Communications in Mathematics and Applications
Vol. 15, No. 1, pp. 73–79, 2024
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v15i1.2402

http://www.rgnpublications.com

Research Article

An Optimal Solution to Multi-Goal Fuzzy Linear Programming Problems Using Elementary Transformations

Arti Shrivastava*[1], Bharti Saxena[1], Vandana Rathore[2] and Ramakant Bharadwaj[3]

[1] *Department of Mathematics, Rabindranath Tagore University, Bhopal, Madhya Pradesh, India*
[2] *School of Engineering and technology Jagran Lakecity University, Bhopal, Madhya Pradesh, India*
[3] *Department of Mathematics, Amity University, Kolkata, West Bengal, India*
***Corresponding author:** arti431980@gmail.com

Received: September 6, 2023 **Accepted:** December 20, 2023

Abstract. This article explores how to solve multipurpose problems to obtain optimal solutions using fuzzy linear programming. Our goal is to minimize production and transportation costs by using the most basic transportation methods and comparing the results to conventional methods. We discuss the results of numerical examples and illustrate this method.

Keywords. Linear Programming Problem (LPP), Multi-goal, Elementary Transformation Method, Triangular Number

Mathematics Subject Classification (2020). 90-XX

1. Introduction

In engineering and management, linear programming can be useful in a variety of fields. Due to the complexity of the real-world problems, fuzzy numbers are commonly used in these applications to represent the parameters of LP. Researchers have been paying much attention to *Fuzzy Linear Programming* (FLP) because of these reasons.

Since fuzzy set theory offers effective solutions to decision-making problems with imprecise data, it has been applied in many research fields in recent years (Tan and Long [17], Tsai and Chen [18], and Zhang *et al*. [20]). According to Delgado *et al*. [3], the parameters of constraints are fuzzy numbers, but the parameters of the objective function are crisp. There is also a general FLP model in Rommelfanger [16] whose main difference from [3] is that the parameters of the objective function are fuzzy. Considering the different hypotheses. Based on fuzzy numbers (van Hop [7]), feasibility degrees (Jiménez *et al*. [9]), satisfaction degrees of constraints (Liu [12]) and confidence intervals (Chiang [2]), some methods are based on superiority and inferiority concepts. Other types of procedures are multi-objective optimization method (van Hop [7]), penalty method (Jamison and Lodwick [8]), and semi-infinite programming method (León and Vercher [11]). Mahdavi-Amiri and Nasseri [14] introduced a new dual algorithm to obtained solution of the FLP problem directly. Ganesan and Veeramani [4] presented a procedure for solving *Fuzzy Linear Programming Problem* (FLPP) without change into crisp LPP. Maleki *et al*. [15] introduced an auxiliary problem in addition to their better method for solving FLP problems in their model.

The literature has recently published several types of a *Fully Fuzzy Linear Programming* (FFLP) problem, where all variables and parameters are represented as fuzzy numbers (Allahviranloo *et al*. [1], Guo and Shang [5], Hashemi *et al*. [6], Kumar *et al*. [10], and Lotfi *et al*. [13]). Different methods for solving FFLP problems with crisp inequality constraints have been proposed by Allahviranloo *et al*. [1], and Hashemi *et al*. [6]. By converting FFLP problems to *Crisp Linear Programming* (CLP) problems, these methods provide fuzzy optimal solutions for FFLP problems. In contrast to Lotfi *et al*. [13], and Kumar *et al*. [10] found fuzzy optimal solutions that satisfy constraints exactly for FFLP problems. Guo and Shang [5] propose a computational model for positive fully fuzzy linear matrix equations and obtain fuzzy approximation solutions using pseudo-inverse equations. However, in most of the previous literature, all limitations of FFLP problems have a definite form.

2. Preliminaries

In this section, some basic definitions and arithmetic operations are reviewed.

Basic Definitions

1. **Fuzzy Set:** A fuzzy set is characterized by a membership function mapping element of a domain, space or universe of discourse X to the unit interval $[0,1]$, i.e., $A = \{(x, \mu_A(x) : x \in X\}$, here $\mu_A : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from $[0,1]$.
2. **Triangular Fuzzy Numbers:** A number $\widetilde{A}$ is a triangular fuzzy number denoted by $\widetilde{A} = (a_1, a_2, a_3)$, where a_1, a_2, a_3 are real numbers and its membership function $\mu_{\widetilde{A}}(x)$ is given

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & \text{for } a < x_1, \\ \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \le x \le a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \le x \le a_3, \\ 0, & \text{for } x > a_3. \end{cases}$$

By using min and max, we have an alternative expression for the proceeding equation:

$$\text{triangle}(x;a,b,c) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right)\right).$$

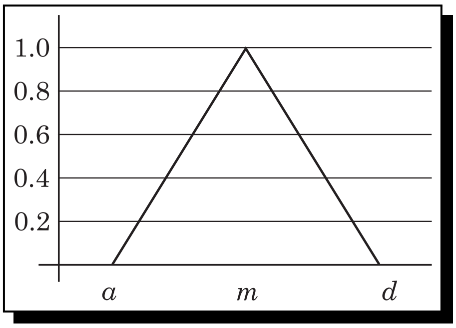

Figure 1. Graphical representation of triangular fuzzy number

Ranking for Triangular Fuzzy Number

If a number $\widetilde{A}$ is a triangular fuzzy number denoted by $A = (a_1, a_2, a_3)$, where a_1, a_2, a_3 are real numbers and its membership-function $R(A)$ is given by

$$R(A) = \frac{a_1 + 2a_2 + a_3}{3}.$$

Multi-Goal Fuzzy Linear Programming Problem

$$\left.\begin{aligned} &\text{Min or Max}\, Z = \sum_{j=1}^{n} c_j^r x_j \\ &\text{subject to constraint } \sum_{j=1}^{n} \widetilde{a}_{ij} x_j \ (\le, =, \ge)\ \widetilde{b}_j, \quad i = 1,2,\dots,m \\ &\qquad\qquad\qquad\qquad x_j \ge 0, \quad j = 1,2,\dots,n, \end{aligned}\right\} \tag{2.1}$$

where

c_j^r = objective values,

x_j = contribution per units,

$\widetilde{a}_{ij}$ = input-output coefficient,

$\widetilde{b}_i$ = total availability of the ith resource.

Elementary Transformation Method

Step 1: Construct of LPP:

$$\text{Min or Max}\, Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{subject to constraints } \sum_{j=1}^{n} \widetilde{a}_{ij} x_j \ (\le, =, \ge)\ b_j, \quad i = 1,2,\dots,m$$

$$x_j \ge 0, \quad j = 1,\ 2,\dots,n$$

Step 2: Subject to constraints considered as system of linear equations:

$$a_{11}x_1 + b_{12}x_2 + c_{13}x_3 = d_1,$$
$$a_{21}x_1 + b_{22}x_2 + c_{23}x_3 = d_2,$$
$$a_{31}x_1 + b_{32}x_2 + c_{33}x_3 = d_3.$$

Step 3: System to equation can be change into the matrix form:

Coefficient matrix $A = \begin{bmatrix} a_{11} & b_{12} & c_{12} \\ a_{21} & b_{22} & c_{22} \\ a_{31} & b_{32} & c_{33} \end{bmatrix}$,

Constant matrix $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$,

Variable matrix $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

Step 4: Construct augmented matrix with the help of coefficient and constant matrix, i.e., $[A : B]$.

Step 5: After solving augmented matrix we have get following two conditions:

Case I: Set of linear equation is called inconsistent if Rank of $A \neq$ Rank of $[A : B]$, than, we have not get basic variables.

Case II: Set of linear equation is called consistent if Rank of $A =$ Rank of $[A : B]$, than, we have get basic variables if rank of augmented matrix is equal to number of unknown variables.

Step 6: To find the optimum solution, the system of equation can be written as $AX = B$.

Numerical Problem

$$\text{Max } Z_1 = (6,9,12)x_1 + (5,8,9)x_2 + (4,7,9)x_3$$
$$\text{Min } Z_2 = (5,7,11)x_1 + (3,7,10)x_2 + (9,11,14)x_3$$

subject to constraints
$$(1,2,4)x_1 + (2,5,6)x_2 + (4,7,9)x_3 \le (24,32,38)$$
$$(5,9,13)x_1 + (2,3,4)x_2 + (3,5,11)x_3 \le (24,34,40)$$
$$(4,8,10)x_1 + (7,11,16)x_2 + (2,4,5)x_3 \le (32,42,49)$$

Solution by Proposed Method

Step 1: After applying ranking formula in above FLPP, we get following LPP:

$$\text{Max } Z_1 = 12x_1 + 107x_2 + 9x_3$$
$$\text{Min } Z_2 = 10x_1 + 9x_2 + 15x_3$$

subject to constraints
$$3x_1 + 6x_2 + 9x_3 \le 42$$
$$12x_1 + 4x_2 + 8x_3 \le 44$$
$$10x_1 + 15x_2 + 5x_3 \le 55$$

Step 2: Subject to constraints considered as system of linear equations:

$$3x_1 + 6x_2 + 9x_3 = 42,$$
$$12x_1 + 4x_2 + 8x_3 = 44,$$
$$10x_1 + 15x_2 + 5x_3 = 55$$

Step 3: System to equation can be change into the matrix form:

$$\text{Coefficient matrix } A = \begin{bmatrix} 3 & 6 & 9 \\ 12 & 4 & 8 \\ 10 & 15 & 5 \end{bmatrix},$$

$$\text{Constant matrix } B = \begin{bmatrix} 42 \\ 44 \\ 55 \end{bmatrix},$$

$$\text{Variable matrix } X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Step 4: Construct augmented matrix with the help of coefficient and constant matrix, i.e.,

$$[A:B] = \begin{bmatrix} 3 & 6 & 9 & : & 42 \\ 12 & 4 & 8 & : & 44 \\ 10 & 15 & 5 & : & 55 \end{bmatrix}.$$

Step 5: After solving augmented matrix, we have

$$[A:B] = \begin{bmatrix} 1 & 2 & 3 & : & 14 \\ 0 & 1 & 5 & : & 17 \\ 0 & 0 & 1 & : & 3 \end{bmatrix}.$$

From above augmented matrix, we found that linear equations are consistent, i.e.,

$$\text{Rank of } A = \text{Rank of } [A:B].$$

Step 6: To find the optimum solution, the system of equation can be written as

$$AX = B \quad \Rightarrow \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 17 \\ 3 \end{bmatrix}.$$

After solving above equation, we get following values $x_1 = 1$, $x_2 = 2$ and $x_3 = 3$, and optimal solution $\text{Max}\, Z_1 = 253$ and $\text{Min}\, Z_2 = 73$, i.e., after applying elementary transformation method and also applying simplex method and Big-M method in our problem we get optimum result for both objective functions as follows:

Table 1. Optimum result for first objective functions

S. No.	Name of method	Variables	Optimum solution
1	Elementary transformation method	$x_1 = 1$, $x_2 = 2$ and $x_3 = 3$	253
2	Simplex method	$x_3 = 3.67$	392.33
3	Big-M method	$x_1 = 1$, $x_2 = 2$ and $x_3 = 3$	253

Table 2. Optimum result for second objective functions

S. No.	Name of method	Variables	Optimum solution
1	Elementary transformation method	$x_1 = 1$, $x_2 = 2$ and $x_3 = 3$	72
2	Simplex method	$x_1 = 0.71$ and $x_3 = 4.43$	73.57
3	Big-M method	$x_1 = 1$, $x_2 = 2$ and $x_3 = 3$	72

3. Conclusion

Multi objective fuzzy linear programming solved by elementary transformation method and obtained result has been compared by simplex method and Big-M methods. In this investigation, we found that proposed method gives the optimum result in comparison of simplex method and provide same result with Big-M method. We also found that proposed method proposed method easy to understand and applicable as comparative to exist methods of LPP. We also observe that Multi Objective FLPP is most effective modal to take decision to handle two or more than two issues at same time for industries along with customers.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

References

[1] T. Allahviranloo, F. H. Lotfi, M. K. Kiasary, N. A. Kiani and L. Alizadeh, Solving fully fuzzy linear programming problem by the ranking function, *Applied Mathematical Sciences* **2**(1-4) (2008), 19 – 32.

[2] J. Chiang, Fuzzy linear programming based on statistical confidence interval and interval-valued fuzzy set, *European Journal of Operational Research* **129**(1) (2001), 65 – 86, DOI: 10.1016/S0377-2217(99)00404-X.

[3] M. Delgado, J. L. Verdegay and M. A. Vila, A general model for fuzzy linear programming, *Fuzzy Sets and Systems* **29**(1) (1989), 21 – 29, DOI: 10.1016/0165-0114(89)90133-4.

[4] K. Ganesan and P. Veeramani, Fuzzy linear programs with trapezoidal fuzzy numbers, *Annals of Operations Research* **143**(1) (2006), 305 – 315, DOI: 10.1007/s10479-006-7390-1.

[5] X. Guo and D. Shang, Fuzzy approximate solution of positive fully fuzzy linear matrix equations, *Journal of Applied Mathematics* **2013** (2013), Article ID 178209, 7 pages, DOI: 10.1155/2013/178209.

[6] S. M. Hashemi, M. Modarres, E. Nasrabadi and M. M. Nasrabadi, Fully fuzzified linear programming, solution and duality, *Journal of Intelligent & Fuzzy Systems* **17**(3) (2006), 253 – 261.

[7] N. van Hop, Solving fuzzy (stochastic) linear programming problems using superiority and inferiority measures, *Information Sciences* **177**(9) (2007), 1977 – 1991, DOI: 10.1016/j.ins.2006.12.001.

[8] K. D. Jamison and W. A. Lodwick, Fuzzy linear programming using a penalty method, *Fuzzy Sets and Systems* **119**(1) (2001), 97 – 110, DOI: 10.1016/S0165-0114(99)00082-2.

[9] M. Jiménez, M. Arenas, A. Bilbao and M. V. Rodríguez, Linear programming with fuzzy parameters: An interactive method resolution, *European Journal of Operational Research* **177**(3) (2007), 1599 – 1609, DOI: 10.1016/j.ejor.2005.10.002.

[10] A. Kumar, J. Kaur and P. Singh, A new method for solving fully fuzzy linear programming problems, *Applied Mathematical Modelling* **35**(2) (2011), 817 – 823, DOI: 10.1016/j.apm.2010.07.037.

[11] T. León and E. Vercher, Solving a class of fuzzy linear programs by using semi-infinite programming techniques, *Fuzzy Sets and Systems* **146**(2) (2004), 235 – 252, DOI: 10.1016/j.fss.2003.09.010.

[12] X. Liu, Measuring the satisfaction of constraints in fuzzy linear programming, *Fuzzy Sets and Systems* **122**(2) (2001), 263 – 275, DOI: 10.1016/S0165-0114(00)00114-7.

[13] F. H. Lotfi, T. Allahviranloo, M. A. Jondabeh and L. Alizadeh, Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution, *Applied Mathematical Modelling* **33**(7) (2009), 3151 – 3156, DOI: 10.1016/j.apm.2008.10.020.

[14] N. Mahdavi-Amiri and S. H. Nasseri, Duality results and a dual simplex method for linear programming problems with trapezoidal fuzzy variables, *Fuzzy Sets and Systems* **158**(17) (2007), 1961 – 1978, DOI: 10.1016/j.fss.2007.05.005.

[15] H. R. Maleki, M. Tata and M. Mashinchi, Linear programming with fuzzy variables, *Fuzzy Sets and Systems* **109**(1) (2000), 21 – 33, DOI: 10.1016/S0165-0114(98)00066-9.

[16] H. Rommelfanger, Fuzzy linear programming and applications, *European Journal of Operational Research* **92**(3) (1996), 512 – 527, DOI: 10.1016/0377-2217(95)00008-9.

[17] Y. Tan and Y. Long, Option-game approach to analyze technology innovation investment under fuzzy environment, *Journal of Applied Mathematics* **2012** (2012), Article ID 830850, 9 pages, DOI: 10.1155/2012/830850.

[18] H.-R. Tsai and T. Chen, A fuzzy nonlinear programming approach for optimizing the performance of a four-objective fluctuation smoothing rule in a wafer fabrication factory, *Journal of Applied Mathematics* **2013** (2013), Article ID 720607, 15 pages, DOI: 10.1155/2013/720607.

[19] H.-C. Wu, Optimality conditions for linear programming problems with fuzzy coefficients, *Computers & Mathematics with Applications* **55**(12) (2008), 2807 – 2822, DOI: 10.1016/j.camwa.2007.09.004.

[20] L. Zhang, X. Xu and L. Tao, Some similarity measures for triangular fuzzy number and their applications in multiple criteria group decision-making, *Journal of Applied Mathematics* **2013** (2013), Article ID 538261, 7 pages, DOI: 10.1155/2013/538261.