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Research Article

Stability of Generalized Quartic Functional Equation in Random Normed Spaces

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Abstract. Aim of this paper is to investigate the Hyers-Ulam stability of generalized quartic functional equation

$$\begin{aligned}\sum_{i=1}^{n}\emptyset\Big(-v_i+\sum_{j=1,i\neq j}^{n}v_j\Big)&=(n-8)\sum_{1=i<j<k<l=n}\emptyset(v_i+v_j+v_k+v_l)-(n^2-12n+28)\sum_{1=i<j<k=n}\emptyset(v_i+v_j+v_k)\\&\quad+\Big(\frac{n^3-15n^2+60n-68}{2}\Big)\sum_{1=i<j=n}\emptyset(v_i+v_j)+2\sum_{1=i<j=n}\emptyset(v_i-v_j)\\&\quad+\sum_{i=1}^{n}\emptyset(3v_i)-\Big(\frac{n^4-17n^3+86n^2-148n+558}{6}\Big)\sum_{i=1}^{n}\emptyset(v_i)\end{aligned}$$

in random normed space.

Keywords. Quartic functional equation, Hyers-Ulam stability, Random normed space

Mathematics Subject Classification (2020). 54E40, 39B82, 6S50

1. Introduction

In 1940, Ulam [20] posed the stability problems of functional equation in group homomorphisms. Next year, Hyers [6] gave an affirmative reply to Ulam's problem for additive groups. After that, many authors e.g., Aoki [2], Czerwol [3], Gajda [4], Gavruta [5], Hyers [6], Hyers *et al*. [7], Jun and Kim [9], Jung [11], Rassias [14], Rassias *et al*. [15], Rassias [16, 17], Rassias and Šemrl [18], and Tamilvanan *et al*. [19] extended the stability theory of functional equations.

Recently, the stability of many functional equations in various spaces like Banach spaces, modular spaces, fuzzy normed spaces and random normed spaces etc. have been established by Alessa *et al*. [1], Aoki [2], Jin and Lee [8], Uthirasamy *et al*. [21], and Vijaykumar *et al*. [22]. Now, in this paper, the Hyers-Ulam stability of quartic function equation,

$$\begin{aligned}\sum_{i=1}^{n}\varnothing\Big(-v_i+\sum_{j=1,i\neq j}^{n}v_j\Big)=(n-8)&\sum_{1=i<j<k<l=n}\varnothing(v_i+v_j+v_k+v_l)\\ &-(n^2-12n+28)\sum_{1=i<j<k=n}\varnothing(v_i+v_j+v_k)\\ &+\Big(\frac{n^3-15n^2+60n-68}{2}\Big)\sum_{1=i<j=n}\varnothing(v_i+v_j)+2\sum_{1=i<j=n}\varnothing(v_i-v_j)\\ &+\sum_{i=1}^{n}\varnothing(3v_i)-\Big(\frac{n^4-17n^3+86n^2-148n+558}{6}\Big)\sum_{i=1}^{n}\varnothing(v_i)\end{aligned} \tag{1.1}$$

introduced by Uthirasamy *et al*. [21], obtain in random normed spaces.

The usual terminology, notions and conventions of the theory of Random normed spaces are adopted as in [1].

Throughout, Δ^+ denotes the distribution functions spaces, i.e., the space of all mapping $f: R\cup(-\infty,\infty)\to[0,1]$ such that f is left continuous and increasing on R, $f(0)=0$ and $f(+\infty)=1$. D^+ subset of Δ^+ consisting of all functions V of $\Delta^+$for which $\ell^- V(+\infty)=1$ where $\ell^- f(s)$ denotes $\ell^- f(s)=\lim_{t\to s^-} f(t)$. The space Δ^+ is partially order by usual wise ordering of functions, i.e., $f=g \iff f(s)=g(s)$, $s\in R$. The maximal element for Δ^+ in this order is the distribution function $\epsilon_0(s)=\begin{cases}0, & \text{if } s=0,\\ 1, & \text{if } s>0.\end{cases}$

Definition 1.1 ([1], t-norm)**.** $T:[0,1]\times[0,1]\to[0,1]$ is a continuous triangular norm (briefly t-norm) if T satisfies the following conditions:

(i) T is commutative and associative;

(ii) T is a continuous;

(iii) $T(x,1)=x$, for all $x\in[0,1]$;

(iv) $T(x,y)\le T(z,w)$, whenever $x\le z$ and $y\le w$, for all $x,y,z,w\in[0,1]$.

Examples of continuous t-norm are $T(x,y)=xy$, $T(x,y)=\max\{x+y-1,0\}$ and $T(x,y)=\min(x,y)$.

Definition 1.2 ([1])**.** A random normed space (RN-space) is a triple (V,Ψ,T), where V is a vector space, T is a continuous t-norm and $\Psi: V\to D^+$ satisfying the following conditions:

(R_1) $\Psi_v(t)=\epsilon_0(t)$, for all $t>0$ if and only if $v=0$,

(R_2) $\Psi_{av}(t)=\Psi_v\big(\frac{t}{|a|}\big)$, for all $v\in V$, $t\ge 0$ and $a\in R$ with $a\neq 0$,

(R_3) $\Psi_{v+u}(t+s)\ge T(\Psi_v(t),\ \Psi_u(s)$, for all $v,u\in V$ and $t,u\ge 0$.

Definition 1.3 ([1])**.** Let (V,Ψ,T) be a RN-space:

(i) A sequence $\{v_n\}$ in V is said to be convergent to $v\in V$ if, for every $t>0$ and $\lambda>0$, there exists a positive integer N such that $\Psi_{v_n-v}(t)>1-\lambda$ whenever $n\ge N$ and write as $\lim_{n\to\infty}\Psi_{v_n-v}(t)=1$, $t>0$.

(ii) A sequence $\{v_n\}$ in V is said to be Cauchy sequence if, for every $t>0$ and $\lambda>0$, there exists a positive integer N such that $\Psi_{v_n-v_m}(t)>1-\lambda$ whenever $n\geq m\geq N$ and write as $\lim\limits_{n\to\infty}\Psi_{v_n-v_m}(t)=1$, $t>0$.

(iii) A RN-space (V,Ψ,T) is complete if every Cauchy sequence in V is convergent to a point in V.

For notational handiness, denote (V,Ψ,T), and (W,Ψ,T) are complete RN spaces and define a mapping $\emptyset:V\to W$ by

$$\begin{aligned}D\emptyset(v_1,v_2,\dots,v_n)=&\sum_{i=1}^{n}\emptyset\Big(-v_i+\sum_{j=1,i\neq j}^{n}v_j\Big)-(n-8)\sum_{1=i<j<k<l=n}\emptyset(v_i+v_j+v_k+v_l)\\&+(n^2-12n+28)\sum_{1=i<j<k=n}\emptyset(v_i+v_j+v_k)\\&-\left(\frac{n^3-15n^2+60n-68}{2}\right)\sum_{1=i<j=n}\emptyset(v_i+v_j)-2\sum_{1=i<j=n}\emptyset(v_i-v_j)\\&-\sum_{i=1}^{n}\emptyset(3v_i)+\left(\frac{n^4-17n^3+86n^2-148n+558}{6}\right)\sum_{i=1}^{n}\emptyset(v_i)\end{aligned}$$

for all $v_1,v_2,\dots,v_n\in V$. (1.2)

2. Hyers-Ulam Stability

Theorem 2.1. *If an even mapping $\emptyset:V\to W$ with $\emptyset(0)=0$ for which there exists a mapping $\Phi:V^n\to D^+$ for some $0<\alpha<8$,*

$$\Phi_{3v_1,3v_2,\dots,3v_n}(\varepsilon)\geq\Phi_{v_1,v_2,\dots,v_n}\left(\frac{\varepsilon}{\alpha}\right) \tag{2.1}$$

and

$$\lim_{t\to\infty}\Phi_{3^tv_1,3^tv_2,\dots,3^tv_n}(3^{4t}\varepsilon)=1, \tag{2.2}$$

for all $v_1,v_2,\dots,v_n\in V$ and all $\varepsilon>0$ such that

$$\Psi_{D_\emptyset}(v_1,v_2,\dots,v_n)(\varepsilon)\geq\Phi_{v_1,v_2,\dots,v_n}(\varepsilon). \tag{2.3}$$

Then, there exists a unique quartic mapping $Q_4:V\to W$ satisfying the functional equation (1.1) *with*

$$\Psi_{Q_4(v)-\emptyset(v)}(\varepsilon)\geq\Phi_{v,0,\dots,0}((3^4-\alpha)2\varepsilon), \tag{2.4}$$

for all $v\in V$ and all $\varepsilon>0$. The mapping $Q_4:V\to W$ is defined by

$$\Psi_{Q_4(v)}(\varepsilon)=\lim_{t\to\infty}\Psi_{\frac{\emptyset(3^tv)}{3^{4t}}}(\varepsilon), \tag{2.5}$$

for all $v\in V$ and all $\varepsilon>0$.

Proof. Replace $(v_1,v_2,\dots,v_n)$ by $(v,0,\dots,0)$ in (2.3), we have

$$\Psi_{\emptyset(3v)-3^4\phi(v)}(\varepsilon)\geq\Phi_{v,0,\dots,0}(\varepsilon),$$

$$\Psi_{\frac{\emptyset(3v)}{3^4}-\phi(v)}(\varepsilon)\geq\Phi_{v,0,\dots,0}(3^4\varepsilon). \tag{2.6}$$

Replace v by $3^t v$ in (2.6), we have

$$\Psi_{\frac{\phi(3^{t+1}v)}{3^4}-\phi(3^t v)}(\varepsilon) \geq \Phi_{3^t v,0,\dots,0}(3^4\varepsilon),$$

$$\Psi_{\frac{\phi(3^{t+1}v)}{3^{4(t+1)}}-\frac{\phi(3^t v)}{3^{4t}}}(\varepsilon) \geq \Phi_{3^t v,0,\dots,0}(3^{4(t+1)}\varepsilon)$$

$$\geq \Phi_{v,0,\dots,0}\left(\frac{3^{4(t+1)}\varepsilon}{\alpha^t}\right), \tag{2.7}$$

for all $v \in V$ and all $\varepsilon > 0$. Since

$$\frac{\phi(3^m v)}{3^{4m}} - \phi(v) = \sum_{i=0}^{m-1} \frac{\phi(3^{i+1}v)}{3^{4(i+1)}} - \frac{\phi(3^i v)}{3^{4i}}. \tag{2.8}$$

From (2.7) and (2.8), we have

$$\Psi_{\frac{\phi(3^m v)}{3^{4m}}-\phi(v)}\left(\sum_{i=0}^{m-1} \frac{\alpha^i}{3^{4(i+1)}}\frac{\varepsilon}{2}\right) \geq \Phi_{v,0,\dots,0}(\varepsilon),$$

$$\Psi_{\frac{\phi(3^m v)}{3^{4m}}-\phi(v)}(\varepsilon) \geq \Phi_{v,0,\dots,0}\left(\frac{2\varepsilon}{\sum_{i=0}^{m-1} \frac{\alpha^i}{3^{4(i+1)}}}\right). \tag{2.9}$$

Replace v by $3^n v$, we get

$$\Psi_{\frac{\phi(3^{m+n}v)}{3^{4(m+n)}}-\frac{\phi(3^n v)}{3^{4n}}}(\varepsilon) \geq \Phi_{v,0,\dots,0}\left(\frac{2\varepsilon}{\sum_{i=n}^{m+n-1} \frac{\alpha^i}{3^{4(i+1)}}}\right),$$

for all $v \in V$ and all $\Delta > 0$. As $\Phi_{v,0,\dots,0}\left(\frac{2\varepsilon}{\sum_{i=n}^{m+n-1} \frac{\alpha^i}{3^{4(i+1)}}}\right) \to 1$ as $m \to \infty$ then $\left\{\frac{\phi(3^n v)}{3^{4n}}\right\}$ is a Cauchy sequence in (W, Ψ, T). Since (W, Ψ, T) is complete RN-space, thus sequence $\left\{\frac{\phi(3^n v)}{3^{4n}}\right\}$ converges to some $Q_4(v) \in W$, i.e., $Q_4(v) = \lim_{n\to\infty} \frac{\phi(3^n v)}{3^{4n}}$.

Fix $v \in V$ and put $n = 0$, we obtain

$$\Psi_{\frac{\phi(3^m v)}{3^{4m}}-\phi(v)}(\varepsilon) \geq \Phi_{v,0,\dots,0}\left(\frac{2\varepsilon}{\sum_{i=0}^{m-1} \frac{\alpha^i}{3^{4(i+1)}}}\right)$$

and so, for every $\delta > 0$, we get

$$\Psi_{Q_4(v)-\phi(v)}(\epsilon+\delta) = T\left(\Psi_{Q_4(v)-\frac{\phi(3^m v)}{3^{4m}}}(\delta), \Psi_{\frac{\phi(3^m v)}{3^{4m}}-\phi(v)}(\varepsilon)\right)$$

$$\geq T\left(\Psi_{Q_4(v)-\frac{\phi(3^m v)}{3^{4m}}}(\delta), \Phi_{v,0,\dots,0}\left(\frac{2\varepsilon}{\sum_{i=0}^{m-1} \frac{\alpha^i}{3^{4(i+1)}}}\right)\right), \tag{2.10}$$

for all $v \in V$ and all $\Delta, \delta > 0$. Taking the limit $m \to \infty$,

$$\Psi_{Q_4(v)-\phi(v)}(\epsilon+\delta) \geq \Phi_{v,0,\dots,0}(2(3^4-\alpha)\varepsilon), \tag{2.11}$$

for all $v \in V$ and all $\varepsilon, \delta > 0$. Since δ was arbitrary, taking $\delta \to 0$,

$$\Psi_{Q_4(v)-\phi(v)}(\epsilon) \geq \Phi_{v,0,\dots,0}(2(3^4-\alpha)\varepsilon), \tag{2.12}$$

for all $v \in V$ and all $\varepsilon > 0$. Replacing $(v_1, v_2, \dots, v_n)$ by $(3^t v_1, 3^t v_2, \dots, 3^t v_n)$ in (2.3), $\Psi_{D\phi(3^t v_1, 3^t v_2, \dots, 3^t v_n)}(\varepsilon) \geq \Phi_{3^t v_1, 3^t v_2, \dots, 3^t v_n}(3^{4t}\varepsilon)$. Since $\lim_{t\to\infty} \Phi_{3^t v_1, 3^t v_2, \dots, 3^t v_n}(3^{4t}\varepsilon) = 1$, so Q_4 satisfies the functional equation (1.1). To prove the uniqueness of quartic mapping Q_4. Assume that there exists another quartic mapping Q_4', which satisfies inequality (2.12). Fix $v \in V$. Clearly,

$Q_4(3^t v) = 3^{4t} Q_4(v)$ and $Q'_4(3^t v) = 3^{4t} Q'_4(v)$, for all $v \in V$, we have

$$\Psi_{Q_4(v)-Q'_4(v)}(\varepsilon) = \lim_{t\to\infty} \Psi_{\frac{Q_4(3^t v)}{3^{4t}} - \frac{Q'_4(3^t v)}{3^{4t}}}(\varepsilon).$$

Consider,

$$\begin{aligned}\Psi_{\frac{Q_4(3^t v)}{3^{4t}} - \frac{Q'_4(3^t v)}{3^{4t}}}(\varepsilon) &\geq T\left(\Psi_{\frac{Q_4(3^t v)}{3^{4t}} - \frac{\emptyset(3^t v)}{3^{4t}}}\left(\frac{\varepsilon}{2}\right), \Psi_{\frac{\emptyset(3^t v)}{3^{4t}} - \frac{Q'_4(3^t v)}{3^{4t}}}\left(\frac{\varepsilon}{2}\right)\right) \\ &\geq \Phi_{3^t v,0,\dots,0}(3^{4t}(3^4-\alpha)\varepsilon) \\ &\geq \Phi_{v,0,\dots,0}\left(\frac{3^{4t}(3^4-\alpha)\varepsilon}{\alpha^t}\right), \end{aligned} \tag{2.13}$$

for all $v \in V$ and $\varepsilon > 0$. Since $\lim_{t\to\infty} \frac{3^{4t}(3^4-\alpha)\varepsilon}{\alpha^t} = \infty$, we have $\lim_{t\to\infty} \Phi_{v,0,\dots,0}\left(\frac{3^{4t}(3^4-\alpha)\varepsilon}{\alpha^t}\right) = 1$. Therefore, $\Psi_{Q_4(v)-Q'_4(v)}(\varepsilon) = 1$, for all $v \in V$ and $\varepsilon > 0$. So, $Q_4(v) = Q'_4(v)$. □

Theorem 2.2. *If an even mapping $\emptyset : V \to W$ with $\emptyset(0) = 0$ for which there exists a mapping $\Phi : V^n \to D^+$ for some $0 < \alpha < 81$,*

$$\Phi_{\frac{v_1}{3},\frac{v_2}{3},\dots,\frac{v_n}{3}}(\varepsilon) \geq \Phi_{v_1,v_2,\dots,v_n}(\alpha\varepsilon)$$

and

$$\lim_{t\to\infty} \Phi_{\frac{v_1}{3^t},\frac{v_2}{3^t},\dots,\frac{v_n}{3^t}}\left(\frac{\varepsilon}{3^{4t}}\right) = 1,$$

for all $v_1, v_2, \dots, v_n \in V$ and all $\varepsilon > 0$ such that

$$\Psi_{D\emptyset(v_1,v_2,\dots,v_n)}(\varepsilon) \geq \Phi_{v_1,v_2,\dots,v_n}(\varepsilon).$$

Then, there exists a unique quartic mapping $Q_4 : V \to W$ satisfying the functional equation (1.1) *with*

$$\Psi_{Q_4(v)-\emptyset(v)}(\varepsilon) \geq \Phi_{v,0,\dots,0}\left(\frac{\varepsilon}{(3^4-\alpha)2}\right),$$

for all $v \in V$ and all $\varepsilon > 0$. The mapping $Q_4 : V \to W$ is defined by

$$\Psi_{Q_4(v)}(\varepsilon) = \lim_{t\to\infty} \Psi_{3^{4t}\emptyset(\frac{v}{3^t})}(\varepsilon),$$

for all $v \in V$ and all $\varepsilon > 0$.

Corollary 2.3. *If an even mapping $\emptyset : V \to W$ with $\emptyset(0) = 0$ for which there exists a mapping $\Phi : V \to D^+$ satisfying*

$$\Psi_{D\emptyset(v_1,v_2,\dots,v_n)}(\varepsilon) \geq \Phi_{\sum_{i=1}^n \|v_i\|^\theta}(\varepsilon).$$

Then, there exists a unique quartic mapping $Q_4 : V \to W$ satisfying the functional equation (1.1) *with*

$$\Psi_{Q_4(v)-\emptyset(v)}(\varepsilon) \geq \Phi_{\|v\|^\theta}((3^4-3^p)2\varepsilon),$$

for all $v \in V$, where $p < 4$ and all $\varepsilon > 0$.

We take $\alpha = 3^{p-4}$ and $v_1, v_2, \dots, v_n = \sum_{i=1}^n \|v_i\|^\theta$ in Theorem 2.1.

3. Conclusion

The Hyers-Ulam stability of generalized quartic functional equation is proved in *Random Normed space*. To get ideas from this equation, researchers can extended and introduced new functional equations and their stability.

Competing Interests

The author declares that she has no competing interests.

Authors' Contributions

The author wrote, read and approved the final manuscript.

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