Communications in Mathematics and Applications
Vol. 14, No. 5, pp. 1669–1678, 2023
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v14i5.2294

http://www.rgnpublications.com

Research Article

An Efficient Twelfth-Order Iterative Method to Solve Nonlinear Equations with Applications

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Received: May 23, 2023 **Accepted:** September 29, 2023

Abstract. This study proposes an effective, derivative-free, four-step iterative method for zero-finding nonlinear equations. There are five functional evaluations needed for this strategy. The suggested approach possesses twelfth-order convergence, according to the convergence study. We use six real-world application issues from physics, chemical engineering, and medical research to demonstrate the applicability of the suggested approach. The numerical outcomes show that the new method outperforms the earlier schemes in the literature regarding performance, adaptability, and efficiency.

Keywords. Nonlinear equations, Efficiency index, Iterative method, Functional evaluations, Order of convergence

Mathematics Subject Classification (2020). 41A25, 65H04, 65H05, 65H20, 65K05

1. Introduction

The development of root-finding techniques has always been prioritized in numerical analysis that gives accurate and efficient approximations of a nonlinear equation. One of the most straightforward and most widely used iterative approaches for these kinds of problems is *Newton's Method* (NM) (Traub [10]), which was given by

$$x_{n+1} = x_n - \frac{h(x_n)}{h'(x_n)}, \quad n = 0, 1, 2, \dots. \tag{1.1}$$

This paper implements an iteration method using the divided difference techniques for solving non-linear equations. This combination provided a twelfth-order strategy that only needed five functional evaluations for each iteration. Analytical derivation of the error equation of the resulting iteration formula demonstrates that our suggested method has twelfth-order convergences. Therefore, our proposed method has an efficiency index is $12^{\frac{1}{5}} = 1.6437$.

We use several real-world application problems to demonstrate the new method's effectiveness. The proposed approach is contrasted with other popular iterative approaches for the same problem. The results show that the new scheme is quicker than others already in use and requires fewer iterations to get to the root.

This article is formatted as follows: In Section 2, it is detailed how the four-step iterative technique was created. Section 3 displays the convergence analysis of the revised scheme. The numerical examples in Section 4 include models from science and engineering. The conclusions are provided in Section 5.

Existing twelfth-order iterative techniques for solving nonlinear equations include the following:

In 2023, Janngam and Comemuang [3] [PJCCM] proposed the iterative approach below:

$$\left.\begin{aligned} y_n &= x_n - \frac{h(x_n)}{h'(x_n)}, \\ z_n &= x_n - \frac{(h(x_n)-h(y_n))h(x_n)}{(h(x_n)-2h(y_n))h'(x_n)}, \\ x_{n+1} &= z_n - \frac{h(z_n)}{h'(z_n)} - \frac{h(z_n)h''(z_n)}{2(h'(z_n))^3}, \end{aligned}\right\} \tag{1.2}$$

where $h''(z_n) = \frac{2}{z_n - y_n}\left(2h'(z_n) + h'(y_n) - 3\frac{h(z_n)-h(y_n)}{z_n - y_n}\right)$.

In 2021, Bawazir *et al*. [1] [HMBM] developed a twelfth-order method as follows:

$$\left.\begin{aligned} y_n &= x_n - \frac{h(x_n)}{h'(x_n)}, \\ z_n &= x_n - \frac{h(y_n)}{h'(y_n)}\left[1 + \frac{h(y_n)(h'(x_n)-h'(y_n))}{2h(x_n)h'(y_n)}\right], \\ x_{n+1} &= z_n - \frac{h(z_n)}{h'(z_n)}\left[1 + \frac{h(z_n)(h'(y_n)-h'(z_n))}{2h(y_n)h'(z_n)}\right]. \end{aligned}\right\} \tag{1.3}$$

In 2013, Liu and Wang [5] [XLXWM] presented the following four-step method:

$$\left.\begin{aligned} y_n &= x_n - \frac{h(x_n)}{h'(x_n)}, \\ z_n &= y_n - \frac{2h(x_n)}{(h'(x_n)+h'(y_n))}, \\ \omega_n &= z_n - \frac{h(z_n)}{h'(z_n)}, \\ x_{n+1} &= \omega_n - \frac{h(z_n)+2h(\omega_n)}{h(z_n)}\frac{h(\omega_n)}{h'(z_n)}. \end{aligned}\right\} \tag{1.4}$$

In 2021, Solaiman and Hasim [9] [OSIHM] suggested an iterative scheme as:

$$\left.\begin{aligned} y_n &= x_n - \frac{h(x_n)}{h'(x_n)}, \\ z_n &= y_n - \frac{h(y_n)}{h'(y_n)} - \frac{2h(y_n)^2 h'(y_n) R[x_n, y_n]}{4(h'(y_n))^4 - 4h(y_n)(h'(y_n))^2 R[x_n, y_n] + (h(y_n))^2 (R[x_n, y_n])^2}, \\ x_{n+1} &= z_n - \frac{h(z_n)}{h'(z_n)}, \end{aligned}\right\} \tag{1.5}$$

where $R[x_n, y_n] = \left(3\frac{h(y_n)-h(x_n)}{y_n - x_n} - 2h'(y_n) - h'(x_n)\right)\frac{2}{x_n - y_n}$ and $h'(y_n) = 2h[y_n, x_n] - h'(x_n)$.

2. Twelfth Order Convergent Method

Consider that the nonlinear equation $h(x) = 0$, which is continuous and has clearly defined first-order derivatives, has an exact root x^*. Let x_n represent the nth approximation's root. Thus

$$x^* = x_n + \varepsilon_n, \tag{2.1}$$

where ε_n is the error. Hence, we obtain

$$h(x^*) = 0. \tag{2.2}$$

From Taylor's series, we have

$$\left.\begin{aligned} h(x^*) &= h(x_n) + (x^* - x_n)h'(x_n) + \frac{(x^* - x_n)^2}{2!}h''(t_n) + \dots, \\ h(x^*) &= h(x_n) + \varepsilon_n h'(x_n) + \frac{\varepsilon_n^2}{2!}h''(x_n) + \dots. \end{aligned}\right\} \tag{2.3}$$

Neglecting ε_n^3 onwards, and using (2.2) and (2.3), we have

$$\left.\begin{aligned} &\varepsilon_n^2 h''(x_n) + 2\varepsilon_n h'(x_n) + 2h(x_n) = 0, \\ &\varepsilon_n = [-2h'(x_n) \pm \sqrt{4h'(x_n) - 8h(x_n)h''(x_n)}] \div 2h''(x_n). \end{aligned}\right\} \tag{2.4}$$

On replacing x^* by x_{n+1} in (2.1) and from (2.4), we get

$$z_n = y_n - H(\tau)\left[\frac{2h(y_n)}{h'(y_n)} \cdot \frac{1}{1 + \sqrt{1 - 2\rho_n}}\right], \tag{2.5}$$

where $\rho_n = \frac{h'(x_n) - h'(y_n)}{h'(x_n)}$, $h'(y_n) = 2h(y_n, x_n) - h'(x_n)$, $H(\tau) = 1 - \tau$ and $\tau = \frac{h(y_n)}{h(x_n)}$ is the weight function.

We build the method by using (1.1) as the first step and (2.5) as the second step, both of which have optimal fourth-order convergence. Then, we extend the above scheme using Newton's variation as the third and fourth steps. In *Step* 3, we used Steffenson's method and in *Step* 4, we used finite difference methods to eliminate first-order derivatives.

Algorithm 1. The iterative scheme is computed as x_{n+1}

Step 1: $y_n = x_n - \frac{h(x_n)}{h'(x_n)}$,

Step 2: $z_n = y_n - H(\tau)\left[\frac{2h(y_n)}{h'(y_n)} \cdot \frac{1}{1+\sqrt{1-2\rho_n}}\right]$, where $\rho_n = \dfrac{h'(x_n)-h'(y_n)}{h'(x_n)}$, $h'(y_n)=2h[y_n,x_n]-h'(x_n)$, $H(\tau)=1-\tau$ and $\tau=\frac{h(y_n)}{h(x_n)}$,

Step 3: $w_n = z_n - \dfrac{h(z_n)}{h'(z_n)}$, where $h'(z_n)=\dfrac{(h(z_n+h(z_n)))-(h(z_n-h(z_n)))}{2h(z_n)}$,

Step 4: $x_{n+1}=w_n-\dfrac{h(w_n)}{h'(w_n)}$; where

$$\begin{aligned} h'(w_n) &= h'(y_n)+(h[y_n,z_n,w_n]+h[y_n,y_n,z_n])(w_n-y_n) \\ &\quad +2(h[y_n,z_n,w_n]-h[y_n,y_n,z_n])(w_n-z_n). \end{aligned} \tag{2.6}$$

This method has four functional evaluations and one of its derivatives is denoted by NKM.

3. Convergence Criteria

Theorem 3.1 ([6]). *Suppose that be a single zero of a sufficiently differentiable function h for an open interval D. If x_0 is in the neighborhood of x^*. Then, the proposed algorithm* (2.6) *has twelfth-order convergence.*

Proof. Let the single zero of $h(x)=0$ be x^* and $x^*=x_n+\varepsilon_n$. Thus,

$$h(x^*)=0.$$

Through Taylor's series, writing $h(x^*)$ about x_n, we get

$$h(x_n)=h'(x^*)(\varepsilon_n+c_2\varepsilon_n^2+c_3\varepsilon_n^3+c_4\varepsilon_n^4+\dots), \tag{3.1}$$

$$h'(x_n)=h'(x^*)(1+2c_2\varepsilon_n+3c_3\varepsilon_n^2+4c_4\varepsilon_n^3+\dots). \tag{3.2}$$

Now, we get $y_n=x^*+Y$

$$Y=c_2\varepsilon_n^2+(2c_3-2c_2^2)\varepsilon_n^3+(3c_4-7c_2c_3+4c_2^3)\varepsilon_n^4+\dots. \tag{3.3}$$

We have,

$$h(y_n)=h'(x^*)(c_2\varepsilon_n^2+(2c_3-2c_2^2)\varepsilon_n^3+(3c_4-7c_2c_3+5c_2^3)\varepsilon_n^4+\dots),$$

$$h'(y_n)=h'(x^*)(1+(2c_2^2-c_3)\varepsilon_n^2+(6c_2c_3-4c_2^3-2c_4)\varepsilon_n^3+\dots) \tag{3.4}$$

and

$$\frac{h(y_n)}{h'(y_n)}=c_2\varepsilon_n^2+(2c_3-2c_2^2)\varepsilon_n^3+(3c_2^3-6c_2c_3+3c_4)\varepsilon_n^4+\dots. \tag{3.5}$$

From (3.2) and (3.4), we get

$$\rho_n=2c_2\varepsilon_n+(4c_3-6c_2^2)\varepsilon_n^2+(6c_4+16c_2^3-20c_2c_3)\varepsilon_n^3+\dots. \tag{3.6}$$

From (3.6), on simplification

$$\begin{aligned} \left(1+\sqrt{1-2\rho_n}\right)^{-1} &= \frac{1}{2}(1+c_2\varepsilon_n+(2c_3-c_2^2)\varepsilon_n^2+(3c_4-2c_2c_3)\varepsilon_n^3 \\ &\quad +(37c_2^4+8c_3^2+12c_2c_4-52c_2^2c_3)\varepsilon_n^4). \end{aligned} \tag{3.7}$$

Using (3.5) and (3.7), we get

$$\frac{2h(y_n)}{h'(y_n)}\left(\frac{1}{1+\sqrt{1-2\rho_n}}\right)=c_2\varepsilon_n^2+(2c_3-c_2^2)\varepsilon_n^3+(3c_4-2c_2c_3)\varepsilon_n^4+\dots \tag{3.8}$$

and

$$H(\tau)=1-\frac{h(y_n)}{h(x_n)}=(1-c_2\varepsilon_n-(2c_3-3c_2^2)\varepsilon_n^2-(3c_4-10c_2c_3+8c_2^3)\varepsilon_2^3+\dots). \tag{3.9}$$

Substituting (3.3), (3.8) and (3.9) are in the second step of (2.6), we get

$$z_n=x^*+Z, \tag{3.10}$$

where

$$Z=(-c_2c_3)\varepsilon_n^4+(c_2c_4-c_3^2+c_2^4)\varepsilon_n^5+(c_2c_5+6c_2^2c_4+4c_2c_3^2+5c_2^3c_3-c_2^5-c_3c_4-13c_2c_3c_4)\varepsilon_n^6+\dots.$$

Expanding $h(z_n)$ about x^* by using Taylor expansion. We obtain

$$h(z_n)=h'(x^*)(Z+c_2Z^2+c_3Z^3+\dots), \tag{3.11}$$

$$h'(z_n)=h'(x^*)(1+2c_2Z+3c_3Z^2+\dots). \tag{3.12}$$

Substituting (3.10), (3.11), and (3.12) are in the third step of (2.6), we get

$$w_n=x^*+W, \tag{3.13}$$

where $W=(-c_2^2c_3^2)\varepsilon_n^8+o(\varepsilon_n^9)$.

Expanding $h(w_n)$ about x^* by applying Taylor's series, we obtain

$$h(w_n)=h'(x^*)(W+c_2W^2+c_3W^3+\dots). \tag{3.14}$$

Consider

$$h[w_n,z_n]=\frac{h(w_n)-h(z_n)}{w_n-z_n}$$
$$=1+c_2(W+Z)+c_3(W^2+WZ+Z^2)+c_4(W^3+W^2Z+WZ^2+Z^3)+\dots, \tag{3.15}$$

$$h[z_n,y_n]=\frac{h(z_n)-h(y_n)}{z_n-y_n}$$
$$=1+c_2(Y+Z)+c_3(Y^2+YZ+Z^2)+c_4(Y^3+Y^2Z+YZ^2+Z^3)+\dots,. \tag{3.16}$$

From (3.15) and (3.16), we obtain

$$h[w_n,z_n,y_n]=\frac{h[w_n,z_n]-h[z_n,y_n]}{w_n-y_n}$$
$$=c_2+c_3W+c_3Z+c_3Y+c_4Z^2+c_4Y^2+c_4ZY+\dots \tag{3.17}$$

and

$$h[y_n,y_n,z_n]=\frac{h[z_n,y_n]-h'(y_n)}{z_n-y_n}$$
$$=c_2+2c_3Y+c_3Z+3c_4Y^2+2c_4ZY+c_4Z^2+5c_5Y^3+5c_5ZY^2\dots. \tag{3.18}$$

From (3.17) and (3.18), we have

$$h[w_n,z_n,y_n]+h[y_n,y_n,z_n]$$
$$=\{2c_2+2c_3Y+c_3W+2c_3Z+c_3Y+3c_4Y^2+2c_4Z^2+2c_4ZY+c_4Y^2+\dots\} \tag{3.19}$$

and

$$h[w_n,z_n,y_n]-h[y_n,y_n,z_n]=\{c_3W-c_3Y-2c_4Y^2-c_4ZY+\dots\}. \tag{3.20}$$

Using (3.19), we get

$$\begin{aligned}&(h[w_n,z_n,y_n]+h[y_n,y_n,z_n])(w_n-y_n)\\&\quad=\{2c_2W+2c_3WY+c_3W^2+2c_3WZ+c_3YW+3c_4Y^2W+2c_4Z^2W+2c_4ZYW+c_4Y^2W\\&\quad-2c_2Y-2c_3Y^2-c_3WY-2c_3ZY-c_3Y^2+3c_4Y^3-2c_4Z^2Y-2c_4ZY^2-c_4Y^3+\ldots\}.\end{aligned} \tag{3.21}$$

From (3.20), we obtain

$$\begin{aligned}&(h[w_n,z_n,y_n]-h[y_n,y_n,z_n])(w_n-y_n)\\&\quad=\{c_3W^2-c_3YW-2c_4Y^2W-c_4ZYW-c_3WY+c_3Y^2+2c_4Y^3+c_4ZY^2+\ldots\}\\&2(h[w_n,z_n,y_n]-h[y_n,y_n,z_n])(w_n-y_n)\\&\quad=\{2c_3W^2-2c_3YW-4c_4Y^2W-2c_4ZYW-2c_3WY+2c_3Y^2+4c_4Y^3+2c_4ZY^2+\ldots\}.\end{aligned} \tag{3.22}$$

From (3.21) and (3.22), we get

$$h'(w_n)=1+5c_5Y_n^2+2c_2Z+c_2Y^2W-6c_5Y_n^4+2c_5Y_n^3W+6c_6Y_n^5+\ldots. \tag{3.23}$$

Putting (3.13), (3.14) and (3.23) are in the fourth step of (2.6), we get

$$\varepsilon_{n+1}=(5c_2^5c_3^2c_5)\varepsilon_n^{12}+o(\varepsilon_n^{13}).$$

The error equation reveals that the technique NKM's order of convergence is twelve and that its efficiency index is $E.I.=12^{\frac{1}{5}}=1.6437$. □

4. Numerical Examples

To verify the effectiveness of the proposed iterative scheme (NKM), we compared the results with some existing methods by taking various regularly encountered application issues from physics, chemistry, and biomedical engineering. We might not be able to get the precise solution using analytical methods because of the nonlinearity. We must, therefore, employ iterative techniques. For testing and comparison reasons, the tolerance is set to $\varepsilon=10^{-199}$ as a stopping condition with 690 decimal places of precision. The mpmath library performs all the computations in PYTHON using an Intel(R) Core (TM) i5-10210U processor running at 2.11 GHz and a 64-bit operating system. Table 1 illustrates the efficiency index analogy. Compare our findings to those of other methods, such as PJCCM, HMBM, XLXWM, and OSIHM, to demonstrate the efficacy of the suggested approach (NKM). Table 2 presents the findings.

Table 1. Comparison of efficiency index

Methods	Order of convergence	Functional evaluations	$E.I.$
PJCCM	12	5	1.6437
HMBM	12	6	1.5130
XLXWM	12	6	1.5130
OSIHM	12	5	1.6437
NKM	12	5	1.6437

Table 2. Comparison of different methods

Method	x_0	n	$\lvert x_{n+1}-x_n\rvert$	$\lvert h(x_{n+1})\rvert$	x_0	n	$\lvert x_{n+1}-x_n\rvert$	$\lvert h(x_{n+1})\rvert$
$h_1(x)$	-0.6				0.1			
PJCCM		4	3.24e-690	2.05e-691		3	1.59e-305	5.29e-305
HMBM		4	0	2.05e-691		3	8.52e-301	2.82e-300
XLXWM		4	1.88e-691	1.37e-691		3	1.22e-298	4.05e-298
OSIHM		4	0	1.37e-691		3	2.21e-323	7.32e-323
NKM		4	0	2.05e-691		3	1.11e-381	3.66e-381
$h_2(x)$	1				10			
PJCCM		11	1.30e-212	1.54e-209		12	1.24e-213	1.46e-210
HMBM		11	5.67e-211	6.72e-208		12	8.32e-212	9.86e-209
XLXWM		11	1.69e-210	2.00e-207		12	3.44e-210	4.07e-207
OSIHM		11	2.48e-224	2.92e-221		12	9.17e-227	1.09e-223
NKM		6	3.99e-408	4.73e-405		6	3.36e-229	3.98e-226
$h_3(x)$	-1				-8			
PJCCM		8	7.56e-688	2.19e-690		8	9.56e-202	1.84e-204
HMBM			Divergent			9	1.13e-687	2.19e-690
XLXWM		8	1.29e-560	2.51e-572			Divergent	
OSIHM		7	7.01e-394	1.35e-396		8	1.35e-687	2.19e-690
NKM		6	3.99e-408	4.73e-405		6	4.06e-241	4.81e-238
$h_4(x)$	2.2				1.8			
PJCCM		9	4.90e-207	6.17e-205		9	7.47e-207	9.41e-205
HMBM		9	2.27e-206	2.86e-204		9	1.14e-206	1.43e-204
XLXWM		9	1.50e-207	1.89e-205		9	2.87e-207	3.62e-206
OSIHM		9	1.61e-209	2.03e-207		9	2.92e-209	3.68e-207
NKM		4	1.66e-217	2.09e-215		4	1.29e-205	1.63e-203
$h_5(x)$	3.8				2			
PJCCM		4	1.66e-518	6.81e-518		3	4.16e-434	1.70e-433
HMBM		4	1.30e-485	5.33e-485		3	1.37e-429	5.63e-429
XLXWM		4	3.33e-416	1.36e-415		3	2.77e-421	1.13e-420
OSIHM		4	1.64e-690	1.59e-689		3	8.98e-449	3.67e-448
NKM		4	6.56e-690	1.59e-689		3	7.51e-465	3.07e-464
$h_6(x)$	5.5				3.8			
PJCCM		4	2.00e-344	1.07e-344		4	1.09e-445	5.82e-446
HMBM		6	1.55e-265	8.26e-266		5	9.89e-380	5.27e-380
XLXWM			Divergent			4	2.13e-464	1.13e-464
OSIHM		7	6.76e-200	3.59e-200		6	1.37e-691	3.42e-691
NKM		4	4.51e-420	2.40e-420		4	7.92e-476	4.21e-476

where x_0 represents the starting approximation, n number of iterations, $\lvert x_{n+1}-x_n\rvert$ represents error and $\lvert h(x_{n+1})\rvert$ represents number of functional evaluations.

Applications

Application 1 (*Blood Rheology Model*, [7])**.** The study of blood's structure and flow characteristics is known as blood rheology. Caisson fluids, which are non-Newtonian fluids, include blood. The model predicts that the flow will move through a tube as a plug with a slight velocity gradient close to the wall. To investigate the plug flow of Caisson fluid flow, we look at the following nonlinear equation:

$$h_1(x) = \frac{1}{441}x^8 - \frac{8}{63}x^5 - 0.0571428571x^4 + \frac{16}{9}x^2 - 3.624489796x + 0.3.$$

The root of the nonlinear equation $h_1(x) = 0$ is 0.0864335580522467.

Application 2 (*Law of Blood Flow*, [7])**.** This law was proposed in 1840 by French physician Jean Louis-Marie Poiseuille. Where v is the blood viscosity, R is the radius, l is the length, P is the pressure, and h is a function of x with the domain $[0,R]$, blood flows via the vein or artery. This law is stated as the nonlinear model shown below by

$$h_2(x) = \frac{P}{vl}(R^2 - x^2),$$

where $P = 4000$, $R = 0.008$, $\eta = 0.027$ and $l = 2$ are taken for the simulations.

Application 3 (*Fluid Permeability in Biogels*, [7])**.** The following nonlinear model can be used to define the relationship between the hydraulic permeability and the pressure gradient to fluid velocity in the extracellular fibre matrix:

$$F_r x^3 - 20H_p(1-x)^2 = 0.$$

F_r is the fibre's radius, H_p is the specific hydraulic permeability, and $x \in [0,1]$ is the medium's porosity. Taking $F_r = 100 \times 10^{-9}$ and $H_p = 0.4655$in the above equation, we obtain the following nonlinear equation:

$$h_3(x) = -100 \times 10^{-9}x^3 + 9.3100 \times x^2 - 18.6200 \times x + 9.3100.$$

The root of the equation is 0.999896376644484. . . , and the results are shown in Table 2.

Application 4 (*Volume from van der Waals Equation*, [4])**.**) Van der Waals' equation of a non-ideal gas is represented by

$$\left(p + \frac{K_1 n^2}{V^2}\right)(V - nK_2) = nRT.$$

Therefore, the above polynomial function becomes

$$h_4(x) = 40x^3 - 95.26535116x^2 + 35.28x - 5.6998368.$$

It has three roots in which one is real, i.e., $x = 1.9707842194070294$.

Application 5 (*Depth of Embedment*, [4, 8])**.** The following nonlinear equation determines the depth of embedment of a sheet-pile wall:

$$h_5(x) = \frac{x^3 + 2.87x^2 - 10.28}{4.62} - x.$$

The approximated root is 2.0021187789538272.

Application 6 (*The Vertical Stress*, [2, 4])**.** Vertical stress σ_x is one of the primary stresses experienced by finite underground structures and is given by

$$\sigma_x = \frac{p}{\pi}x + \cos x \sin x,$$

where the vertical stress σ_x is one fourth of the footing stress p and it can be rewritten above to be equal to 25% of p:

$$h_6(x) = \frac{x + \cos x \sin x}{\pi} - \frac{1}{4}.$$

The root of the nonlinear equation $h_6(x) = 0$ is 0.4160444988100767043.

5. Conclusion

We presented a twelfth-order iterative method using the divided difference approximation to solve nonlinear equations. The five function evaluations used in this method have a twelve-step convergence order. NKM surpasses comparison approaches in six applications, including three medical science problems, two physics problems, and one chemical engineering issue. Theoretical and computational order of convergence are proven in the problems under consideration. Table 2 shows that the suggested method performs well in terms of accuracy, speed, iterations, and computational order of convergence.

Acknowledgments

The authors would like to be thankful to GITAM (Deemed to be University) for their support and providing the resources.

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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