Communications in Mathematics and Applications
Vol. 14, No. 3, pp. 1181–1188, 2023
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
http://www.rgnpublications.com
DOI: 10.26713/cma.v14i3.2382

Special Issue
Recent Trends in Mathematics and Applications
Proceedings of the International Conference of Gwalior Academy of Mathematical Sciences 2022
***Editors*: Vinod P. Saxena and Leena Sharma**

Research Article

A Simple Mathematical Model for Covid-19: Case Study of Goa

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Received: February 1, 2023 **Accepted:** May 5, 2023

Abstract. This paper analyses a simple SEIR model (*Susceptible-Exposed-Infected-Removed*) to study the impact of the Covid-19 pandemic on Goa. A system of non-linear ODE has been used. The system is analysed using stability analysis. Novel Corona Virus (Covid-19) was declared a pandemic on 11th March 2020. Thereon India has witnessed three Covid-19 waves from March 2020 to March 2022. To control the subsequent impact on our livelihoods, it is of utmost need to study the spread of the virus. The findings of the model will then be compared to the recorded data of the 1st and 2nd waves of the neighbouring states of Maharashtra and Karnataka. Further, it will be shown how the probability of effective exposure and the per-capita contact rate influences the healthy and infected population. The proposed model can be used to make predictions for subsequent waves if any, in Goa.

Keywords. COVID-19, First wave, Second wave, SEIR-Compartmental epidemic model

Mathematics Subject Classification (2020). 92D30

1. Introduction

The novel corona virus (SARS-CoV-2) originated in China in the beginning of December 2019 and rapidly spread throughout the world in the consequent months (Abou-Ismail [1]).

It is a highly contagious disease which resulted from zoonotic transmission from bats. India's first case of Covid 19, a virus originated in China was reported in Kerela, India on 27 January 2020 (Andrews *et al*. [2]). Since then the country has seen two waves from January 2020 and February 2021 respectively (Sarkar *et al*. [4]). According to WHO, India has reported 4,33,31,645 cases and 5,24,903 deaths as of 23rd June 2022[1]. Goa is a small state in the west coast of India, having Karnataka and Maharashtra as its neighboring states. Goa is a tourist destination and has an average of 53,80,000 tourists annually[2]. Hence it is of great importance to study the effects of this disease as it is a touristic state and has a lot of movement of masses. Many researchers have studied the corona virus, so as to curb its effect on the economic and human loss. During a pandemic, factors like rapid rate of spread and measures to curb it is essential to be determined (Abou-Ismail [1]).

This paper aims to demonstrate the predictions of corona virus using an SEIR (*Susceptible-Exposed-Infected-Removed*) compartmental epidemic model in the state of Goa for two periods. The first wave which is studied in Period 1 (4th July 2020 to 20th January 2021) and the second wave in Period 2 (19th March 2021 to 9th October 2021). The validity of the results is tested by comparing the outcomes with government recorded data of Karnataka and Maharashtra. The results can be further used to implement control strategies and act against the spread of the disease for future waves by the respective Government body.

2. Assumptions

In the following model we have assumed that there are no new births and deaths caused apart from that of Covid-19. Further we have assumed that individuals who have been infected once are immune to the virus.

3. Mathematical Model

The SEIR model is a dynamic, deterministic model composed of a system of non linear ordinary differential equations. To examine the spread of the disease there are many factors involved. The parameters used are as follows: $S(t)$ refers to the susceptible population at time t. $E(t)$ refers to the exposed population at time t, which is the group of infected individuals which are asymptomatic and undiagnosed, but capable of infecting. $I(t)$ refers to the symptomatic, diagnosed infected population which spreads the disease and lastly, $R(t)$ refers to the removed, which represents the dead and recovered population. The total population at any given time t is assumed to be constant.

We consider the following system of equations based on the work of Mellian *et al*. [3]. Therefore,

$$N = S(t) + E(t) + I(t) + R(t),$$

$$S'(t) = -\beta\frac{S(t)I(t)}{N} - \beta_1\frac{S(t)E(t)}{N}, \tag{3.1a}$$

[1]WHO, *India Situation*, World Health Organisation (2022), URL: https://covid19.who.int/region/searo/country/in, accessed 15 June 2022.

[2]S. Keelery, Tourist arrivals to Goa India 2012-2021, *Statista*, 6 May 2022, URL: https://www.statista.com/statistics/1027205/india-tourist-arrivals-in-goa-by-type/#:$\sim$:text=ln%202020%2C%20the%20domestic%20tourist,accounted%20for%20over%20300%20thousand, accessed 17 June 2022.

[3]S. Melliani, A. El Allaoui and L. S. Chadli, A simple mathematical model for Coronavirus (COVID-19), *medRxiv* (2020), DOI: 10.1101/2020.04.23.20076919.

$$E'(t) = \beta \frac{S(t)I(t)}{N} + \beta_1 \frac{S(t)E(t)}{N} - \gamma E(t), \tag{3.1b}$$

$$I'(t) = \gamma E(t) - \sigma I(t), \tag{3.1c}$$

$$R'(t) = \sigma I(t), \tag{3.1d}$$

where

β : transmission rate from susceptible to infected;
β_1 : transmission rate from susceptible to exposed;
γ : per capita Infectious rate;
σ : per capita death rate.

Further,

$$\beta = cb \quad \text{and} \quad \beta_1 = c_1 b_1,$$

where

c : effective contact rate of an infected person per day;
b : transmission risk from susceptible to infected;
c_1 : effective contact rate of an exposed person per day;
b_1 : transmission risk from Susceptible to exposed.

The boundary conditions for the model is given in Table 1.[4] [5]

Table 1

	Period 1	Period 2
Population N	15,21,875	15,28,375
$S(0) = 0.9 \times N$	13,69,688	13,75,540
$I(0)$	800	899
Deaths	6	2
$R(0)$	59	62
$E(0)$	1,51,328	1,51,877
b	0.08351	0.08234
b_1	0.08351	0.08234
$\beta = cb,\ c = 5$	0.41755	0.4117
$\beta_1 = c_1 b_1,\ c_1 = 10$	0.8351	0.8234
σ	0.0075	0.0022
$\gamma = \frac{I(0)}{E(0)}$	0.0052	0.0059

4. Stability

To find the stable solution, we have to check the stability at the equilibrium points. The equilibrium points is $(0,0,0,0)$.

[4] *Goa Population 2011-2022*, Population Census Data (2022), URL: https://www.census2011.co.in/census/state/goa.html, accessed 15 June 2022.

[5] *Covid 19*, Government of Goa, India (2022), URL: https://www.goa.gov.in/covid-19/, accessed 16 June 2022.

To evaluate the other equilibrium point, we solve the following equations:

$$S'(t) = E'(t) = I'(t) = R'(t) = 0.$$

Hence from Period 1,

$$\begin{aligned}
0 &= -\beta\frac{S(t)I(t)}{N} - \beta_1\frac{S(t)E(t)}{N},\\
0 &= \beta\frac{S(t)I(t)}{N} + \beta_1\frac{S(t)E(t)}{N} - \gamma E(t),\\
0 &= \gamma E(t) - \sigma I(t),\\
0 &= \sigma I(t)\\
\Rightarrow \quad & I(t) = E(t) = 0.
\end{aligned}$$

The Jacobian is given by

$$J = \begin{bmatrix} -\beta\frac{I(t)}{N} - \beta_1\frac{E(t)}{N} & -\beta_1\frac{S(t)}{N} & -\beta\frac{S(t)}{N} & 0\\ \beta\frac{I(t)}{N} + \beta_1\frac{E(t)}{N} & \beta_1\frac{S(t)}{N} - \gamma & \beta\frac{S(t)}{N} & 0\\ 0 & \gamma & -\sigma & 0\\ 0 & 0 & \sigma & 0 \end{bmatrix}.$$

To calculate the Jacobian at the equilibrium point, we take $I(t) = E(t) = 0$.

$$J = \begin{bmatrix} 0 & -\beta_1\frac{S(t)}{N} & -\beta\frac{S(t)}{N} & 0\\ 0 & \beta_1\frac{S(t)}{N} - \gamma & \beta\frac{S(t)}{N} & 0\\ 0 & \gamma & -\sigma & 0\\ 0 & 0 & \sigma & 0 \end{bmatrix}.$$

The eigen values are given by $\lambda_1 = \lambda_2 = 0$

$$\lambda_3 = \frac{\beta_1 S(t) - N(\gamma+\sigma)}{2N} - \frac{\sqrt{N^2(\gamma-\sigma)^2 + 2NS(t)(2\gamma\beta - \beta_1\gamma - \beta_1\sigma) + S^2(t)\beta_1^2}}{2N},$$

$$\lambda_4 = \frac{\beta_1 S(t) + N(\gamma+\sigma)}{2N} - \frac{\sqrt{N^2(\gamma-\sigma)^2 + 2NS(t)(2\gamma\beta - \beta_1\gamma - \beta_1\sigma) + S^2(t)\beta_1^2}}{2N}.$$

For Period 1, we have

$$\lambda_3 = \frac{0.8351S(t) - 19327.8125}{3043750} - \frac{\sqrt{122521.5976 - 19063.7671S(t) - 0.6974S^2(t)}}{3043750},$$

$$\lambda_4 = \frac{0.8351S(t) - 19327.8125}{3043750} + \frac{\sqrt{122521.5976 - 19063.7671S(t) - 0.6974S^2(t)}}{3043750}.$$

From further calculations we can verify, that $\lambda_4 > 0$ and $\lambda_3 \le 0$ iff $S(0) \ge 18993.1145$.
Thus, a sufficient condition for λ_3 to be negative is $S(0) \ge 18993.1145$.
Similarly for Period 2, we have

$$\lambda_3 = \frac{0.8234S(t) - 19257.525}{3056750} - \frac{\sqrt{1494995.29 - 16863.4172S(t) - 0.6779S^2(t)}}{3056750},$$

$$\lambda_4 = \frac{0.8234S(t) - 19257.525}{3056750} + \frac{\sqrt{1494995.29 - 16863.4172S(t) - 0.6779S^2(t)}}{3056750}.$$

From further calculations we can verify, that $\lambda_4 > 0$ and $\lambda_3 \le 0$ iff $S(0) \ge 24872.7532$.

5. Graphical Solution

To trace the solution of the proposed model the table below gives the following estimations of the coefficients. Here $t = 0$ for Period 1 and Period 2 refers to starting date of the respective timelines. The model will be predicted for a duration of 200 days, i.e., in the interval $[0,200]$. The transmission risk b for Period 1 is 0.08351 and for Period 2, b_1 is 0.08234.

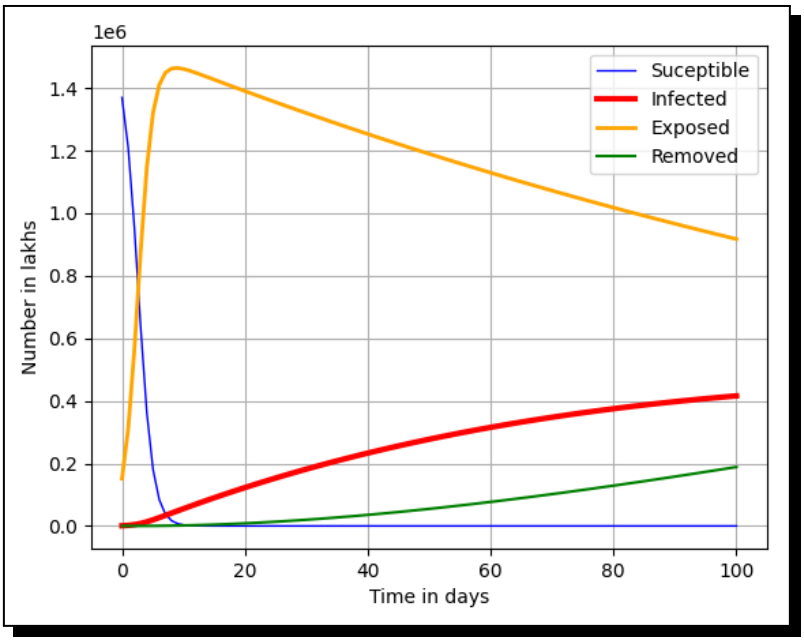

Figure 1. Model prediction for Period 1

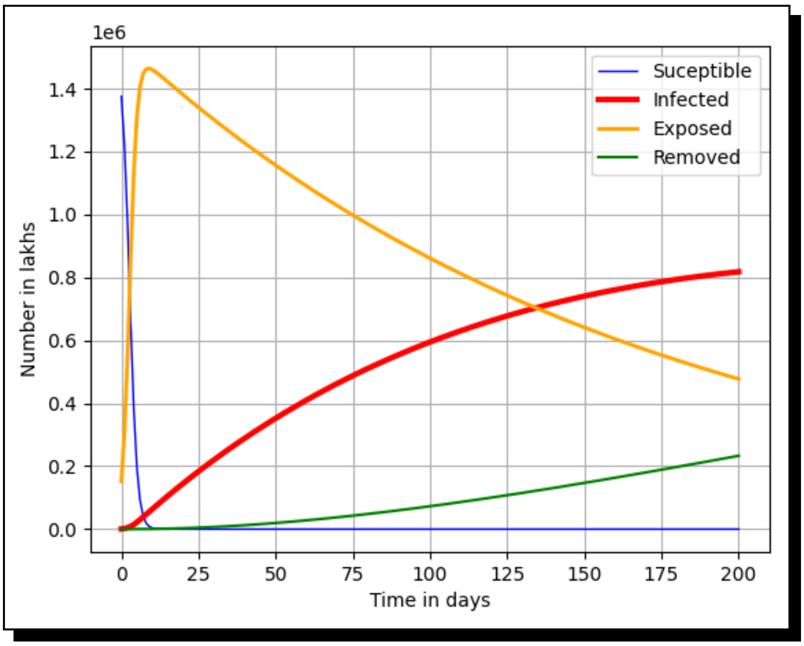

Figure 2. Model prediction for Period 2

6. Results and Discussion

The proposed model estimates the infected cases for Goa for Period 1 and Period 2. Here $t = 0$ for Period 1 and Period 2 refers to start date of the respective timelines. The model predicts for a duration of 200 days, i.e., in the interval $[0, 200]$. In the above graphs, the model fitting is done in Python program and the data graphs for Maharashtra[6] and Karnataka[7] are obtained from their respective official government websites.

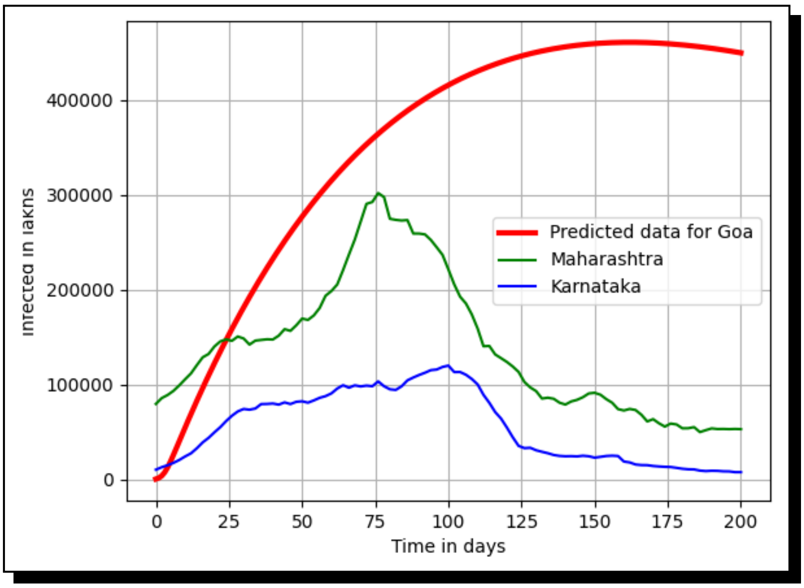

Figure 3. Comparison for Period 1

Figure 3 gives us the comparison for Period 1. In this period, we see that the number of infected cases in Goa surpasses Karnataka on day 4 with a predicted reading of 19,000 active cases. It later exceeds cases in Maharashtra on day 24 with prediction of 1,49,000 cases. Further, the infected cases dip at day 75 and day 100 for Maharashtra and Karnataka, respectively. The wave of Goa, dips much later on day 160 with a maximum reading of 4,62,000 cases.

Similarly in Period 2, we observe from Figure 4 that the number of active infected cases in Goa exceeds cases in Maharashtra on day 60 with prediction of 4,20,000 cases. It later surpasses Karnataka on day 65 with a predicted reading of 4,47,000 active cases. In addition, the active cases for Maharashtra and Karnataka dip at day 37 and day 57, respectively. In this period, the predicted wave of Goa does not achieve its peak before 9th October 2021 (day 200).

In both of the periods, we can deduce that the estimated cases of Goa are much higher the recorded data of Karnataka and Goa and waves attain its peak much later in the period of 200 days studied.

[6] *Covid 19*, URL: https://prsindia.org/covid-19/cases, accessed 16 June 2022.
[7] *Media Bulletin*, URL: https://covid19.karnataka.gov.in/govt_bulletin/en, accessed 16 June 2022.

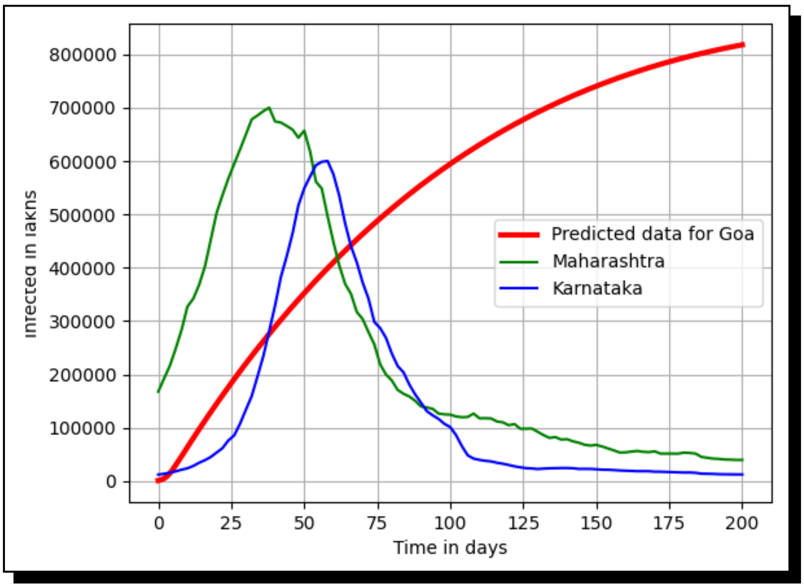

Figure 4. Comparison for Period 2

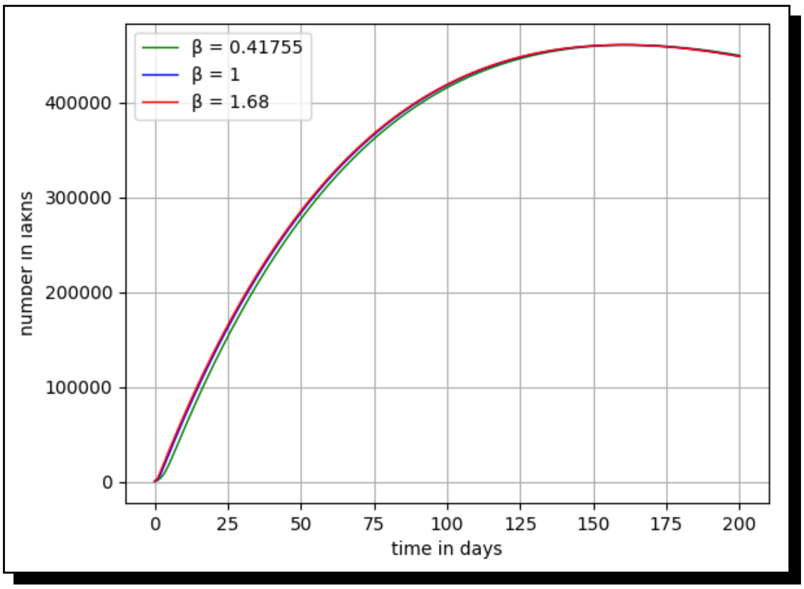

Figure 5. Model prediction for different values of β for Period 1

In Figure 5, we can see that, the coefficient of transmission β is directly proportional to the number of infected. Hence to reduce the number of infected we have to minimize the value of β. Since, the parameter is defined by the product of the probability of transmission b and the contact rate per day, it is imperative to reduce these two parameters which rely on contact. Hence it is necessary for the government to impose regulations that work towards decreasing contacts to curb the infection spread.

7. Conclusion

The merit of our model is that we considered some essential elements like transmission rates and governmental actions. The model predicts the number of infected cases for a period of

200 days taken during Covid 19's 1st and 2nd waves. The results obtained by the solution of the model are compared to the data of the actual spread in the case of Maharashtra and Karnataka. This model can be considered as a baseline model for further improvement.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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