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Research Article

Impact of Multiple Mild Stenoses on the Movement of Casson Fluid in a Non-Uniform Tube

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Abstract. The impact of multiple stenoses on the stream of a Casson fluid over a non-uniform pipe has been studied. Under the assumption of mild-stenosis, explicit solutions have been derived to calculate the flow resistance and shear-stress on the wall. The consequences of numerous relevant factors on these flow qualities are deliberated. It is experiential that the flow resistance declines with a change of radius in the plug region, where as the shear stress increases with the same. Further, both the above two flow characteristics upsurge with the elevations of the stenoses. The impacts of boundaries identified with statures of stenoses on shear-stress may support in improved recognition of the fluid mechanical viewpoints in stenotic stream area, which thusly would support in further comprehension of the advancement and improvement of 'Arteriosclerosis' in biological frameworks.

Keywords. Stenosis, Casson fluid, Non-Newtonian fluid, Mathematica, Resistance to flow, Shear stress

Mathematics Subject Classification (2020). 76Z05, 92C10, 76A05

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1. Introduction

Stenosis is the word usually used when narrowing is brought about by lesion that decreases the space of lumen. This decrease occurs due to abnormal and unnatural growth of the plaques along the walls of the lumen. As stenosis is been a widespread disease in many countries, it is necessary to study the flow field in a constricted stenotic tube in detail. This study may help us in better thoughtful of the illnesses occur in arteries.

In view of this, to realize the influence of stenosis on flow in the narrowed section of an artery, a good deal theoretical analysis has been done under different conditions. Young [18] considered axially symmetric mild stenosis and analyzed the result of stenotic growth on shear stress and impedance. Forrester and Young [5] considered the steady stream of an incompressible fluid in a contracting and expanding axisymmetric tube theoretically and determined the velocity profiles, pressure and shearing stresses. Chaturani and Ponnalagarsamy [2] inspected the impacts of inlet velocity profiles on the 2D flow of blood over a stenosed tube. However, in these studies a Newtonian fluid was considered to compensate the blood by ignoring its suspension nature. There are experimental evidences which exhibit the non-Newtonian comportment of blood at modest shear rates in pipes of tiny widths (Huckaba and Hahn [6]). Therefore, numerous authors have premeditated the flow of blood in stenotic region by considering blood as a non-Newtonian fluid. For instance, Shukla *et al*. [15] investigated the consequences of stenosis on movement of the blood (non-Newtonian) in an artery, Jung *et al*. [7] studied irregular streams of non-Newtonian fluids in symmetric constricted artery. Also, there are several authors who considered blood as non-Newtonian fluid (Kiran *et al*. [8], Sreenadh *et al*. [16], Kiran *et al*. [10], Kiran and Radhakrishnamacharya [11], Das and Mandal [3], Kiran *et al*. [9], and Kumar *et al*. [12]).

The Casson fluid model is renowned for recognizing the existence of yield stress and also exhibits a plug flow. In view of its importance, various researchers have considered the flow of Casson fluid in the study of biological systems (Mishra and Pandey [13], Vajravelu *et al*. [17], and Amlimohamadi *et al*. [1]).

Motivated by this, the main purpose of this article is to deliberate the effect of multiple stenoses on the flow of a Casson fluid through a tube of non-uniform cross section. The non-Newtonian character of blood is represented by a Casson fluid and computationally derived closed form expressions to examine the deviation of velocity profiles, the shear stress near the wall and also the resistive impedance in the flow. The results are presented graphically.

2. Mathematical Model of the Problem

Contemplate a 2-dimensional stream of an incompressible Casson fluid in a non-uniform tube with two stenoses. Polar cylindrical coordinate system is preferred to facilitate the z-axis overlaps with center line of tube. The stenoses are considered as mild and grow in a symmetric way about axis. The geometry of the problem is kept as proposed by Prasad and

Radhakrishnamacharya [14],

$$h(z)=\begin{cases} R_0, & 0\le z\le d_1,\\ R_0-\frac{\delta_1}{2}\left(1+\cos\frac{2\pi}{L_1}\left[z-d_1-\frac{L_1}{2}\right]\right), & d_1\le z\le d_1+L_1,\\ R_0, & d_1+L_1\le z\le B_1-\frac{L_2}{2},\\ R_0-\frac{\delta_1}{2}\left(1+\cos\frac{2\pi}{L_2}[z-B_1]\right), & B_1-\frac{L_2}{2}\le z\le B_1,\\ R^*(z)-\frac{\delta_2}{2}\left(1+\cos\frac{2\pi}{L_2}[z-B_1]\right), & B_1\le z\le B_1+\frac{L_2}{2},\\ R^*(z), & B_1+\frac{L_2}{2}\le z\le B, \end{cases} \tag{1}$$

where B is the total channel length, L_1 and δ_1 are the length and extreme height of the primary stenosis, L_2 and δ_2 are the length and extreme height of the secondary stenosis (Figure 1).

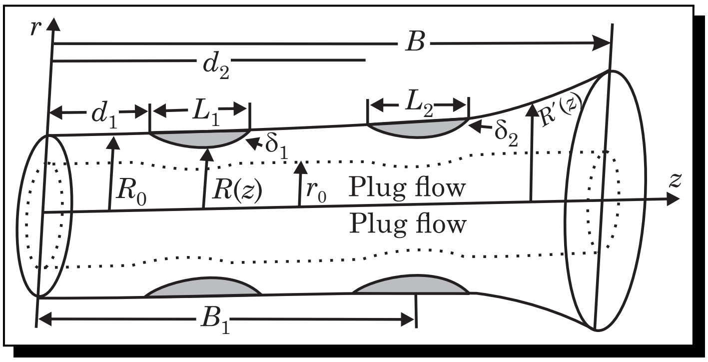

Figure 1. Physical geometry of the considered problem

The governing equation of the flow of Casson fluid is given by

$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})=-\frac{1}{\mu}\frac{\partial p}{\partial z}, \tag{2}$$

where

$$\sqrt{\tau_{rz}}=\begin{cases}\sqrt{\mu}\sqrt{\frac{-\partial u}{\partial r}}+\sqrt{\tau_0}, & \text{if } \tau\ge\tau_0,\\ 0, & \text{if } \tau\le\tau_0.\end{cases} \tag{3}$$

Considering the forces in the plug region, we have $2\pi r_0 B\tau_0 = P\pi r_0^2 B$ which can be simplified as

$$\tau_0=\frac{Pr_0}{2}, \tag{4}$$

where $P=\frac{\partial p}{\partial z}$.

The boundary conditions are:

$$\tau_{rz} \text{ is finite at } r=0, \tag{5}$$

$$u=0 \text{ at } r=h. \tag{6}$$

Taking the constraint of mild stenoses and evaluating eq. (2) using eqs. (5) and (6), the velocity is obtained as

$$u=\frac{P}{2\mu}\left[\frac{4}{3}r_0^{\frac{1}{2}}(r^{\frac{3}{2}}-h^{\frac{3}{2}})-\frac{1}{2}(r^2-h^2)-r_0(r-h)\right]. \tag{7}$$

Substituting $r = r_0$ in eq. (7), we get the plug velocity as

$$u_p = \frac{P}{2\mu}\left[\frac{-1}{6}r_0^2 - \frac{4}{3}r_0^{\frac{1}{2}}h^{\frac{3}{2}} + \frac{1}{2}h^2 + hr_0\right]. \tag{8}$$

3. Analysis of the Solution

The fluid flux Q is given by:

$$Q = \int_0^{r_0} 2u_p r dr + 2\int_{r_0}^{h} u r dr \tag{9}$$

$$= \frac{P}{\mu}\left[\frac{-1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}}h^{\frac{7}{2}} + \frac{h^4}{8} + \frac{1}{6}r_0h^3\right]. \tag{10}$$

Using the following non dimensional quantities,

$$\left.\begin{aligned} &\delta_1' = \frac{\delta_1}{R_0},\ \delta_2' = \frac{\delta_2}{R_0},\ H = \frac{h}{R_0},\ p\frac{\mu UB}{R_0^2},\ z' = \frac{z}{B},\ L_1' = \frac{L_1}{B},\ L_2' = \frac{L_2}{B},\ B_1' = \frac{B_1}{B},\\ &d_1' = \frac{d_1}{B},\ d_2' = \frac{d_2}{B},\ Q' = \frac{Q}{UR_0^2},\ R^*(z') = \frac{R^*(z)}{R_0},\ r_0' = \frac{r_0}{R_0}\ \text{ and }\ r' = \frac{r}{R_0} \end{aligned}\right\} \tag{11}$$

in eq. (10), we get (ignoring the primes)

$$Q = -\frac{\partial p}{\partial z}\left[\frac{-1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}}H^{\frac{7}{2}} + \frac{H^4}{8} + \frac{1}{6}r_0H^3\right]. \tag{12}$$

From eq. (12),

$$\frac{\partial p}{\partial z} = \frac{-Q}{\left[\frac{-1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}}H^{\frac{7}{2}} + \frac{H^4}{8} + \frac{1}{6}r_0H^3\right]}, \tag{13}$$

$$\Delta p = \int_0^1 \frac{-Q}{\left[\frac{-1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}}H^{\frac{7}{2}} + \frac{H^4}{8} + \frac{1}{6}r_0H^3\right]}dz. \tag{14}$$

The resistance to flow, denoted by

$$\lambda = \frac{\Delta P}{Q}. \tag{15}$$

By considering eqs. (14) and (15) together, we get

$$\lambda = \int_0^1 \frac{-1}{\left[\frac{-1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}}H^{\frac{7}{2}} + \frac{H^4}{8} + \frac{1}{6}r_0H^3\right]}dz. \tag{16}$$

In case of ignored stenoses ($H = 1$), the pressure change denoted by Δp_n is obtained from eq. (14)

$$\Delta p_n = \int_0^1 \frac{-Q}{\left[\frac{-1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}} + \frac{1}{8} + \frac{1}{6}r_0\right]}dz. \tag{17}$$

The flow resistance in the nonappearance of stenoses λ_n is demarcated by

$$\lambda_n = \frac{\Delta p_n}{Q}. \tag{18}$$

By means of eq. (17) in eq. (18), we get

$$\lambda_n = \int_0^1 \frac{-1}{\left[\frac{-1}{168}r_0^4 - \frac{2}{7}r_0^{\frac{1}{2}} + \frac{1}{8} + \frac{1}{6}r_0\right]} dz. \tag{19}$$

The normalized flow resistance $\bar{\lambda}$ is given by

$$\tau_w = -\mu \frac{\partial u}{\partial r}\bigg|_{r=h}. \tag{20}$$

Applying dimensionless quantities in eq. (11) collected with

$$\tau'_w = \frac{\tau_w}{\left(\frac{\mu U}{R_0}\right)}. \tag{21}$$

Eq. (20) reduces to

$$\tau'_w = \frac{-\partial u'}{\partial r'}. \tag{22}$$

Using eq. (7) in non-dimensional form and eq. (13) in eq. (22) (after dropping the primes), we get

$$\tau_w = \frac{Q}{2}\left[\frac{2r_0^{\frac{1}{2}}H^{\frac{1}{2}} - H - r_0}{\frac{1}{168}r_0^4 + \frac{2}{7}r_0^{\frac{1}{2}}H^{\frac{7}{2}} - \frac{1}{8}H^4 - \frac{1}{6}r_0H^3}\right]. \tag{23}$$

The shear-stress near the wall when the constriction is ignored ($H = 1$), denoted by $(\tau_w)_n$ can be found from eq. (23) as

$$\tau_{w_n} = \frac{Q}{2}\left[\frac{2r_0^{\frac{1}{2}} - 1 - r_0}{\frac{1}{168}r_0^4 + \frac{2}{7}r_0^{\frac{1}{2}} - \frac{1}{8} - \frac{1}{6}r_0}\right]. \tag{24}$$

The normalized shear-stress at the wall $\bar{\tau}_w$ is given by

$$\bar{\tau}_w = \frac{\tau_w}{(\tau_w)_n}. \tag{25}$$

4. Numerical Results and Discussion

The flow resistance and wall shear stress are two significant attributes in the examination of blood flow over a constricted stenosed artery. The mathematical equations for flow resistance and wall shear are specified by the eqs. (23) and (25), respectively. These have been numerically calculated using MATHEMATICA for diverse values of the pertinent parameters and displayed graphically. Moreover, it is assumed $\frac{R^*(z)}{R_0} = \exp[\beta B^2(z - B_1)^2]$ (Prasad and Radhakrishnamacharya [14]).

Further, it is also assumed that $d_1 = 0.2$, $d_2 = 0.6$, $L_1 = 0.2$, $L_2 = 0.2$, $B_1 = 0.3$, $\beta = 0.1$.

Figures 2-5 demonstrate the impacts of numerous parameters on the resistance to flow. It is detected that the resistance to flow $\bar{\lambda}$ decreases with the increase in the radius of the plug region, i.e., the resistance decreases with non-Newtonian character of the fluid. It is also noticed that the resistance increases with the elevations of both the primary and secondary stenoses δ_1 and δ_2. This result agrees with the earlier results obtained by Young [18], Shukla *et al.* [15], and Prasad and Radhakrishnamacharya [14].

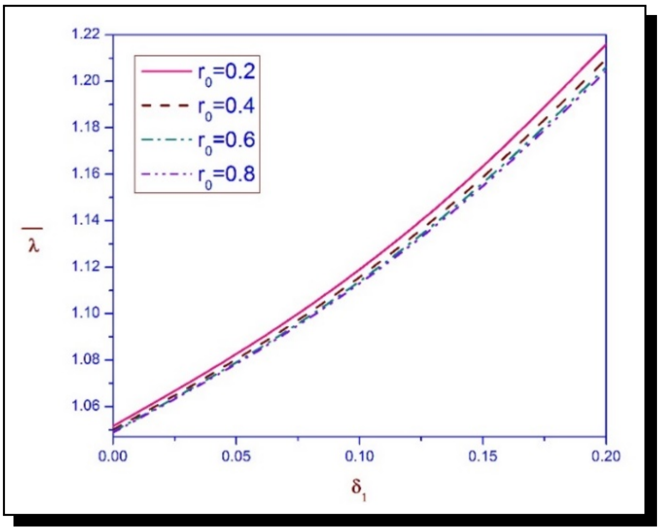

Figure 2. Impact of r_0 on resistance $\bar{\lambda}$ ($\delta_2 = 0.2$)

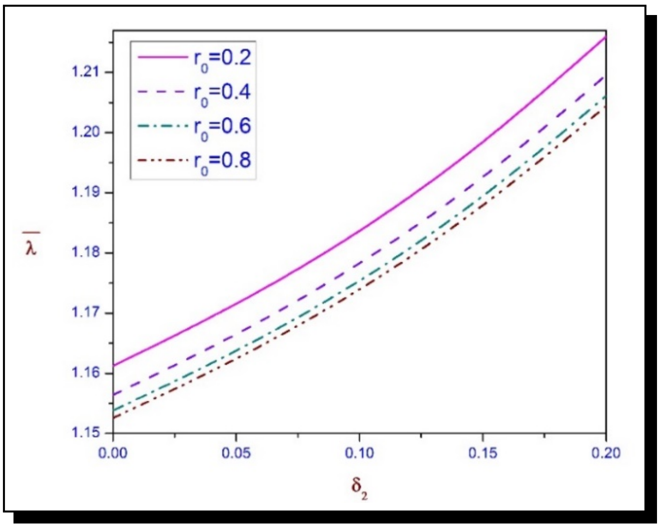

Figure 3. Impact of r_0 on resistance $\bar{\lambda}$ ($\delta_1 = 0.2$)

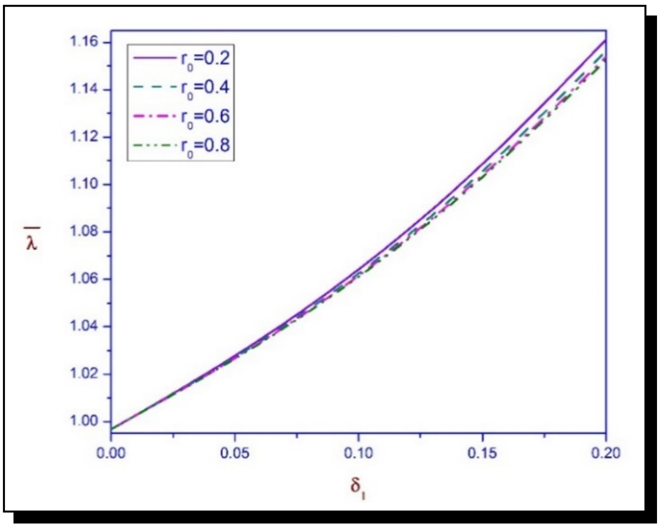

Figure 4. Impact of r_0 on resistance $\bar{\lambda}$ ($\delta_2 = 0$)

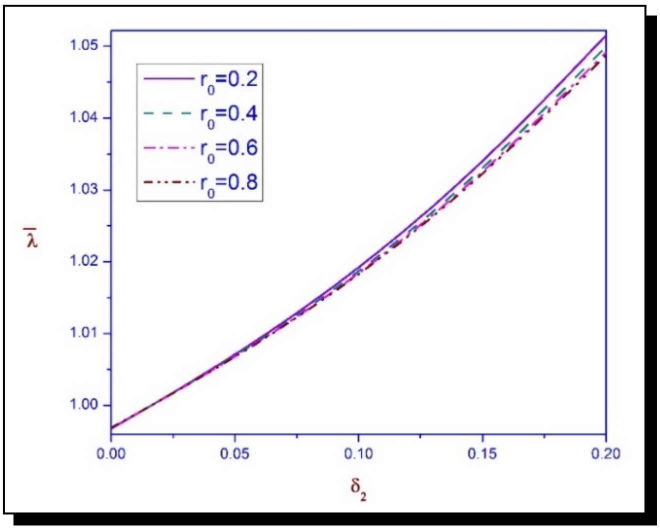

Figure 5. Impact of r_0 on resistance $\bar{\lambda}$ ($\delta_1 = 0$)

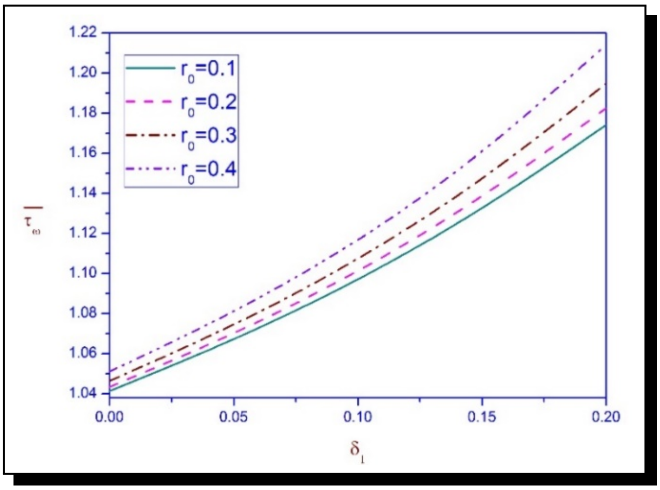

Figure 6. Impact of $r_0$on shear stress $\bar{\tau}_w$ ($\delta_2 = 0.2$)

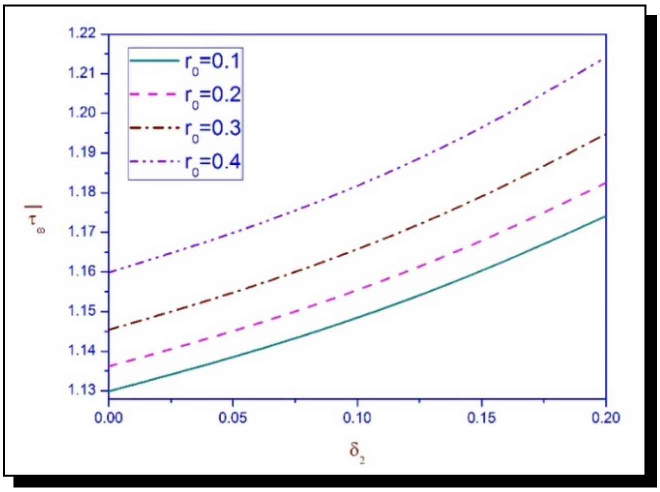

Figure 7. Impact of $r_0$on shear stress $\bar{\tau}_w$ ($\delta_1 = 0.2$)

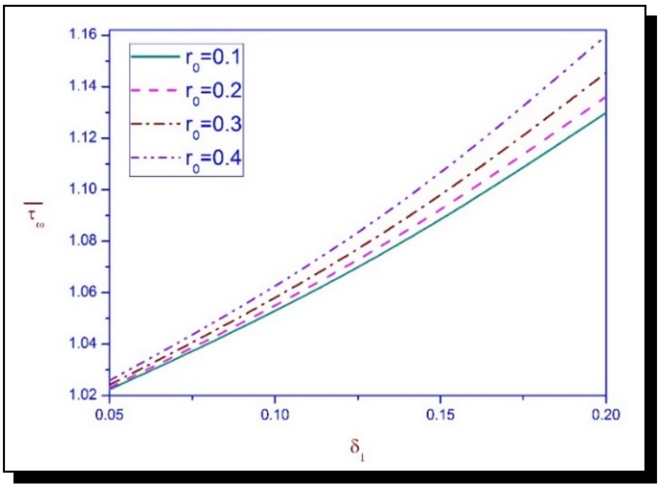

Figure 8. Impact of r_0 on shear stress $\bar{\tau}_w$ ($\delta_2 = 0$)

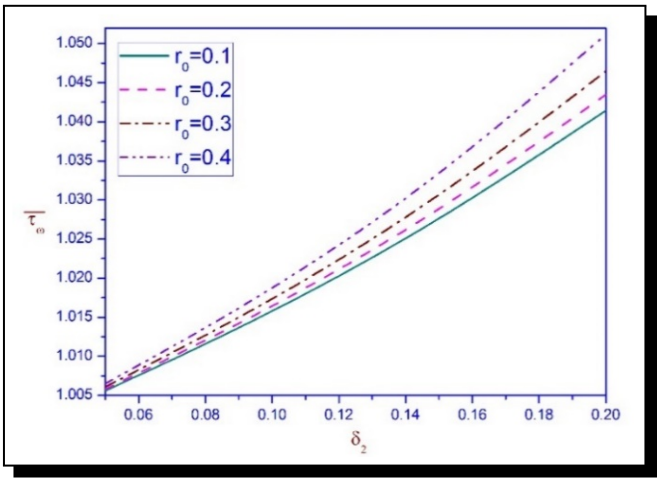

Figure 9. Impact of r_0 on shear stress $\bar{\tau}_w$ ($\delta_1 = 0$)

It is observed that the effects of diverse parameters on the wall shear stress is portrayed in Figure 6 - Figure 9. The stress ($\bar{\tau}_w$) increases with the heights of the stenoses δ_1 and δ_2. This result also agrees with the results of Young [18], and Shukla *et al*. [15]. It is also seen that shear stress increases with plug region radius r_0.

Further, the changes in resistance to the flow and wall shear stress are more significant in the case of two stenoses (Figures 2, 3, 6 and 7) than those of single stenosis (Figures 4, 5, 8 and 9).

Competing Interests

The authors declare that they have no competing interests.

Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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