Communications in Mathematics and Applications
Vol. 15, No. 1, pp. 111–132, 2024
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v15i1.2531

http://www.rgnpublications.com

Research Article

A Two Server Poisson Queue With State Dependent Hybrid Service Discipline With Variant Breakdown

R. Kalyanaraman* and B. Anurathi

Department of Mathematics, Annamalai University, Annamalainagar 608002, Tamil Nadu, India
***Corresponding author:** r.kalyan24@rediff.com

Received: December 22, 2023 **Accepted:** March 18, 2024

Abstract. A Poisson queue with two servers and with system breakdown has been considered in this paper. In addition, the servers are in homogeneous mode upto serving of N customers. After which the servers changed to heterogeneous mode. If the system is busy failure may occur to the system. As in the case of service policy, in a similar way two different breakdown policies are assumed. At the instant of breakdown, if there are N or less than N customers in the system the system is completely shutdown. Otherwise, the server provides service with different service rates. The number of arrivals and the number of service completions follow different Poisson distributions. The interbreakdown periods follow negative exponential distributions. Immediately the repair process takes place. The repair periods are random variables, and follow a negative exponential distribution. This model is defined and the time independent solutions are derived. Also, some system performance measures are obtained. To show the practical applicability of the model some numerical illustrations are provided. The corresponding cost model is defined and analyzed.

Keywords. Markovian queue, Homogeneous mode of service, Variant breakdown, Repair, Time independent solution, System measures

Mathematics Subject Classification (2020). 90B22, 60K25, 60K30

1. Introduction

In practical waiting line situations, it can be seen that same work can be done by more than one server separately, e.g., in bank, post office, railway station and airport, there are more than one counter for the same task. In the queueing literature, such a systems are called multi-server systems. If the servers are human beings even though, they have same types of work, but their skill and efficiency varies, i.e., their service rates vary. Such a queueing system is called queues with heterogeneous servers. The research on heterogeneous models is comparatively less than their counter part, homogeneous models. In practical queueing situations, the study of model with heterogeneous servers are more appropriate than model with homogeneous servers. Some earlier works in the area of queueing system with heterogeneous servers are Kendall [18], Kiefer and Wolfowitz [19], Karlin and McGregor [17], Krishnamoorthi [20], Singh [28], Lin and Kumar [23], and Abou-El-Ata and Shawky [1].

Alves *et al*. [2] analysed heterogeneous multi server queue. Ammar [3] analyzed the time dependent behaviour of a two heterogeneous server system with catastrophes. Efrosinin *et al*. [7] analysed a two server heterogeneous queueing system with threshold control policy. Sudhesh *et al*. [30] studied the transient analysis of a two heterogeneous servers queue with system disaster, server repair and customers impatience. Kalyanaraman and Senthilkumar [16] studied a heterogeneous two server Poisson queue with switching of service modes.

Laxami and Kassahun [21] studied multiserver queueing system. Pradhan [27] studied a buffer queueing system. Gupta and Agarwal [13] analyzed a cost model for machine repair man problem. Sudhesh and Azhagappan [29] analysed an M/M/c queue with heterogeneous servers, balking and reneging.

In real life, manufacturing systems, computer systems, communication and network systems sudden failure is always possible. In the literature, such situations are theoretically studied using queue with breakdown. Some earlier works in this area are White and Christie [38], Gaver [11], Avi-Itzhak and Naor [4], Thiruvengadam [31], Federgruen and Green [8], and van Dijk [32]. Single server queueing system with unreliable server that is, breakdown of server have been studied by many researchers including Feller [9], Federgruen and Green [8], Li *et al*. [22], Nakdimon and Yechiali [25], Wang *et al*. [35, 36], Choudhury and Tadj [5], to mention a few. But the systems multi server counterpart is more flexible in practice. However, due to their analytical complexity, there have been only a few studies in this direction. Miltrany and Avi-Itzhak [24] studied an M/M/N queue with server break down and ample repair capacity. In their study, they obtain the moment generating function of the queue size by using the transformation method. Vinod [33] considered the same model using the matrix-geometric method. For $N = 1$, Vinod [34] imposed some restrictions on the server down-periods (either independent of the queue length or only occurring when the server is active). Neuts and Lucantoni [26], and Wartenhorst [37] extended the models studied by Miltrany and Avi-Itzhak [24], and Vinod [33, 34] by considering limited repair capacity. Kalyanaraman and Kalaiselvi [15] have analyzed a heterogeneous server queue with breakdown and a threshold on slower server.

One of the major issues in waiting line model is the server breakdown. It is clear that breakdown in service mechanism add delay in serving the customers. To overcome this, a spare server can be used to serve the customers temporarily or the damage to the server is minimum and the server has the capacity to serve the customers irrespective of breakdown the server can be utilized but with lower service rate. In this paper, we have the assumption that in the case of system breakdown the servers has the capacity to serve the queue with lower service rates. Fiems *et al*. [10] analyzed a queueing system with different types of server interruptions. Kalidas and Kasturi [14] considered a queue with working breakdown.

In queueing analysis, in particular the behavioral operation investigations, the service times are affected by the load or otherwise, there is a clear impact of workload on the service speed (Delasay *et al*. [6]), with this in mind we developed the model discussed in this paper. The model discussed in this paper is more versatile related to real life situations. In this paper, we consider a two server Markovian queue with partial breakdown. Also, the servers are in homogeneous mode upto serving of N customers. After which the servers changed to heterogeneous mode. During the system busy, the system may breakdown. Immediately the repair process takes place. During repair, the servers give service to the waiting customers, if any with lower service rate. The inter breakdown periods and the repair periods are negative exponential distributions. The structure of the paper is given below: The model definition and the correlated mathematical notations are introduced in Section 2. The steady state analysis of the model is given in Section 3. Some existing models are obtained as particular models in Section 4. Some performance measures are obtained in Section 5. A numerical study is carried out in Section 6. A cost structure is given in Section 7. Finally, a conclusion is given in Section 8.

2. The Model

In this work, we consider a two server Markovian queue, it has the following characteristics:

- The arrival process follows Poisson process with parameter λ.
- The arriving customer waits in the queue of infinite capacity, if the service is not immediate.
- In the queue the First-In-First-Out queue discipline is applied for service.
- The two servers serve the customers, the service periods follows exponential distribution.
- The service rule is if the number of customers in the system does not exceed threshold value N ($N \geq 1$) both the servers gives service with same rate μ (homogeneous mode). On the other hand, if the number exceeds N the services are given using different rates. The rates are μ_1 for Server 1 and μ_2 for Server 2 with $\mu_1 > \mu_2$.
- If an arrival finds both the servers are idle, the customer selects any one of the server for service. On the other hand, if both the servers are busy the arrival waits for the free server.
- During busy period (both the servers may be busy or any one of the server may be busy), the system may breakdown. The breakdown period follows negative exponential with rate θ. Immediately the repair is carried out, the repair period follows exponential with rate β.
- At the point of breakdown if the system has N or less than N customers, the system is completely shutdown. Otherwise if there are more than N customers in the system then the servers doing service with rates μ_3 (Server 1) and μ_4 (Server 2) independent of the repair process (partial breakdown).

For the mathematical analysis the following random variables are introduced:

At time t, $X(t)$ be the number of customers in the system, $Y(t)$ be the service mode of the servers and $Z(t)$ be the system state. $Y(t)$ and $Z(t)$ has the following random representations:

$$Y(t) = \left\{\begin{matrix} 1, & \text{the servers are in homogeneous mode} \\ 2, & \text{the servers are in heterogeneous mode} \end{matrix}\right\},$$

$$Z(t) = \left\{\begin{matrix} 0, & \text{the system is in working condition} \\ 1, & \text{the system is in breakdown condition} \\ 2, & \text{the system is in partial breakdown condition} \end{matrix}\right\}.$$

Let $\{(Z(t), Y(t), X(t) : t \geq 0)\}$ be a continuous time Markov process whose state space

$$S = \{(0,1,j); j = 0,1,2\ldots N\} \cup \{(1,1,j); j = 1,2\ldots N\}$$
$$\cup \{(0,2,j); j = N+1, N+2,\ldots\} \cup \{(2,2,j); j = N+1, N+2,\ldots\}.$$

The corresponding probability distributions are:

$$p_{ijn} = Pr\{Z(t) = i, Y(t) = j, X(t) = N\}, \quad i,j,N \in S.$$

In steady state, $p_{ijn} = \lim_{t\to\infty} p_{ijn}(t)$.

Let X_0 be the number of customers in the system when the server is in breakdown mode, let X_1 be the number of customers in the system when the server is in partial breakdown mode, let X_2 be the number of customers in the system when the server is on homogeneous mode, let X_3 be the number of customers in the system when the server is on heterogeneous mode and let X be the number of customers in the system. Let $E(X_0)$, $E(X_1)$, $E(X_2)$, $E(X_3)$, $E(X)$ are the corresponding expected values.

The schematic representation is given below:

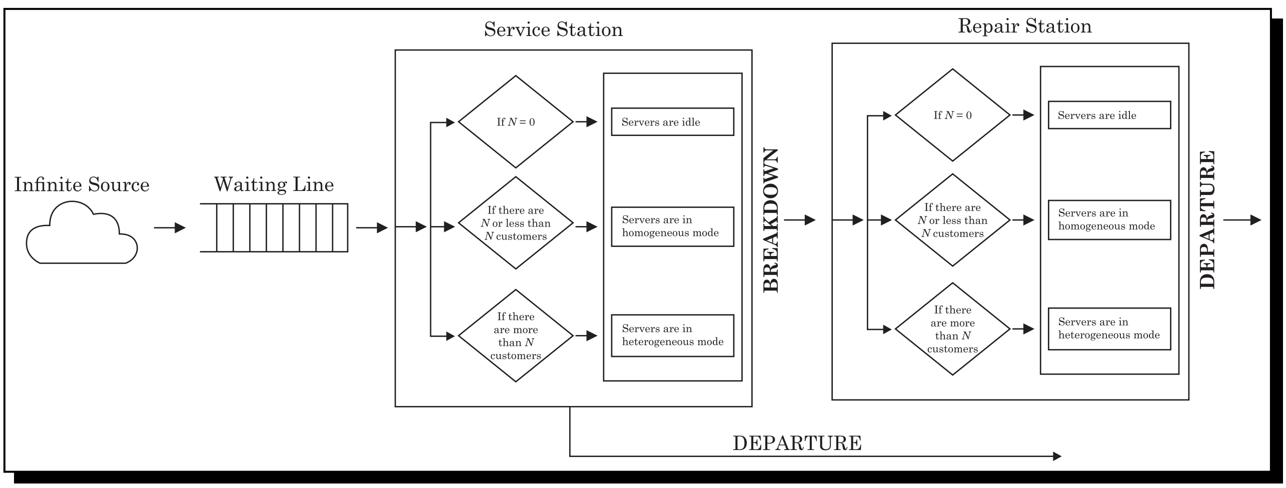

Figure 1. Schematic representation of the model

3. The Analysis

In this section, we analyze the model defined in Section 2, in steady state by obtaining the difference equations.

Using birth-death arguments, the following difference equations are:

$$\lambda p_{010} = 2\mu p_{011}, \tag{1}$$

$$(\lambda+\theta+2\mu)p_{01i} = \lambda p_{01i-1} + 2\mu p_{01i+1} + \beta p_{11i}, \quad i=1,2,\dots,N-1, \tag{2}$$

$$(\lambda+\theta+2\mu)p_{01N} = \lambda p_{01N-1} + (\mu_1+\mu_2)p_{02N+1} + \beta p_{11N}, \tag{3}$$

$$(\lambda+\theta+\mu_1+\mu_2)p_{02N+1} = \lambda p_{01N} + (\mu_1+\mu_2)p_{02N+2} + \beta p_{22N+1}, \tag{4}$$

$$(\lambda+\theta+\mu_1+\mu_2)p_{02i} = \lambda p_{02i-1} + (\mu_1+\mu_2)p_{02i+2} + \beta p_{22i}, \quad i=N+1,N+2,\dots, \tag{5}$$

$$(\lambda+\beta)p_{111} = \theta p_{011}, \tag{6}$$

$$(\lambda+\beta)p_{11i} = \lambda p_{11i-1} + \theta p_{01i}, \quad i=2,3,\dots,N-1, \tag{7}$$

$$(\lambda+\beta)p_{11N} = \lambda p_{11N-1} + \theta p_{01N} + (\mu_3+\mu_4)p_{22N+1}, \tag{8}$$

$$(\lambda+\beta+\mu_3+\mu_4)p_{22N+1} = \lambda p_{11N} + \theta p_{02N+1} + (\mu_3+\mu_4)p_{22N+2}, \tag{9}$$

$$(\lambda+\beta+\mu_3+\mu_4)p_{22i} = \lambda p_{22i-1} + \theta p_{02n} + (\mu_3+\mu_4)p_{22i+1}, \quad i=N+2,N+3,\dots. \tag{10}$$

Theorem 3.1. *Under the stability condition, $\rho_2 < 1$, the steady state results are:*

$$p_{011} = A_1 p_{010},$$

$$p_{012} = A_2 p_{010},$$

$$p_{01i} = [\alpha A_{i-1} - \beta A_{i-2} - \gamma p_{11i-1}], \quad i=3,4,\dots,N,$$

$$p_{02N+1} = [\alpha_1 A_N - \beta_1 A_{N-1} - \gamma_1 p_{11N}]p_{010},$$

$$p_{02N+2} = [\alpha_2 B_{N+1} - \beta_1 A_N - \gamma_1 p_{22N+1}]p_{010},$$

$$p_{02i} = [\alpha_2 B_{i-1} - \beta_1 B_{i-2} - \gamma_1 p_{22i-1}]p_{010}, \quad i=N+3,N+4,\dots,$$

$$p_{111} = \delta A_1 p_{010},$$

$$p_{11i} = [\delta A_i + \xi p_{11i-1}], \quad i=2,3,\dots,N,$$

$$p_{22N+1} = [\eta p_{11N} - \delta_1 A_N - \beta_2 p_{11N-1}],$$

$$p_{22N+2} = [\eta_1 p_{22N+1} - \beta_2 p_{11N} - \delta_1 B_{N+1}],$$

$$p_{22i} = [\eta_1 p_{22i-1} - \beta_2 p_{22i-2} - \delta_1 B_{i-1}], \quad i=N+3,N+4,\dots.$$

Proof. Solving equation (1),

$$p_{011} = A_1 p_{010}. \tag{11}$$

Solving equation (2), for $i=2$

$$p_{012} = A_2 p_{010}. \tag{12}$$

Solving equation (2) successively, for $i \geq 3$

$$p_{01i} = [\alpha A_{i-1} - \beta A_{i-2} - \gamma p_{11i-1}]p_{010}, \quad i=3,4,\dots,N. \tag{13}$$

Solving equation (3),

$$p_{02N+1} = [\alpha_1 A_N - \beta_1 A_{N-1} - \gamma_1 p_{11N}]p_{010}. \tag{14}$$

Solving equation (4),

$$p_{02N+2} = [\alpha_2 B_{N+1} - \beta_1 A_N - \gamma_1 p_{22N+1}]p_{010}. \tag{15}$$

Solving equation (5) successively,

$$p_{02i} = [\alpha_2 B_{i-1} - \beta_1 B_{i-2} - \gamma_1 p_{11i-1}]p_{010}, \quad i = N+3, N+4, \dots. \tag{16}$$

Solving equation (6),

$$p_{111} = \delta A_1 p_{010}. \tag{17}$$

Solving equation (7) successively,

$$p_{11i} = [\delta A_i + \xi p_{11i-1}]p_{010}, \quad i = 2, 3, \dots, N. \tag{18}$$

Solving equation (8),

$$p_{22N+1} = [\eta p_{11N} - \delta_1 A_N - \beta_2 p_{11N-1}]p_{010}. \tag{19}$$

Solving equation (9),

$$p_{22N+2} = [\eta_1 p_{22N+1} - \beta_2 p_{11N} - \delta_1 B_{N+1}]p_{010}. \tag{20}$$

Solving equation (10) successively,

$$p_{22i} = [\eta_1 p_{22i-1} - \beta_2 p_{22i-2} - \delta_1 B_{i-1}]p_{010}. \tag{21}$$

p_{010} is obtained using the normalization condition,

$$p_{010} + \sum_{i=1}^{N} p_{01i} + \sum_{i=N+1}^{\infty} p_{02i} = 1. \tag{22}$$

Using (11), (12), (13), (14), (15) and (16), we get

$$p_{010} = \left\{1 + A_1 + A_2 + \sum_{i=3}^{N} A_i + B_{N+1} + B_{N+2} + \sum_{i=N+3}^{\infty} B_i\right\}^{-1}, \tag{23}$$

where

$$A_1 = \left(\frac{\lambda}{2\mu}\right),$$

$$A_2 = [\alpha A_1 - \beta - \gamma p_{111}],$$

$$A_i = [\alpha A_{i-1} - \beta A_{i-2} - \gamma p_{11i-1}], \quad i = 3, \dots, N,$$

$$B_{N+1} = [\alpha_1 A_N - \beta_1 A_{N-1} - \gamma_1 p_{11N}],$$

$$B_{N+2} = [\alpha_2 B_{N+1} - \beta_1 A_N - \gamma_1 p_{22N+1}],$$

$$B_i = [\alpha_2 B_{i-1} - \beta_1 B_{i-2} - \gamma_1 p_{11i-1}], \quad i = N+3, N+4, \dots,$$

$$\alpha = \left(\frac{\lambda+\theta+2\mu}{2\mu}\right), \qquad \beta = \left(\frac{\lambda}{2\mu}\right), \qquad \gamma = \left(\frac{\beta}{2\mu}\right),$$

$$\delta = \left(\frac{\theta}{\lambda+\beta}\right), \qquad \alpha_1 = \left(\frac{\lambda+\theta+2\mu}{\mu_1+\mu_2}\right), \qquad \alpha_2 = \left(\frac{\lambda+\theta+\mu_1+\mu_2}{\mu_1+\mu_2}\right),$$

$$\beta_1 = \left(\frac{\lambda}{\mu_1+\mu_2}\right), \qquad \gamma_1 = \left(\frac{\beta}{\mu_1+\mu_2}\right), \qquad \xi = \left(\frac{\lambda}{\lambda+\beta}\right),$$

$$\eta = \left(\frac{\lambda+\beta}{\mu_3+\mu_4}\right), \qquad \eta_1 = \left(\frac{\lambda+\beta+\mu_3+\mu_4}{\mu_3+\mu_4}\right), \qquad \delta_1 = \left(\frac{\theta}{\mu_3+\mu_4}\right),$$
$$\beta_2 = \left(\frac{\lambda}{\mu_3+\mu_4}\right).$$

4. Particular Models

In this section, some particular models related to the model discussed in this paper are given below:

(i) As $N \to \infty$, $\mu_1, \mu_2 = \mu$ the model coincides with M/M/2 model (Gross *et al*. [12] for $c = 2$), $\theta = \beta = 0$, $\mu_3 = \mu_4 = 0$:
$$p_n = \frac{\lambda^n}{2^{n-1}\mu^n} p_0, \quad n \geq 1,$$
$$p_0 = \frac{2\mu+\lambda}{2\mu-\lambda}.$$

(ii) For $j \geq N+1$, $\lambda = 0$, the model coincides with M/M/2/N model (Gross *et al*. [12] for $c = 2$, $K = N$), $\theta = \beta = 0$, $\mu_3 = \mu_4 = 0$:
$$p_n = \frac{\lambda^n}{2^{n-1}\mu^n} p_0, \quad 1 \leq N \leq N,$$
$$p_0 = \left(1 + \frac{\lambda}{\mu} + \frac{r^2}{2!}\frac{1-\rho^{N-1}}{1-\rho}\right)^{-1}, \quad \rho = \frac{\lambda}{2\mu} \neq 1,$$
$$p_0 = \left(1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{2\mu^2}(N-1)\right)^{-1}, \quad \rho = 1.$$

5. Some Performance Measures

Using straight forward calculations, the following performance measures are obtained:

(i) Idle probability $p_{010} = \left\{1 + A_1 + A_2 + \sum_{i=3}^{N} A_i + B_{N+1} + B_{N+2} + \sum_{i=N+3}^{\infty} B_i\right\}^{-1}$.

(ii) Probability that the servers are in homogeneous mode, are busy $p_{hob} = \sum_{n=1}^{N} p_{01n}$.

(iii) Probability that the servers are in heterogeneous mode, are busy $p_{htb} = \sum_{n=N+1}^{\infty} p_{02n}$.

(iv) Probability that the system is in partial breakdown mode $p_{pbd} = \sum_{n=N+1}^{\infty} p_{22n}$.

(v) Probability that the system is in breakdown mode $p_{bd} = \sum_{n=1}^{N} p_{11n}$.

(vi) Mean number of customers in the system when the servers are in homogeneous mode $E(X_0) = \sum_{n=1}^{N} n p_{01n}$.

(vii) Mean number of customers in the system when the servers are in heterogeneous mode $E(X_1) = \sum_{n=N+1}^{\infty} n p_{02n}$.

(viii) Mean number of customers in the system when the servers are in partial breakdown mode $E(X_2) = \sum_{n=N+1}^{\infty} Np_{22n}$.

(ix) Mean number of customers in the system when the servers are in breakdown mode $E(X_3) = \sum_{n=1}^{N} Np_{11n}$.

(x) Expected number of customers in the system mode

$$E(X) = E(X_0) + E(X_1) + E(X_2) + E(X_3),$$

$$E(X) = \sum_{n=1}^{N} np_{01n} + \sum_{n=N+1}^{\infty} np_{02n} + \sum_{n=N+1}^{\infty} Np_{22n} + \sum_{n=1}^{N} Np_{11n}.$$

6. Numerical Illustrations

In this section, we presents some numerical illustrations related to the model discussed in this paper by varying some parameter values and fixing the remaining parameters. The results corresponding to the formulas given in Section 3 and Section 5 are obtained and are tabulated or shown in figures. In this study, the value of λ is taken as 1.1, 1.4, 1.6 and 2, the value of N is taken as 5, 10 and 15, and the value of μ is taken as 1.1, 1.4, 1.6 and 2. The value of λ, μ, μ_1, μ_2, μ_3, μ_4 and θ is fixed as 2, 5, 4, 3, 2, 1 and 2.5, respectively. The probabilities are calculated and are shown in Tables 1-4. The system performance measures such as probability that servers are idle p_{010}, probability that the servers are busy (homogeneous mode) p_{hob} and probability that the servers are busy (heterogeneous mode) p_{htb} are listed in Tables 5-8.

From these tables, it is clear that the analytical results derived in this paper are numerically tractable. Tables 5 and 6 shows that (i) for increasing values of arrival rate λ (for increasing values of N), the idle probability p_{010} decreases, that (ii) for increasing values of arrival rate λ (for increasing values of N), the Probability that the servers are busy (homogeneous mode) p_{hob} increases (increases) and (iii) for increasing values of arrival rate λ (for increasing values of N), the Probability that the servers are busy (heterogeneous mode) p_{htb} increases (increases).

Tables 7 and 8 shows that (i) for increasing values of service rate μ (for increasing values of N) the idle probability p_{010} increases (decreases). (ii) for increasing values of service rate μ the Probability that the servers are busy (homogeneous mode) p_{hob} increases for some values of N and decreases for some values of N. We have the trend if we have change the value of N. (iii) for increasing values of service rate μ (for increasing values of N) the probability that the servers are busy (heterogeneous mode) p_{htb} decreases (increases).

The mean number of customers in the system is given in Figures 2-7. The graphs in the figures shows that as arrival rate increases the mean number also increases whereas the service rate increases the mean number decreases.

Figures 8-10 represents arrival rate verses, the expected waiting time. From the graphs, it is clear that as arrival rate increases there is decline in the expected waiting time. Figures 11-13 represents expected waiting time against service. Here also, we have the same trend.

Table 1. Probabilities ($\mu = 5$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

$p_{j,i,n}$	$\lambda = 1.1$			$\lambda = 1.4$		
	$N = 5$	$N = 10$	$N = 15$	$N = 5$	$N = 10$	$N = 15$
$p_{0,1,0}$	0.686825	0.491719	0.307886	0.619201	0.411066	0.237801
$p_{0,1,1}$	0.075550	0.054089	0.033867	0.086688	0.057549	0.033292
$p_{0,1,2}$	0.025481	0.018243	0.011422	0.031838	0.021136	0.012227
$p_{0,1,3}$	0.024203	0.017328	0.010849	0.029604	0.019653	0.011369
$p_{0,1,4}$	0.027618	0.019773	0.012380	0.033733	0.022394	0.012955
$p_{0,1,5}$	0.032002	0.022912	0.014345	0.039288	0.026082	0.015088
$p_{0,1,6}$	-	0.026579	0.016642	-	0.030440	0.017609
$p_{0,1,7}$	-	0.030836	0.019307	-	0.035533	0.020555
$p_{0,1,8}$	-	0.035775	0.022399	-	0.041479	0.023995
$p_{0,1,9}$	-	0.041505	0.025987	-	0.048420	0.028010
$p_{0,1,10}$	-	0.048152	0.030150	-	0.056522	0.032698
$p_{0,1,11}$	-	-	0.034979	-	-	0.038169
$p_{0,1,12}$	-	-	0.040581	-	-	0.044556
$p_{0,1,13}$	-	-	0.047081	-	-	0.052012
$p_{0,1,14}$	-	-	0.054623	-	-	0.060716
$p_{0,1,15}$	-	-	0.063371	-	-	0.070876
$p_{0,2,6}$	0.053035	-	-	0.065504	-	-
$p_{0,2,7}$	0.075281	-	-	0.094141	-	-
$p_{0,2,8}$	0.0	-	-	0.0	-	-
$p_{0,2,9}$	0.0	-	-	0.0	-	-
$p_{0,2,10}$	0.0	-	-	0.0	-	-
$p_{0,2,11}$	0.0	0.079807	-	0.0	0.094258	-
$p_{0,2,12}$	0.0	0.113284	-	0.0	0.135468	-
$p_{0,2,13}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,14}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,15}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,16}$	0.0	0.0	0.105031	0.0	0.0	0.118194
$p_{0,2,17}$	0.0	0.0	0.149089	0.0	0.0	0.169870
$p_{0,2,18}$	0.0	0.0	0.0	0.0	0.0	0.0
TP	0.999	1	0.999	0.999	1	0.999

Table 2. Probabilities ($\mu = 5$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

$p_{j,i,n}$	$\lambda = 1.6$			$\lambda = 2$		
	$N = 5$	$N = 10$	$N = 15$	$N = 5$	$N = 10$	$N = 15$
$p_{0,1,0}$	0.577672	0.365553	0.201633	0.502374	0.290199	0.146786
$p_{0,1,1}$	0.092427	0.058488	0.032261	0.100474	0.058040	0.029357
$p_{0,1,2}$	0.035794	0.022651	0.012493	0.042930	0.024799	0.012543
$p_{0,1,3}$	0.032958	0.020856	0.011504	0.039102	0.022588	0.011425
$p_{0,1,4}$	0.037520	0.023743	0.013096	0.044449	0.025676	0.012987
$p_{0,1,5}$	0.043845	0.027746	0.015304	0.052290	0.030206	0.015278
$p_{0,1,6}$	-	0.032515	0.017934	-	0.035698	0.018056
$p_{0,1,7}$	-	0.038116	0.021024	-	0.042213	0.021352
$p_{0,1,8}$	-	0.044682	0.024646	-	0.049922	0.025251
$p_{0,1,9}$	-	0.052380	0.028892	-	0.059039	0.029862
$p_{0,1,10}$	-	0.061405	0.033869	-	0.069820	0.035316
$p_{0,1,11}$	-	-	0.039705	-	-	0.041765
$p_{0,1,12}$	-	-	0.046545	-	-	0.049392
$p_{0,1,13}$	-	-	0.054565	-	-	0.058413
$p_{0,1,14}$	-	-	0.063965	-	-	0.069080
$p_{0,1,15}$	-	-	0.074986	-	-	0.081696
$p_{0,2,6}$	0.073403	-	-	0.088282	-	-
$p_{0,2,7}$	0.106375	-	-	0.130095	-	-
$p_{0,2,8}$	0.0	-	-	0.0	-	-
$p_{0,2,9}$	0.0	-	-	0.0	-	-
$p_{0,2,10}$	0.0	-	-	0.0	-	-
$p_{0,2,11}$	0.0	0.102834	-	0.0	0.117959	-
$p_{0,2,12}$	0.0	0.149030	-	0.0	0.173841	-
$p_{0,2,13}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,14}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,15}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,16}$	0.0	0.0	0.125579	0.0	0.0	0.138022
$p_{0,2,17}$	0.0	0.0	0.181992	0.0	0.0	0.203410
$p_{0,2,18}$	0.0	0.0	0.0	0.0	0.0	0.0
TP	0.999	1	0.999	0.999	1	0.999

Table 3. Probabilities ($\lambda = 2$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

$p_{j,i,n}$	$\mu = 1.1$			$\mu = 1.4$		
	$N = 5$	$N = 10$	$N = 15$	$N = 5$	$N = 10$	$N = 15$
$p_{0,1,0}$	0.022128	0.000580	0.000015	0.045330	0.002339	0.000117
$p_{0,1,1}$	0.020116	0.000528	0.000013	0.032379	0.001671	0.000083
$p_{0,1,2}$	0.034003	0.000892	0.000023	0.044430	0.002293	0.000115
$p_{0,1,3}$	0.068282	0.001791	0.000046	0.076661	0.003956	0.000198
$p_{0,1,4}$	0.141111	0.003701	0.000095	0.138297	0.007137	0.000358
$p_{0,1,5}$	0.292864	0.007681	0.000198	0.251323	0.012970	0.000650
$p_{0,1,6}$	-	0.015952	0.000412	-	0.023598	0.001184
$p_{0,1,7}$	-	0.033130	0.000857	-	0.042943	0.002154
$p_{0,1,8}$	-	0.068808	0.001781	-	0.078147	0.003921
$p_{0,1,9}$	-	0.142908	0.003699	-	0.142212	0.007136
$p_{0,1,10}$	-	0.296807	0.007683	-	0.258799	0.012986
$p_{0,1,11}$	-	-	0.015957	-	-	0.023632
$p_{0,1,12}$	-	-	0.033141	-	-	0.043007
$p_{0,1,13}$	-	-	0.068831	-	-	0.078264
$p_{0,1,14}$	-	-	0.142958	-	-	0.142426
$p_{0,1,15}$	-	-	0.296911	-	-	0.259189
$p_{0,2,6}$	0.191145	-	-	0.182903	-	-
$p_{0,2,7}$	0.230349	-	-	0.228677	-	-
$p_{0,2,8}$	0.0	-	-	0.0	-	-
$p_{0,2,9}$	0.0	-	-	0.0	-	-
$p_{0,2,10}$	0.0	-	-	0.0	-	-
$p_{0,2,11}$	0.0	0.193739	-	0.0	0.188386	-
$p_{0,2,12}$	0.0	0.233484	-	0.0	0.235548	-
$p_{0,2,13}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,14}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,15}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,16}$	0.0	0.0	0.193807	0.0	0.0	0.188669
$p_{0,2,17}$	0.0	0.0	0.233565	0.0	0.0	0.235903
$p_{0,2,18}$	0.0	0.0	0.0	0.0	0.0	0.0
TP	1	1	0.999	0.999	1	0.999

Table 4. Probabilities ($\lambda = 2$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

$p_{j,i,n}$	$\mu = 1.6$			$\mu = 2$		
	$N = 5$	$N = 10$	$N = 15$	$N = 5$	$N = 10$	$N = 15$
$p_{0,1,0}$	0.065514	0.004787	0.000335	0.114836	0.014336	0.001669
$p_{0,1,1}$	0.040946	0.002992	0.000209	0.057418	0.007168	0.000834
$p_{0,1,2}$	0.049964	0.003651	0.000256	0.057418	0.007168	0.000834
$p_{0,1,3}$	0.079538	0.005812	0.000407	0.080386	0.010036	0.001168
$p_{0,1,4}$	0.133864	0.009781	0.000686	0.121727	0.015197	0.001769
$p_{0,1,5}$	0.227462	0.016620	0.001165	0.186954	0.023340	0.002717
$p_{0,1,6}$	-	0.028286	0.001984	-	0.035933	0.004184
$p_{0,1,7}$	-	0.048153	0.003377	-	0.055343	0.006444
$p_{0,1,8}$	-	0.081976	0.005750	-	0.085245	0.009926
$p_{0,1,9}$	-	0.139559	0.009789	-	0.131303	0.015289
$p_{0,1,10}$	-	0.237589	0.016666	-	0.202248	0.023550
$p_{0,1,11}$	-	-	0.028373	-	-	0.036274
$p_{0,1,12}$	-	-	0.048304	-	-	0.055873
$p_{0,1,13}$	-	-	0.082234	-	-	0.086063
$p_{0,1,14}$	-	-	0.139998	-	-	0.132564
$p_{0,1,15}$	-	-	0.238337	-	-	0.204190
$p_{0,2,6}$	0.176968	-	-	0.164472	-	-
$p_{0,2,7}$	0.225744	-	-	0.216789	-	-
$p_{0,2,8}$	0.0	-	-	0.0	-	-
$p_{0,2,9}$	0.0	-	-	0.0	-	-
$p_{0,2,10}$	0.0	-	-	0.0	-	-
$p_{0,2,11}$	0.0	0.184905	-	0.0	0.178015	-
$p_{0,2,12}$	0.0	0.235889	-	0.0	0.234667	-
$p_{0,2,13}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,14}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,15}$	0.0	0.0	-	0.0	0.0	-
$p_{0,2,16}$	0.0	0.0	0.185487	0.0	0.0	0.581685
$p_{0,2,17}$	0.0	0.0	0.236632	0.0	0.0	0.416645
$p_{0,2,18}$	0.0	0.0	0.0	0.0	0.0	0.0
TP	0.999	1	0.999	1	0.999	0.999

Table 5. System state probabilities ($\mu = 5$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

SSP	$\lambda = 1.1$			$\lambda = 1.4$		
	$N = 5$	$N = 10$	$N = 15$	$N = 5$	$N = 10$	$N = 15$
p_{010}	0.686825	0.491719	0.307886	0.619201	0.411066	0.237801
p_{hob}	0.184856	0.315190	0.437992	0.221152	0.359209	0.474133
p_{htb}	0.128317	0.193091	0.254121	0.159645	0.229726	0.288064

Table 6. System state probabilities ($\mu = 5$, $\mu_1 = 4$,$\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

SSP	$\lambda = 1.6$			$\lambda = 2$		
	$N = 5$	$N = 10$	$N = 15$	$N = 5$	$N = 10$	$N = 15$
p_{010}	0.577672	0.365553	0.20163320	0.502374	0.290199	0.1467869
p_{hob}	0.242547	0.382583	0.49079490	0.279247	0.418001	0.5117800
p_{htb}	0.179779	0.251864	0.30757186	0.218378	0.291800	0.3414330

Table 7. System state probabilities ($\lambda = 2$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

SSP	$\mu = 1.1$			$\mu = 1.4$		
	$N = 5$	$N = 10$	$N = 15$	$N = 5$	$N = 10$	$N = 15$
p_{010}	0.022128	0.000580	0.0000150	0.045330	0.002339	0.0001173
p_{hob}	0.556377	0.572197	0.5726110	0.543089	0.573727	0.5753096
p_{htb}	0.421495	0.427223	0.4273731	0.411580	0.423934	0.4245730

Table 8. System state probabilities ($\lambda = 2$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

SSP	$\mu = 1.6$			$\mu = 2$		
	$N = 5$	$N = 10$	$N = 15$	$N = 5$	$N = 10$	$N = 15$
p_{010}	0.065514	0.004787	0.000335	0.114836	0.014336	0.0016690
p_{hob}	0.531774	0.574419	0.577543	0.503903	0.572981	0.5816854
p_{htb}	0.402711	0.420794	0.422120	0.381261	0.412682	0.4166452

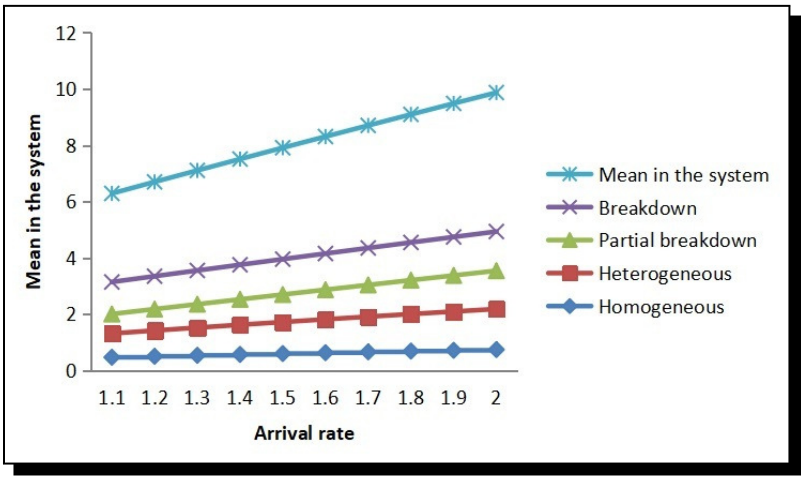

Figure 2. Mean number of customers in the system ($N = 5$, $\mu = 5$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

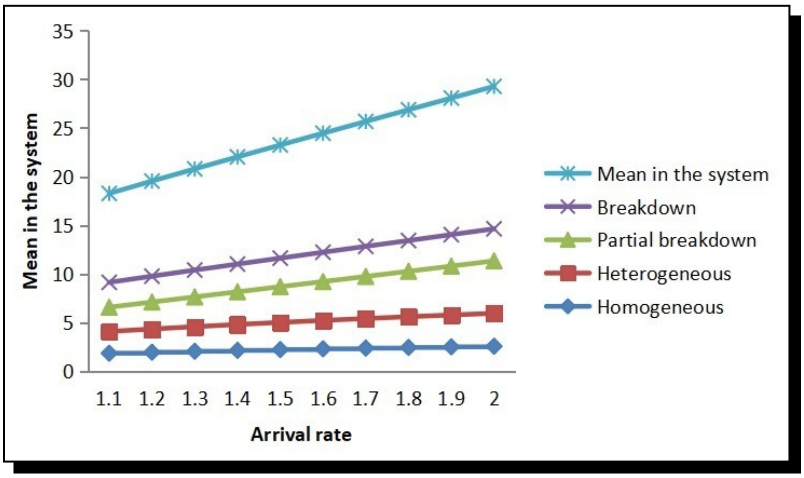

Figure 3. Mean number of customers in the system ($N = 10$, $\mu = 5$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

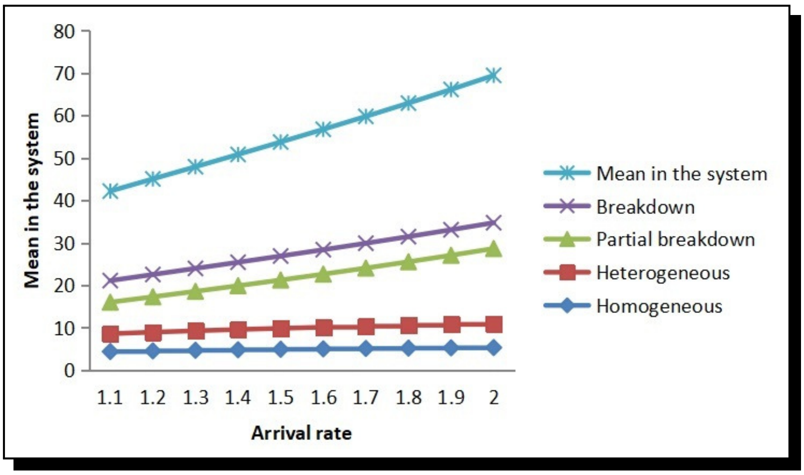

Figure 4. Mean number of customers in the system ($N = 15$, $\mu = 5$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

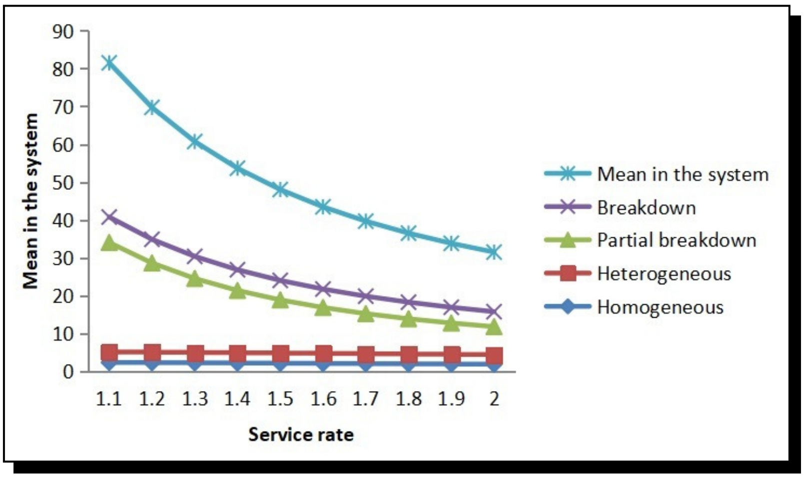

Figure 5. Mean number of customers in the system ($N = 5$, $\lambda = 2$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

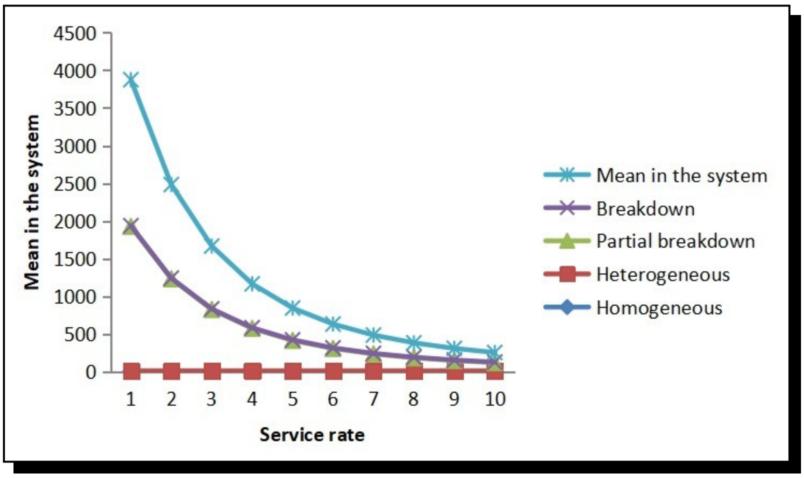

Figure 6. Mean number of customers in the system ($N = 10$, $\lambda = 2$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

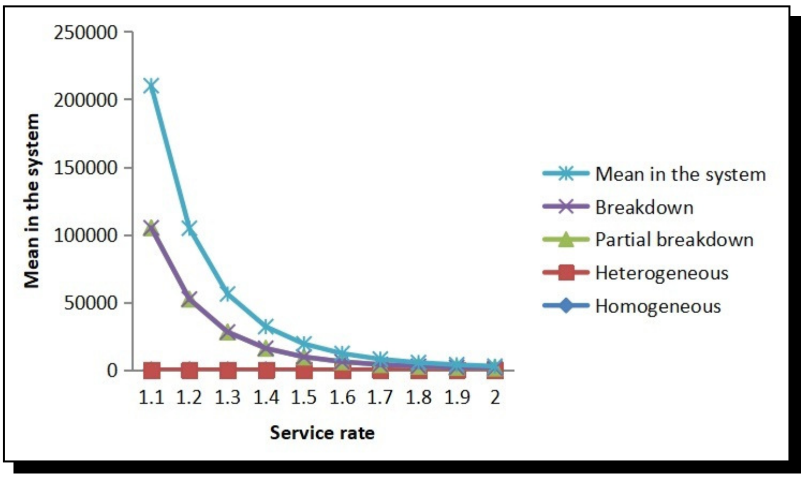

Figure 7. Mean number of customers in the system ($N = 15$, $\lambda = 2$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

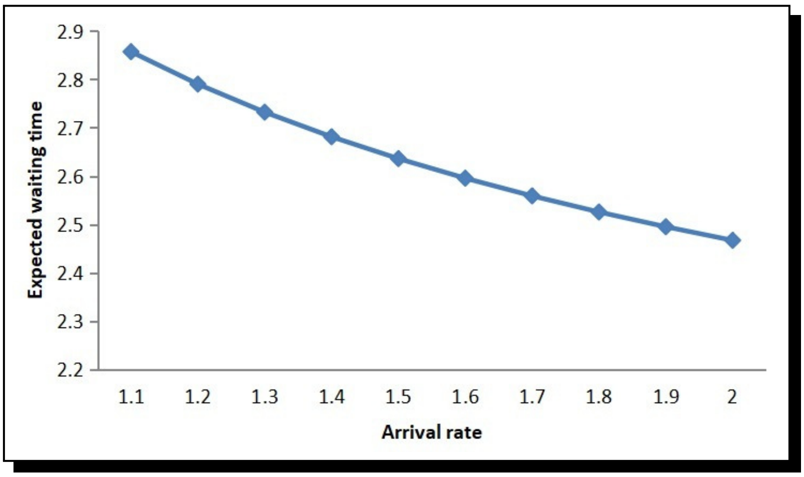

Figure 8. Expected waiting time ($N = 5$, $\mu = 5$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

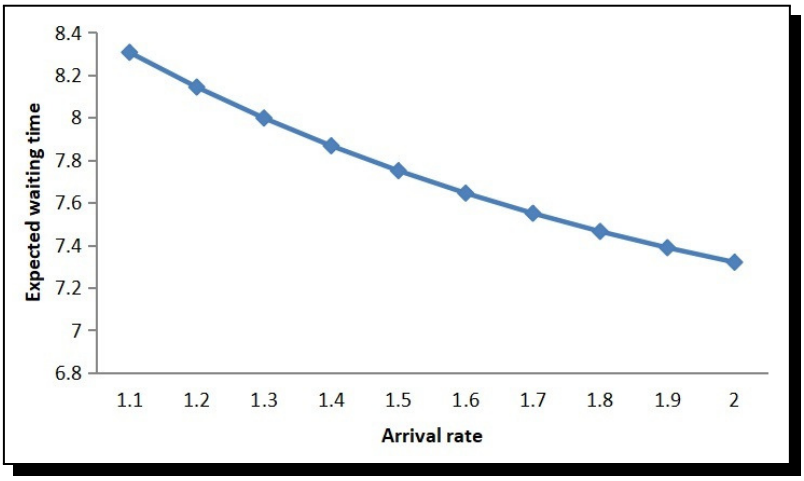

Figure 9. Expected waiting time ($N = 10$, $\mu = 5$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

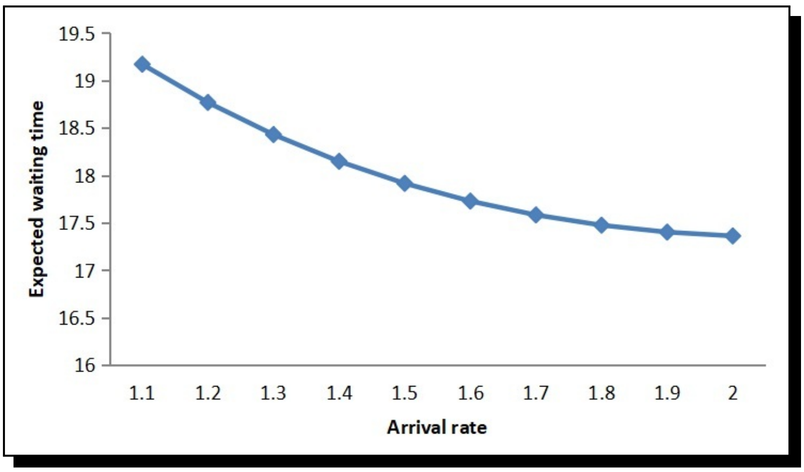

Figure 10. Expected waiting time ($N = 15$, $\mu = 5$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

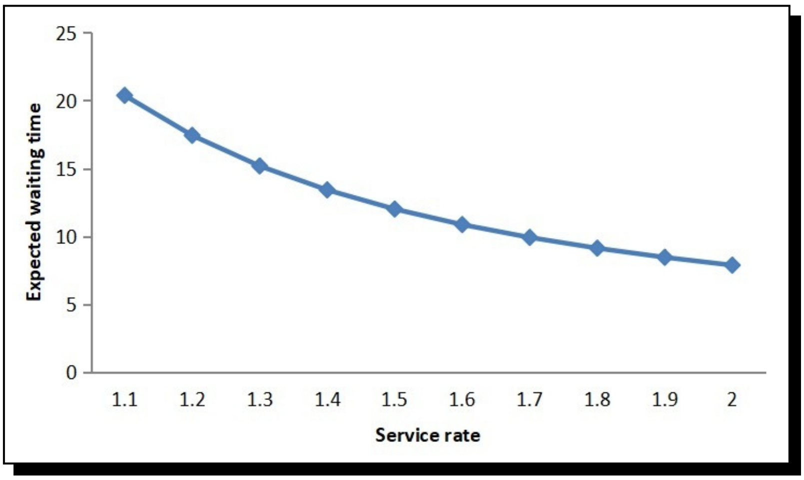

Figure 11. Expected waiting time ($N = 5$, $\lambda = 2$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

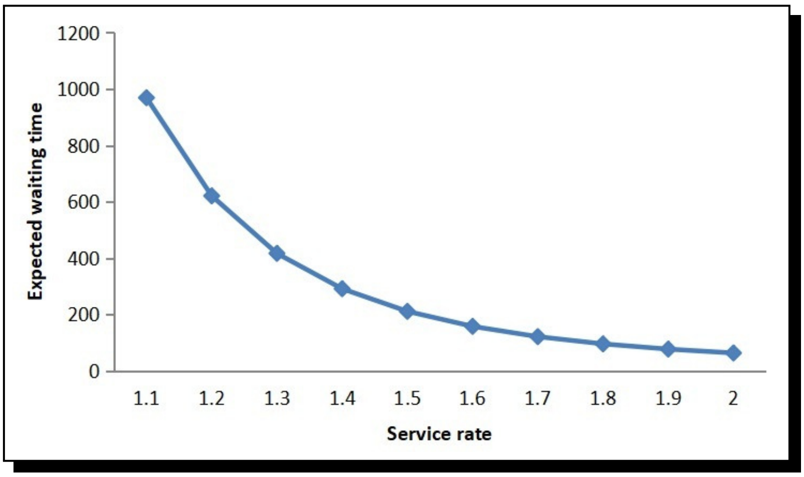

Figure 12. Expected waiting time ($N = 10$, $\lambda = 2$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

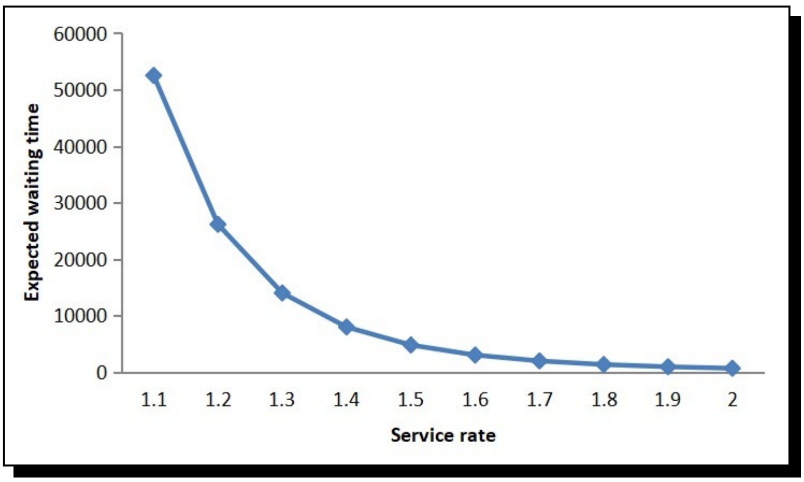

Figure 13. Expected waiting time ($N = 15$, $\lambda = 2$, $\mu_1 = 4$, $\mu_2 = 3$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 2.5$)

7. Cost Sensitivity Analysis

In this section, we define the total cost function by defining cost elements and utilizing the system measures of the model discussed in this paper.

Let TC be the total cost per unit time. The total expected cost is

$$E(TC) = c_{hob}p_{hob} + c_{htb}p_{htb} + c_hE(X) + c_s\mu + c_{s_1}\mu_1 + c_{s_2}\mu_2 + c_{s_3}\mu_3 + c_{s_4}\mu_4 + c_wE(T) + c_bp_{bd} + c_{pb}p_{pbd}, \tag{24}$$

where the cost elements are defined below:

c_{hob}	-	Cost per unit time when the homogeneous servers are busy
c_{htb}	-	Cost per unit time when the heterogeneous servers are busy
c_h	-	Unit's holding cost per unit time
c_s	-	Cost per service by homogeneous servers per unit time
c_{s_1}	-	Cost per service by Server 1 (heterogeneous) per unit time
c_{s_2}	-	Cost per service by Server 2 (heterogeneous) per unit time
c_{s_4}	-	Cost per service by Server 2 (heterogeneous) per unit time
c_w	-	Waiting cost per unit time when one customer is waiting for service
c_b	-	Cost per unit time when the server is in breakdown state
c_{pb}	-	Cost per unit time when the server is in partial breakdown state

Figures 14 and 15 shows the graph of total expected cost function with respect to arrival rate and service rate, respectively. The minimum total expected cost are obtained and are given in the figures itself.

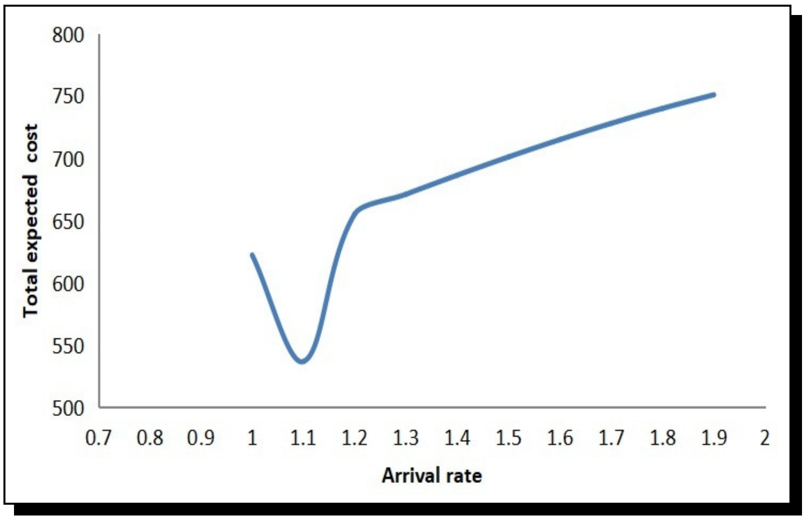

Figure 14. Total expected cost ($N = 5$, $\mu = 2.5$, $\mu_1 = 5$, $\mu_2 = 4$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 6$, $c_1 = 10$, $c_2 = 20$, $c_3 = 18$, $c_4 = 15$, $c_5 = 7$, $c_6 = 16$, $c_7 = 3$, $c_8 = 14$, $c_9 = 2$, $c_{10} = 12$, $c_{11} = 1$)
Minimum point: **1.1**, Cost: **536.769**

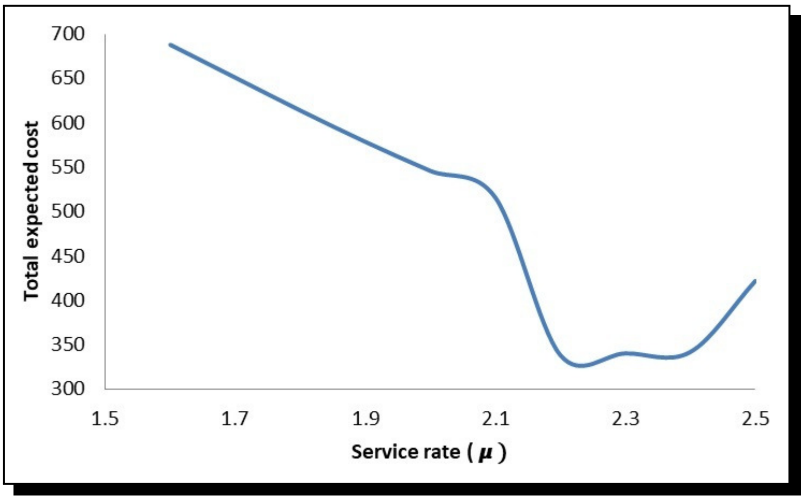

Figure 15. Total expected cost ($N = 5$, $\lambda = 2.5$, $\mu_1 = 5$, $\mu_2 = 4$, $\mu_3 = 2$, $\mu_4 = 1$, $\theta = 6$, $c_1 = 8$, $c_2 = 1$, $c_3 = 7$, $c_4 = 9$, $c_5 = 5$, $c_6 = 4$, $c_7 = 3$, $c_8 = 2$, $c_9 = 6$, $c_{10} = 10$, $c_{11} = 1$)
Minimum point: **2.2**, Cost: **338.695**

8. Conclusion

In this paper, we have analyzed a two server queue with the inter-arrival time follows negative exponential distribution and the two servers serve the customers based on exponential distributions. The customers wait in a line of infinite capacity based on the order of their arrival, if the service is not immediate. If the number of customers in the system does not exceed a threshold value N ($N \leq 1$), the services are given in homogeneous mode and the service rate for both the servers are equal. On the other hand, if the number in the system exceeds the threshold value N, the services are given in heterogeneous mode and the service rates are different. When the servers are busy, the system may breakdown, immediately the repair process starts. At the breakdown instant if the number in the system is less than or equal to N the system is completely shutdown. On the other hand, if the number exceeds N the servers serve the customers with lower service rates. The inter-breakdown period and the repair period follow exponential distributions. If an arrival finds, both the servers are idle, the customer selects any one of the server. On the other hand, if an arrival finds both the Servers are busy, the arrival waits for the first free server. This model is analyzed in time independent domain. This model can be utilized in related real life situation. We carried out the study state analysis for the model. Also, we provide some numerical illustrations. Further study can be carried out by assuming general distribution instead of exponential distribution where ever possible.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] M.O. Abou-El-Ata and A.I. Shawky, A simple approach for the slower server problem, *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics* **48** (1999), 1 – 6, DOI: 10.1501/Commua1_0000000390.

[2] F.S.Q. Alves, H.C. Yehia, L.A.C. Pedrosa, F.R.B. Cruz and L. Kerbache, Upper bounds on performance measures of heterogeneous $M/M/c$ queues, *Mathematical Problems in Engineering* **2011** (2011), Article ID 702834, 18 pages, DOI: 10.1155/2011/702834.

[3] S.I. Ammar, Transient behavior of a two-processor heterogeneous system with catastrophes, server failures and repairs, *Applied Mathematical Modelling* **38**(7-8) (2014), 2224 – 2234, DOI: 10.1016/J.APM.2013.10.033.

[4] B. Avi-Itzhak and P. Naor, Some queuing problems with the service station subject to breakdown, *Operations Research* **11**(3) (1963), 303 – 320, DOI: 10.1287/opre.11.3.303

[5] G. Choudhury and L. Tadj, An M/G/1 queue with two phases of service subject to the server breakdown and delayed repair, *Applied Mathematical Modelling* **33**(6) (2009), 2699 – 2709, DOI: 10.1016/j.apm.2008.08.006.

[6] M. Delasay, A. Ingolfsson, B. Kolfal and K. Schultz, Load effect on service times, *European Journal of Operational Research* **279**(3) (2019), 673 – 686, DOI: 10.1016/j.ejor.2018.12.028.

[7] D. Efrosinin, J. Sztrik, M. Farkhadov and N. Stepanova, Reliability analysis of a two-server heterogeneous unreliable queueing system with a threshold control policy, In: *Information Technologies and Mathematical Modelling. Queueing Theory and Applications* (ITMM 2017), A. Dudin, A. Nazarov and A. Kirpichnikov (editors), Communications in Computer and Information Science series, Vol. 800. Springer, Cham., (2017), DOI: 10.1007/978-3-319-68069-9_2.

[8] A. Federgruen and L. Green, Queueing systems with service interruptions, *Operations Research* **34**(5) (1986), 752 – 768, DOI: 10.1287/opre.34.5.752.

[9] W. Feller, *An Introduction to Probability Theory and Its Applications*, 3rd edition, Vol. 1, John Wiley & Sons, Inc., New York (1957).

[10] D. Fiems, T. Maertens and H. Bruneel, Queueing systems with different types of server interruptions, *European Journal of Operational Research* **188**(3) (2008), 838 – 845, DOI: 10.1016/j.ejor.2007.05.010.

[11] D. P. Gaver, Jr., A waiting line with interrupted service, including priorities, *Journal of the Royal Statistical Society: Series B* (*Methodological*) **24**(1) (1962), 73 – 90, DOI: 10.1111/j.2517-6161.1962.tb00438.x.

[12] D. Gross, J.F. Shortle, J.M. Thompson and C.M. Harris, *Fundamentals of Queueing Theory*, John Wiley & Sons, Inc., New York, xiii + 509 pages (2008), DOI: 10.1002/9781118625651.

[13] R. Gupta and D. Agarwal, Cost analysis of N-policy vacation machine repair problem with optional repair, *International Journal of Mathematics in Operational Research* **19**(3) (2021), 354 – 374, DOI: 10.1504/IJMOR.2021.116964.

[14] K. Kalidass and R. Kasturi, A queue with working breakdowns, *Computers & Industrial Engineering* **63**(4) (2012), 779 – 783, DOI: 10.1016/j.cie.2012.04.018.

[15] R. Kalyanaraman and S. Kalaiselvi, Heterogeneous server queue with breakdown and a threshold on slow server, *AIP Conference Proceedings* **2177** (2019), 020032, DOI: 10.1063/1.5135207.

[16] R. Kalyanaraman and R. Senthilkumar, Heterogeneous server Markovian queue with switching of service modes, *Annamalai University Science Journal* **51** (2018), 1 – 8.

[17] S. Karlin and J.L. McGregor, Many server queueing processes with Poisson input and exponential service times, *Pacific Journal of Mathematics* **8**(1) (1958), 87 – 118, DOI: 10.2140/PJM.1958.8.87.

[18] D.G. Kendall, Stochastic processes occurring in the theory of queues and their analysis by the method of the imbedded Markov chain, *The Annals of Mathematical Statistics* **24**(3) (1953), 338 – 354, DOI: 10.1214/aoms/1177728975.

[19] J. Kiefer and J. Wolfowitz, On the theory of queues with many servers, *Transactions of the American Mathematical Society* **78**(1) (1955), 1 – 18, DOI: 10.2307/1992945.

[20] B. Krishnamoorthi, On Poisson queue with two heterogeneous servers, *Operations Research* **11**(3) (1963), 321 – 330, DOI: 10.1287/opre.11.3.321.

[21] P.V. Laxmi and T.W. Kassahun, Transient analysis of multi-server Markovian queueing system with synchronous multiple working vacations and impatience of customers, *International Journal of Mathematics in Operational Research* **16**(2) (2020), 217 – 237, DOI: 10.1504/IJMOR.2020.105880.

[22] W. Li, D. Shi and X. Chao, Reliability analysis of M/G/1 queueing systems with server breakdowns and vacations, *Journal of Applied Probability* **34**(2) (1997), 546 – 555, DOI: 10.2307/3215393.

[23] W. Lin and P. Kumar, Optimal control of a queueing system with two heterogeneous servers, *IEEE Transactions on Automatic Control* **29**(8) (1984), 696 – 703, DOI: 10.1109/TAC.1984.1103637.

[24] I.L. Miltrany and B. Avi-Itzhak, A many-server queue with service interruptions, *Operations Research* **16**(3) (1968), 628 – 638, DOI: 10.1287/opre.16.3.628.

[25] O. Nakdimon and U. Yechiali, Polling systems with breakdowns and repairs, *European Journal of Operational Research* **149**(3) (2003), 588 – 613, DOI: 10.1016/S0377-2217(02)00451-4.

[26] M.F. Neuts and D.M. Lucantoni, A Markovian queue with N servers subject to breakdowns and repairs, *Management Science* **25**(9) (1979), 849 – 861, DOI: 10.1287/mnsc.25.9.849.

[27] S. Pradhan, On the distribution of an infinite-buffer queueing system with versatile bulk-service rule under batch-size-dependent service policy: $M/G_n^{(a,Y)}/1$, *International Journal of Mathematics in Operational Research* **16**(3) (2020), 407 – 434, DOI: 10.1504/IJMOR.2020.106908.

[28] V.P. Singh, Queue-dependent servers, *Journal of Engineering Mathematics* **7** (1973), 123 – 126, DOI: 10.1007/BF01535357.

[29] R. Sudhesh and A. Azhagappan, Analysis of an M/M/c queue with heterogeneous servers, balking and reneging, *International Journal of Operational Research* **36**(3) (2019), 293 – 309, DOI: 10.1504/IJOR.2019.103120.

[30] R. Sudhesh, P. Savitha and S. Dharmaraja, Transient analysis of a two-heterogeneous servers queue with system disaster, server repair and customers' impatience, *TOP* **25** (2017), 179 – 205, DOI: 10.1007/S11750-016-0428-X.

[31] K. Thiruvengadam, Queuing with breakdowns, *Operations Research* **11**(1) (1963), 62 – 71, DOI: 10.1287/opre.11.1.62.

[32] N.M. van Dijk, Simple bounds for queueing systems with breakdowns, *Performance Evaluation* **8**(2) (1988), 117 – 128, DOI: 10.1016/0166-5316(88)90017-X.

[33] B. Vinod, Unreliable queueing systems, *Computers & Operations Research* **12**(3) (1985), 323 – 340, DOI: 10.1016/0305-0548(85)90031-0.

[34] B. Vinod, Exponential queues with server vacations, *Journal of the Operational Research Society* **37**(10) (1986), 1007 – 1014, DOI: 10.1057/jors.1986.170.

[35] J. Wang, B. Liu and J. Li, Transient analysis of an M/G/1 retrial queue subject to disasters and server failures, *European Journal of Operational Research* **189**(3) (2008), 1118 – 1132, DOI: 10.1016/j.ejor.2007.04.054.

[36] K.-H. Wang, T.-Y. Wang and W.L. Pearn, Optimal control of the N policy M/G/1 queueing system with server breakdowns and general startup times, *Applied Mathematical Modelling* **31**(10) (2007), 2199 – 2212, DOI: 10.1016/j.apm.2006.08.016.

[37] P. Wartenhorst, N parallel queueing systems with server breakdown and repair, *European Journal of Operational Research* **82**(2) (1995), 302 – 322, DOI: 10.1016/0377-2217(94)00266-F.

[38] H. White and L. Christie, Queueing with preemptive priorities or with breakdown, *Operations Research* **6**(1) (1958), 79 – 95, DOI: 10.1287/opre.6.1.79.