij}) \text{ if } j \in J); (\min V_{ij} \text{ if } j \in J')\},$$

$$NIS = A^- = \{V_1^-, V_2^-, \dots, V_n^-\}, \text{ where } V_j^- = \{(\min(V_{ij}) \text{ if } j \in J); (\max V_{ij} \text{ if } j \in J')\},$$

where J is associated with the beneficial criteria and J' is associated with the cost criteria.

Step 5: *Calculate the separation distance of each competitive scenario from the positive and negative ideal solution*:

$$SM^+ = \sqrt{\sum_{j=1}^{n} (V_{ij} - V_j^+)^2},$$

$$SM^- = \sqrt{\sum_{j=1}^{n} (V_{ij} - V_j^-)^2}.$$

Step 6: *Measure the relative closeness of each location to the ideal solution*:
For each competitive scenario find the relative closeness with respect to the ideal solution:

$$C_i = \frac{SM^-}{SM^- + SM^+}.$$

Step 7: *Rank the preference order*:
The highest value of C_i show that the solution is at the farthest distance from negative ideal solution. Rank the scenarios in the decreasing order of the relative closeness from ideal solution.

In this context, a response is collected from a group of 5 different peoples from academia as well as from industry which are aggregated in the Table 6 and Table 7. The people selected for this response are serving the acadamia and industry since last more than 15 years. The academia persons are at the minimum level of associate professor and the industry persons are at least at the level of senior manager. Table 6 shows the aggregate weights of criteria.

Table 6

	Total cost (C1)	Product-miles (C2)	Fill Rate (C3)	Cycle service level (C4)
Criteria weights	0.4	0.2	0.2	0.2

Table 6 shows the aggregate ratings of scenarios against the 4 criterions.

From the above two matrices (Table 6 and Table 7), normalized decision matrix, weighted normalized decision matrix, positive ideal solution, negative ideal solution, measure of separation and closeness coefficient is calculated. Finally, the rank is provided to the closeness coefficient and best scenario of inventory policy is selected. The closeness coefficients for all scenarios are shown in Figure 7.

Scenario 12 has the highest closeness coefficient which provides a fill rate of more than 99% and the customer service level up to 95%.

Table 7. Responses from experts

	Total cost (C1)	Product-miles (C2)	Fill Rate (C3)	Cycle service level (C4)
S1	9	2	2	1
S2	8	2	2	1
S3	7	2	2	1
S4	8	2	2	1
S5	9	2	2	1
S6	9	2	2	1
S7	8	2	2	1
S8	7	2	2	1
S9	9	2	2	1
S10	9	2	2	1
S11	5	2	2	1
S12	2	1.5	1.5	1
S13	15	1.5	1.5	1
S14	8	1.5	1.5	1
S15	8	1.5	1.5	1
S16	8	3	2	2
S17	9	3	2	2
S18	8	3	2	2
S19	8	3	2	2
S20	8	3	2	2

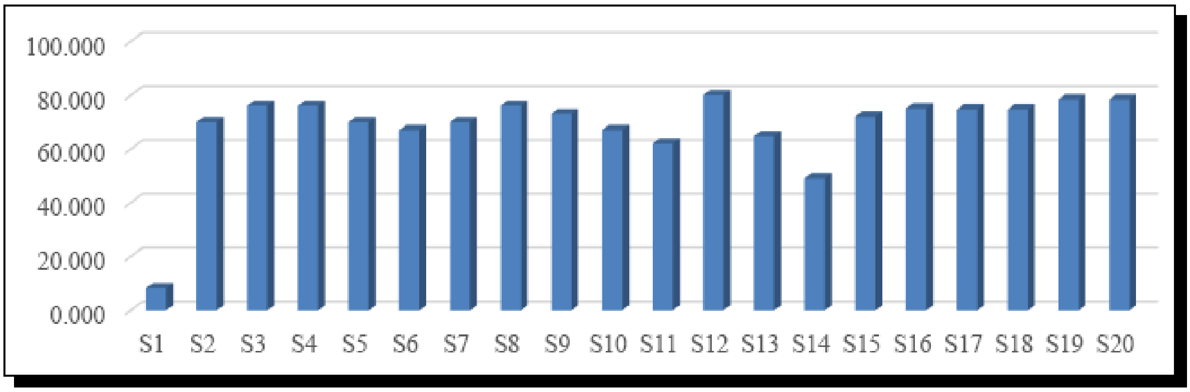

Figure 7. Closeness coefficient of all scenarios

3.6 Discussion

From the previous section, scenario 12 achieved the highest value of closeness coefficient (C_i) which is 0.800829. So it is at the farthest distance from negative ideal solution. Therefore, only scenario 12 is considered for result analysis. The scenario 12 is shown in Figure 8.

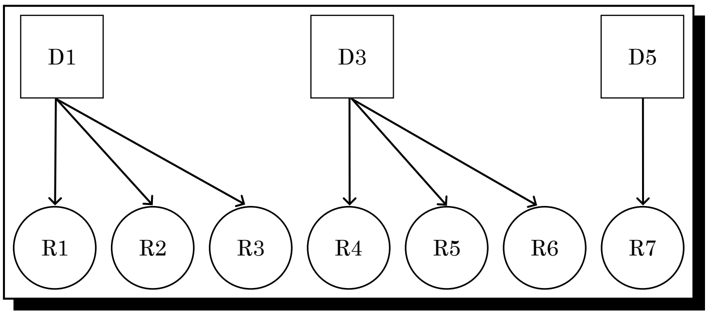

Figure 8. Scenario 12

In this scenario, retail stores R1, R2 and R3 are aggregated for wholesaler D1. Retail stores R4, R5, and R6 are aggregated for D3. The demand of R7 is only fulfilled by D5. In this case there is no need of wholesaler D4. Hence this facility may be closed. For discussion the results, four levels of uncertainty are considered. In this section, the effect of demand uncertainty, the effect of supply uncertainty and the combined effect of both types of uncertainty are examined on the performance parameters.

3.6.1 Effect of Demand Uncertainty on Performance Parameters

For evaluating the effect of demand uncertainty on performance parameters, four levels of uncertainty are considered. The levels of uncertainties are 0.2, 0.4, 0.6 and 0.8. The parameters considered for analysis are "reordering point", "safety stock", "anticipated shortages per cycle"; "fill rate", "average inventory" and the "expected total cost". The effects of four levels of demand uncertainty on performance parameters are shown in Figure 9.

3.6.2 Effect of Supply Uncertainty on Performance Parameters

For examining the effect of supply uncertainty on performance parameters, same levels of uncertainty and same parameters are considered as discussed in the previous case. The effects of four levels of supply uncertainty on performance parameters are shown in Figure 10.

3.6.3 Combined Effect of Demand and Supply Uncertainties on Performance Parameters

The same levels of uncertainty and the same performance parameters are taken for evaluating the combined effect of both types of uncertainty (demand and supply) on performance parameters. The main observation is that there is a large increment in the performance parameters as compared to the individual demand and supply uncertainty. The combined effect of both types of uncertainty on performance parameters is shown in Figure 11.

In this section, various important inventory parameters are also evaluated under various uncertainty levels. The common observation is that except fill rates, all parameters increase as the level of uncertainty increase. But the most important thing is that whatever be the level of uncertainty, the fill rates is always greater than 98% under the cycle service level of 95%. Another important fact here is that the increment level of average inventory is very slow as the level of uncertainty increases as a result the inventory holding cost will be minimum. It comes into the observation that; there is a very small change in the total cost of system as the uncertainty level increase. The trend of the results show that the inventory aggregation approach is very much suitable to reduce safety stocks, to maintain proper service level, to obtain optimized inventory policies and to achieve minimum cost under various types of uncertainty. Therefore, this approach can be implemented to achieve desired service level under various types of uncertainty.

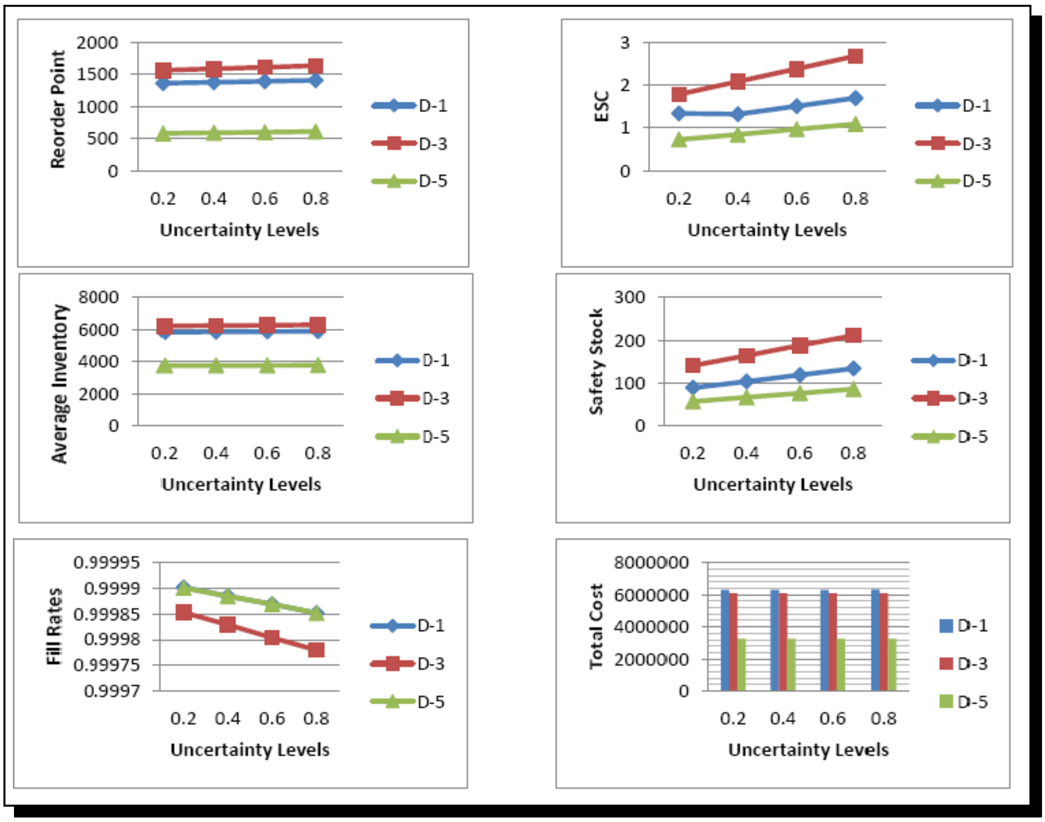

Figure 9. Effect of demand uncertainty level on various key performance parameters

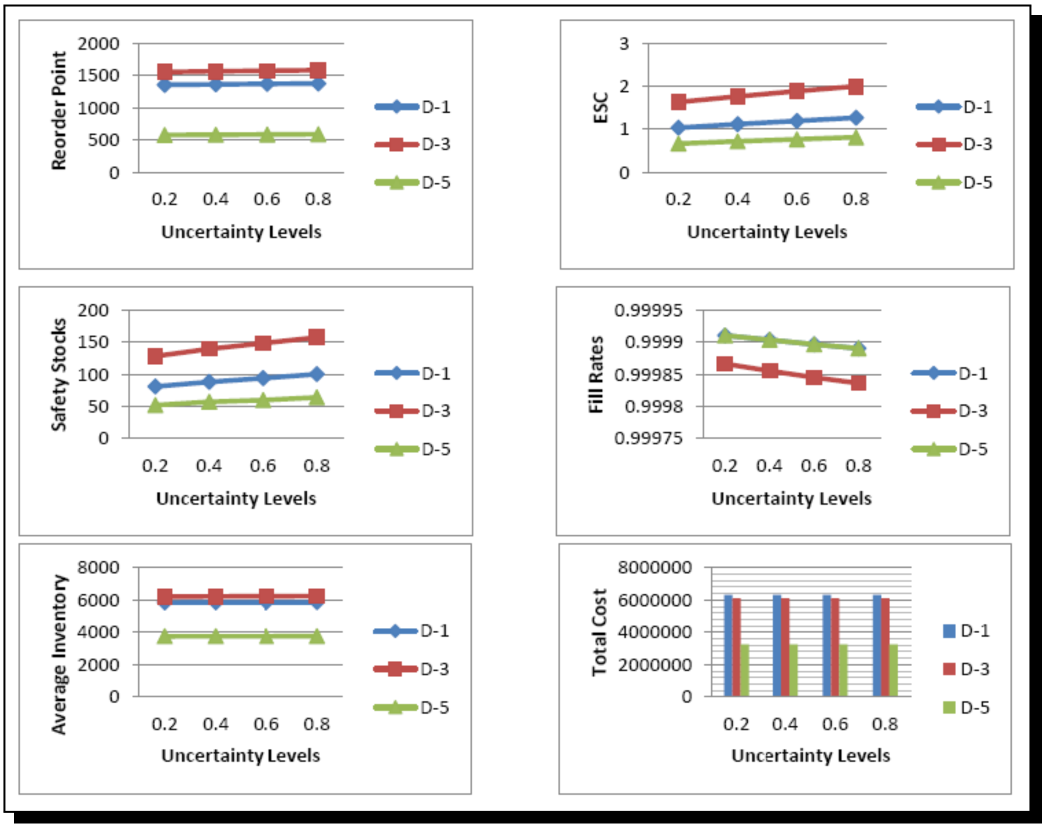

Figure 10. Effect of supply uncertainty level on various key performance parameters

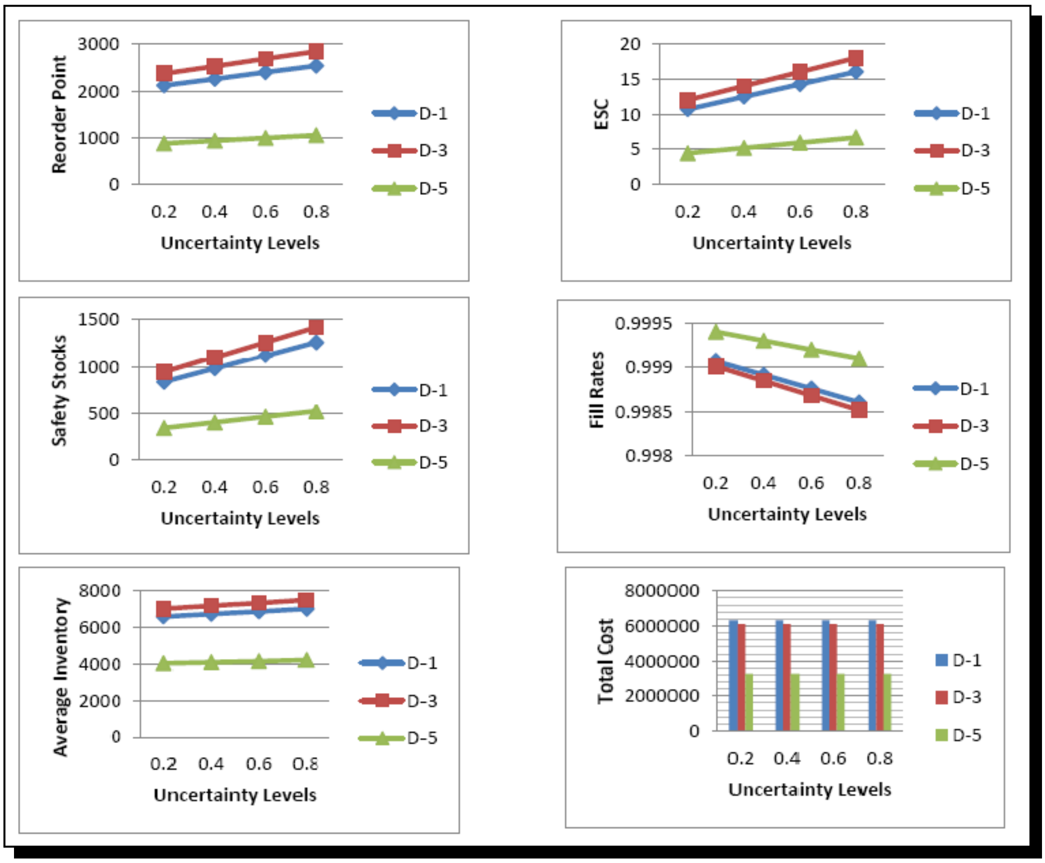

Figure 11. Effect of combined uncertainty level on various key performance parameter

4. Conclusion

Inventory aggregation is one of the best ways to keep the safety stock minimum and it is the most suitable method to maintain the proper balance between the costs and service level. In this study, a distribution network is considered among the various wholesalers and retail stores in which inventory aggregation is applied at the retail store stage. In this problem, different scenarios of distribution networks are generated by considering the inventory aggregation and the partial fulfillment are not allowed for retail stores. In this study, a two-stage problem is formulated. In the first one a non-linear mathematical model is developed using inventory aggregation approach. In this stage, non-linear stochastic programming is done in order to obtain the optimum inventory policy along with minimized expected costs for all scenarios. Some performance parameters via safety stock, fill rate, expected shortages per cycle and average inventory are also calculated in this stage for all scenarios. As it is discussed earlier that, different scenarios are considered of distribution network considering inventory aggregation and the first stage provides the desired result for all scenarios. In second stage, the best scenario is to be identified by considering the proper weightage between cost and service level. Therefore, TOPSIS approach is implemented in between performance parameters (cost and service levels) and the different. This stage provides the best scenario of distribution network along with cost, service level and performance parameters. At the end of this study, the effects of demand uncertainty, supply uncertainty and the combined effect of demand and supply uncertainty are also examined on the performance parameters. Results show that the values of performance parameters increase with the increase in the level of uncertainty except that of the fill rate.

This study pertains to vast ambit for different types of products in the distribution network. This study has some limitations also; it is not an exception as well. A small distribution network is considered, while a large and complex network may be considered.

Furthermore, inventory management tools enhance order fulfillment processes by automating order processing and reducing the likelihood of errors. Integration with shipping and logistics systems enables businesses to efficiently manage the movement of goods from warehouse to customer, ensuring timely deliveries and customer satisfaction.

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Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] P. Agrawal, Effect of uncertain and turbulent environment on organizational design, *Economics & Business Journal: Inquires & Perspectives* **5**(1) (2014), 11 – 24, URL: https://nebeconandbus.org/journal/EBJIP2014v5n1Agrawal-p11.pdf.

[2] E. Arikan and L. Silbermayr, Risk pooling via unidirectional inventory transshipments in a decentralized supply chain, *International Journal of Production Research* **56**(17) (2017), 5593 – 5610, DOI: 10.1080/00207543.2017.1394586.

[3] J. R. T. Arnold, S. N. Chapman and L. M. Clive, *Introduction to Materials Management*, 6th edition, Pearson Prentice Hall, 515 pages (2008).

[4] S. M. Belenson and K. C. Kapur, An algorithm for solving multicriterion linear programming problems with examples, *Journal of the Operational Research Society* **24**(1) (1973), 65 – 77, DOI: 10.1057/jors.1973.9.

[5] F. Bernstein, G. A. DeCroix and Y. Wang, The impact of demand aggregation through delayed component allocation in an assemble-to-order system, *Management Science* **57**(6) (2011), 1154 – 1171, URL: https://www.jstor.org/stable/25835765.

[6] B. Brunaud, J. M. Laínez-Aguirre, J. M. Pinto and I. E. Grossmann, Inventory policies and safety stock optimization for supply chain planning, *AIChE Journal* **65**(1) (2018), 99 – 112, DOI: 10.1002/aic.16421.

[7] P.-L. Chang and C.-T. Lin, On the effect of centralization on expected costs in a multi-location newsboy problem, *Journal of the Operational Research Society* **42**(11) (1991), 1025 – 1030, DOI: 10.1057/jors.1991.193.

[8] C. J. Charles and K. Rajaram, A generalization of the inventory pooling effect to nonnormal dependent demand, *Manufacturing & Services Operation Management* **8**(4) (2006), 351 – 358, DOI: 10.1287/msom.1060.0117.

[9] O. Cosma, P. Pop and C. Sabo, Solving the two-stage supply chain network design problem with risk-pooling and lead times by an efficient genetic algorithm, in: *15th International Conference on Soft Computing Models in Industrial and Environmental Applications* (SOCO 2020), Advances in Intelligent Systems and Computing series, Vol. 1268, Á. Herrero, C. Cambra, D. Urda, J. Sedano, H. Quintián and E. Corchado (editors), Springer, Cham., DOI: 10.1007/978-3-030-57802-2_49.

[10] C. Edirisinghe and D. Atkins, Lower bounding inventory allocations for risk pooling in two-echelon supply chains, *International Journal of Production Economics* **187** (2017), 159 – 167, DOI: 10.1016/j.ijpe.2017.02.015.

[11] G. D. Eppen, Note–Effects of centralization on expected costs in a multi-location newsboy problem, *Management Science* **25**(5) (1979), 413 – 501, DOI: 10.1287/mnsc.25.5.498.

[12] A. Eynan and T. Fouque, Benefiting from the risk-pooling effect: internal (component commonality) vs. external (demand reshape) efforts, *International Journal of Services and Operations Management* **1**(1) (2005), 90 – 99, DOI: 10.1504/IJSOM.2005.006320.

[13] S. Gaur and A. R. Ravindran, A bi-criteria model for the inventory aggregation problem under risk pooling, *Computers & Industrial Engineering* **51**(1) (2006), 482 – 501, DOI: 10.1016/j.cie.2006.08.009.

[14] R. N. Giri, S. K. Kumar and M. Maiti, Joint ordering inventory policy for deteriorating substitute products with price and stock dependent demand, *International Journal of Industrial Engineering* **27**(1) (2020), 58 – 71, DOI: 10.23055/ijietap.2020.27.1.3413.

[15] N. Hu, J.-Y. (Fisher) Ke, L. Liu and Y. Zhang, Risk pooling, supply chain hierarchy, and analysts' forecasts, *Production and Operations Management* **28**(2) (2019), 276 – 291, DOI: 10.1111/poms.12904.

[16] C. L. Hwang and K. Yoon, Methods for multiple attribute decision making, in: *Multiple Attribute Decision Making*, *Lecture Notes in Economics and Mathematical Systems*, Vol. 186, Springer, Berlin — Heidelberg (1981), DOI: 10.1007/978-3-642-48318-9_3.

[17] K. Inderfurth, Safety stock optimization in multi-stage inventory systems, *International Journal of Production Economics* **24**(1-2) (1991), 103 – 113, DOI: 10.1016/0925-5273(91)90157-O.

[18] M. Monenikiyai, S. Ebrahiminejad and B. Vahdani, A bi-objective mathematical model for inventory-distribution-routing problem under risk pooling effect: Robust metaheuristic approach, *Economic Computation and Economic Cybernetics Study and Research* **52**(4) (2018), 257 – 274, DOI: 10.24818/18423264/52.4.18.17.

[19] P. Oeser and P. Romano, Exploring risk pooling in hospitals to reduce demand and lead time uncertainty, *Operations Management Research* **14** (2021), 78 – 94, DOI: 10.1007/s12063-020-00171-y.

[20] M. S. Puga and J.-S. Tancrez, A heuristic algorithm for solving large location–inventory problems with demand uncertainty, *European Journal of Operational Research* **259**(2) (2017), 413 – 423, DOI: 10.1016/j.ejor.2016.10.037.

[21] D. S. Putman, The scope of risk pooling, in: *Selected Presentation at the 2020 Agricultural & Applied Economics Association Annual Meeting* (Kanas City, Missouri, July 26-28, 2020), AgEcon Search, 44 pages, DOI: 10.22004/ag.econ.304480.

[22] S. Routroy and R. Kodali, Differential evolution algorithm for supply chain inventory planning, *Journal of Manufacturing Technology Management* **16**(1) (2005), 7 – 17, DOI: 10.1108/17410380510574059.

[23] F. Salimi and B. Vahdani, Designing a bio-fuel network considering links reliability and risk-pooling effect in bio-refineries, *Reliability Engineering & System Safety* **174** (2018), 96 – 107, DOI: 10.1016/j.ress.2018.02.020.

[24] A. J. Schmitt, S. A. Sun, L. V. Synder and Z.-J. Shen, Centralization versus decentralization: Risk pooling, risk diversification, and supply chain disruptions, *Omega* **52** (2015), 201 – 212, DOI: 10.1016/j.omega.2014.06.002.

[25] D. Singh, H. Singh and K. Singh, Distinct and joint price approaches for multi-layer, multi-channel selling price by manufacturer, *Journal of Optimization and Supply Chain Management* **1**(1) (2024), 50 – 62, DOI: 10.22034/ISS.2024.7892.1010.

[26] D. Singh, A. Jayswal, M. G. Alharbi and A. A. Shaikh, An investigation of a supply chain model for co-ordination of finished products and raw materials in a production system under different situations, *Sustainability* **13**(22) (2021), 12601, DOI: 10.3390/su132212601.

[27] K. Singh, K. C. Sharma and D. Singh, An analysis of the supply chain model for production system under selling price demand rate, controllable deterioration rate with several market demand, *Communications in Mathematics and Applications* **14**(2) (2023), 845 – 860, DOI: 10.26713/cma.v14i2.2216.

[28] K. Singh, K. C. Sharma, D. Singh and Hariom, An integrated two-layer supply chain inventory with declining and price sensitive demand for deteriorating items, *International Journal of Research and Analytical Reviews* **9**(4) (2022), 496 – 506, DOI: 10.1729/Journal.32484.

[29] L. V. Synder, M. Daskin and C.-P. Teo, The stochastic location model with risk pooling, *European Journal of Operations Research* **179**(3) (2007), 1221 – 1238, DOI: 10.1016/j.ejor.2005.03.076.

[30] G. Tagaras, Effects of pooling on the optimization and service levels of two-location inventory systems, *IIE Transactions* **21**(3) (1989), 250 – 257, DOI: 10.1080/07408178908966229.

[31] G. Tagaras, Pooling in multi-location periodic inventory distribution systems, *Omega* **27**(1) (1999), 39 – 59, DOI: 10.1016/S0305-0483(98)00030-9.

[32] E. B. Trikoalaee, J. Mahmoodkhami, M. R. Baurani and R. T. Moghaddam, A self-learning particle swarm optimization for robust multi-echelon capacitated location-allocation-inventory problem, *Journal of Advanced Manufacturing System* **18**(4) (2019), 677 – 694, DOI: 10.1142/S0219686719500355.

[33] P. Vats, G. Soni and A. P. S. Rathore, A review of multi-objective inventory control problem, *International Journal of Intelligent Enterprises* **5**(3) (2018), 213 – 230, DOI: 10.1504/IJIE.2018.093396.

[34] P. Vats, G. Soni, A. P. S. Rathore and S. P. Prakash, A demand aggregation approach for inventory control in two echelon supply chain under uncertainty, *OPSEARCH* **56** (2019), 840 – 868, DOI: 10.1007/s12597-019-00389-w.

[35] B. M. Vishkaei, S. T. A. Niaki, E. Khorram and M. Farhangi, A bi-objective inventory model to minimize cost and stock out time under backorder shortages and screening, *International Journal of Industrial Engineering* **26**(5) (2019), 707 – 718, DOI: 10.23055/ijietap.2019.26.5.3431.

[36] C. A. Weber, J. R. Current and W. C. Benton, Vendor selection criteria and methods, *European Journal of Operational Research* **50**(1) (1991), 2 – 18, DOI: 10.1016/0377-2217(91)90033-R.

[37] Z. K. Weng, Risk-pooling over demand uncertainty in the presence of product modularity, *International Journal of Production Economics* **62**(1-2) (1999), 75 – 85, DOI: 10.1016/S0925-5273(98)00226-6.

[38] K. Xu and P. T. Evers, Managing single echelon inventories through demand aggregation and the feasibility of a correlation matrix, *Computers & Operations Research* **30**(2) (2003), 297 – 308, DOI: 10.1016/S0305-0548(01)00097-1.

[39] K. P. Yoon and C.-H. Hwang, *Multiple Attribute Decision Making: An Introduction*, Sage Publications, Inc., (1995), DOI: 10.4135/9781412985161.

[40] F. You and I. E. Grossman, Balancing responsiveness and economics in process supply chain design with multi-echelon stochastic inventory, *AIChE Journal* **57**(1) (2009), 178 – 192, DOI: 10.1002/aic.12244.

[41] M. Zeleny, A concept of compromise solutions and the method of the displaced ideal, *Computers & Operations Research* **1**(3-4) (1974), 479 – 496, DOI: 10.1016/0305-0548(74)90064-1.

[42] Y. Zhang, G. Hua, T.C.E. Cheng, J. Zhang and V. Fernandez, Risk pooling through physical probabilistic selling, *International Journal of Production Economics* **219** (2020), 295 – 311, DOI: 10.1016/j.ijpe.2019.04.014.