**Communications in Mathematics and Applications**
Vol. 14, No. 5, pp. 1759–1766, 2023
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v14i5.2194

http://www.rgnpublications.com

Research Article

# On the Square Free Detour Number of Windmill Graphs

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**Received:** January 25, 2023 **Accepted:** May 5, 2023

**Abstract.** The set $S$ of vertices is said to be a square free detour set of $G^* = (V^*, E^*)$ if $I_{D_{\square f}}[S] = V^*$. The square free detour number of $G^*$ is the cardinality of the minimum proper square free detour subset of $V^*$. The square free detour number $dn_{\square f}(G^*)$, the connected square free detour number $cdn_{\square f}(G^*)$ and the vertex square free detour number $dn_{\square f_u}(G^*)$ of $G^*$ are defined. Also, we determine the square free detour number, the connected square free detour number and the vertex-square free detour number of windmill graphs.

**Keywords.** Square free detour number, Connected square free detour number, Vertex square free detour number

**Mathematics Subject Classification (2020).** 05C12



## 1. Introduction

Over the past two decades, the detour concept, applied widely in communication networks has pervaded the field of graph theory. Chartrand *et al.* [3, 4] began a new page in the history of mathematics with his invention on the detour concept based on the longest distance $D(u,v)$ for any two vertices $u,v \in G^*$. They offered many wonderful results on the longest distance that acquired the largest access and paved a path to further investigation of diverse parameters in detour number (see e.g., Ali and Ali [1], Elakkiya and Abhishek [5], John and Kumar [6], Narayan and Sunitha [7], Ramalingam *et al.* [10], Santhakumaran and Athisayanathan [11], Santhakumaran *et al*. [12], and Titus and Ganesamoorthy [13]). Santhakumaran and Athisayanathan [11] defined a novel detour parameter called connected detour number and sparked a lot of interest of the successors, who studied the connected weak edge detour number (Prabhakar and Athisayanathan [9]) and the upper connected vertex detour monophonic number (Arumugam *et al*. [2]). Inspired by these great minds, we define square free detour number $dn_{\square_f}(G^*)$ of $G^*$. Further, we extend it to connected and vertex-square free detour parameters based on the longest path in which no four vertices induce a square, called square free detour. Also, we investigate the various square free detour parameters for windmill graph and Dutch windmill graph. We also exhibit the observed properties of those parameters. The graph $G^* = (V^*, E^*)$ considered in this paper is a simple connected graph of finite order $n$. We refer *connected graph* as *con*-graph. We abbreviate *square free detour* to *sfd* and refer to Chartrand *et al*. [3], and Pacifica and Rani [8] for the basic terminologies.

## 2. Preliminaries

We give some basic ideas that are useful for proving theorems in the following sections.

**Theorem 2.1** ([3])**.** *If G is a con-graph with cut-vertex v and a detour set S of G, then every component of G − v consists of a vertex of S.*

**Theorem 2.2** ([11])**.** *If G is a con-graph with cut-vertices, then all the cut-vertices of G are contained in every con-detour set of G.*

**Theorem 2.3** ([12])**.** *If u is any vertex of a con-graph G, then all the end-vertices of G except u are contained in all the vertex detour sets. cut-vertices of G are never contained in a vertex detour set.*

## 3. Square Free Detour Number of the Windmill Graphs

**Definition 3.1.** The set $S$ of vertices in a *con*-graph $G^*$ is said to be an *sfd*-set of $G^*$ if $I_{D_{\square_f}} = V^*$, where the interval $I_{D_{\square_f}}[u,v]$ contains every vertex appearing on any *sfd* : $u - v$. The minimum cardinality of its *sfd*-sets is the *sfd*-number $dn_{\square_f}(G^*)$ of $(G^*)$. An *sfd*-set $S$ is called *sfd*-basis of $(G^*)$ if $dn_{\square_f}(G^*)$.

**Example 3.2.** Consider the graph $G_1$ pictured in Figure 1, a set $S_1 = \{v_1, v_4\}$ is an *sfd*-basis of $G_1$. Consequently, $dn_{\square_f}(G_1) = 2$. Moreover, the existence of the sets $S_2 = \{v_2, v_6\}$, $S_3 = \{v_3, v_4\}$, $S_4 = \{v_2, v_5\}$, $S_5 = \{v_1, v_6\}$ and $S_6 = \{v_3, v_5\}$ as the *sfd*-bases exhibits that *sfd*-basis of $G_1$ need not be unique.

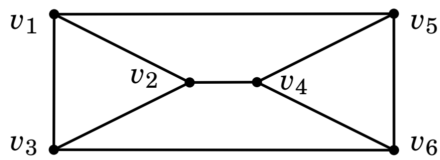


**Figure 1.** $G_1$

**Observation 3.3.** In any *con*-graph $G^*$ with order $n$,

(i) $2 \le dn(G^*) \le dn_{\Box_f}(G^*) \le n$,

(ii) every *sfd*-set is also a detour set.

**Theorem 3.4.** *Let $G^* = (V^*, E^*)$ be a windmill $W_n^{(m)}$ containing $m$ copies of $K_n$ $(m, n \ge 2)$ with a common vertex $x$, then a set of vertices $S^*$ is an sfd-basis of $G^*$ if $S^*$ consists of $m$ vertices, exactly one vertex from each copy of $K_n$.*

*Proof.* Suppose $G^* = W_n^{(m)}$ is a windmill containing $m$ copies of $K_n$ $(n \ge 2)$ with the common vertex $x$ and of order $m(n-1)+1$, where $V^*(G^*) = \{x, x_{pq} \mid p = 1,2,3,\dots,m; q = 1,2,3,\dots,n-1\}$. Let $S^*$ be a set containing $m$ vertices of $G^*$, extracted one vertex each from every copy of $K_n^{(m)}$ $(m, n \ge 2)$. Then all the vertices of $G^*$ lie on any $sfd: x_{p_1q} - x_{p_2q_2}$ $(1 \le p_1 < p_2 \le m; 1 \le q_1 \le q_2 \le n-1)$ of length 4. Thus every vertex of $W_n^{(m)}$ is positioned on any $sfd: x_{p_1q_1}, x_{p_1j_1}, x, x_{p_2j_2}, x_{pq_2}$, where $j_1$ and $j_2$ are distinct from $q_1$ and $q_2$ such that $j_1 \in V(C_n^{(p_1)})$, $j_2 \in V(C_n^{(p_2)})$. Then by Theorem 2.1, each component $W_n^{(m)} - x$ includes a vertex from $S^*$, an *sfd*-set of $W_n^{(m)}$. Since $|S^*| = m$ and so $S^*$ is an *sfd*-basis of $W_n^{(m)}$. □

**Theorem 3.5.** *If $G^* = (V^*, E^*)$ is a Dutch windmill $D_n^{(m)}$ $(n \ge 3, m \ge 2)$ consisting of $m$ copies $C_n$ with a common vertex $x$, then $S \subseteq V^*(G^*)$ of vertices is an sfd-basis of $G^*$ if $S$ contains $m$ vertices exactly one from each copy of $C_n (n \ge 3)$ in $D_n^{(m)}$.*

*Proof.* Consider $G^* = D_n^{(m)}$, a Dutch windmill graph of order $m(n-1)+1$ consisting of $m$ copies of $C_n$ $(n \ge 3)$ with the common vertex $x$. Let $V^*(G^*) = \{x, x_{p_iq_j} \mid i = 1,2,3,\dots,m; j = 1,2,3,\dots,n-1\}$. Let a set $S$ contain $m$ vertices of $D_n^{(m)}$. Then, we have three cases:

*Case* 1: Consider $n \ge 3$ and $n$ is odd. Suppose $S = \{x_{pq} \mid p = 1,2,3,\dots,m; q = 1 \text{ or } (n-1)\}$ is a set of $m$ adjacent vertices of $x$, exactly one vertex from each copy of $C_n$. Then all the vertices of $G$ appear on any square free detour $x_{p_1s} - x_{p_2s}$ $(1 \le p_1 < p_2 \le m-1; s \in \{x_1, x_{n-1}\})$, which admits $x$ as the central vertex with $D_{\Box_f}(x, x_{p_1s}) = D_{\Box_f}(x, x_{p_2s}) = n-1$. Since $x$ is a cut-vertex in $G$ by Theorem 2.1, each component of $D_n^{(m)} - x$ includes a vertex of $S$. Therefore, $S$ is a square free detour set of $D_n^{(m)}$.

*Case* 2: Consider $n \ge 6$ and $n$ is even. Suppose $S = \{x_{pq} \mid 1 \le p \le m;\ q = 1 \text{ or } \frac{n}{2} \text{ or } n-1\}$ contains either $m$ adjacent vertices or antipodal vertices of $x$. Then each vertex in $V^*(G^*)$ lies on $x_{r_1j} - x_{r_2j}$ $(1 \le r_1 < r_2 \le m-1; j \in \{x_1, x_{\frac{n}{2}}, x_{n-1}\})$ square free detour. Consider $u = x_{k_1j}$ and $v = x_{k_2j}$. We have three subcases:

*Case* 2.1: Let $S$ contain the vertices adjacent to $x$ and let $j = 1$ or $n-1$. Then $D_{\Box_f}(u,x) = D_{\Box_f}(v,x) = j$ and all the vertices of $D_n^{(m)}$ lie on a $u - v$ *sfd* of length $2(n-1)$. Thus $S$ is an *sfd*-set of $D_n^{(m)}$.

*Case* 2.2: Let $S$ contain the vertices antipodal to $x$ and let $j = \frac{n}{2}$. Then $D_{\Box_f}(u,x) = D_{\Box_f}(v,x) = j$ and all the vertices of $D_n^{(m)}$ lie on a $u - v$ *sfd* of length $n$. Therefore, $S$ is an *sfd* set of $D_n^{(m)}$.

*Case* 2.3: Let $S$ contain the vertices either adjacent or antipodal to $x$. Consider $u = x_{p(n-1)}$ and $v = x_{(p+1)\frac{n}{2}}$. Then $D_{\Box_f}(u,x) = n-1$, $D_{\Box_f}(v,x) = \frac{n}{2}$ and all vertices of $D_n^{(m)}$ lie on a $u - v$ square free detour of length $\frac{3n-2}{2}$. Hence $S$ is an *sfd*-set of $D_n^{(m)}$.

*Case* 3: Let $n = 4$ and $S$ consist of $m$ antipodal vertices of $x$ from each copy of $C_4^{(r)}$ : $x_{r1}, x_{r2}, x_{r3}, x_{r4}, x_{r1}$ $(1 \leq r \leq m)$. Clearly, every vertex of $D_4^{(m)}$ appears on $x_{p2} - x_{p+1}$ $(1 \leq p \leq m-1)$ square free detour of length 4 where $D_{\square_f}(x, x_{p2}) = D_{\square_f}(x, x_{(p+1)2}) = 2$. Thus $S$ is an *sfd*-set of $D_n^{(m)}$.

All three cases, discussed above shows that $|S| = m$ and so $S$ is an *sfd*-basis of $G^*$. □

## 4. Connected Square Free Detour Number of the Windmill Graphs

**Definition 4.1.** The set $S$ of vertices in a *con*-graph $G^*$ is said to be an *sfd*-set of $G^*$ if $S$ is an *sfd*-set of $G^*$ and the subgraph $G^*[S]$ induced by $S$ is connected. The minimum cardinality of its $csfd$-sets is the $csfd$-number $cdn_{\square_f}(G^*)$ of $G^*$. A $csfd$-set $S$ is called $csfd$-basis of $G^*$ if $|S| = cdn_{\square_f}(G^*)$.

**Example 4.2.** Look at the $G_2$ pictured in Figure 2, the subsets $S_1 = \{v_1, v_6\}$, $S_2 = \{v_2, v_5\}$ and $S_3 = \{v_3, v_4\}$ are the three *sfd*-bases of $G_2$ so that $dn_{\square_f}(G_2) = 2$. Thus any subset of $V^*(G_2)$ with two vertices cannot be a $csfd$-set of $G$. However, $S_4 = \{v_2, v_4, v_6\}$ is the $csfd$-basis of $G_2$ so that $cdn_{\square_f}(G_2) = 3$.

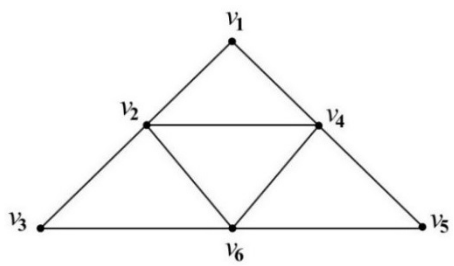


**Figure 2.** $G_2$

**Observation 4.3.** In any *con*-graph $G^*$ with order $n$,

(i) $2 \leq cdn(G^*) \leq cdn_{\square_f}(G^*) \leq n$,

(ii) every $csfd$-set is also a *con*-detour set.

**Theorem 4.4.** *If $G^* = (V^*, E^*)$ is a windmill $W_n^{(m)}$ consisting of $m$ copies of $K_n (m, n \geq 2)$ with a common vertex $x$, then a set $S$ of $V^*$ is a $csfd$-basis of $G^*$ if $S$ contains $m$ vertices, exactly one vertex from each copy of $K_n$ with the common vertex.*

*Proof.* Suppose $G^* = W_n^{(m)}$ is a windmill containing $m$ copies of $K_n$ $(n \geq 2)$ with the common vertex $x$ and of order $m(n-1)+1$, where $V^*(W_n^{(m)}) = \{x, x_{pq} \mid p = 1,2,3,\dots,m;\ q = 1,2,3,\dots,n-1\}$. Let $S^\#$ be a subset of $V^*(W_n^{(m)})$ with $m$ vertices, extracted one vertex each from every copy of $K_n^{(m)}$ $(m, n \geq 2)$. Then by Theorem 3.4, $S^\#$ is an *sfd*-basis of $G^*$. Since $G^*[S^\#]$ is not connected, consider the set $S = S^\# \cup \{x\}$. Clearly, $|S| = m+1$ and so $S$ is a $csfd$-basis of $W_n^{(m)}$ $(m, n \geq 2)$. □

**Theorem 4.5.** *If $G^* = (V^*, E^*)$ is a Dutch windmill $D_n^{(m)}$ $(n \geq 3, m \geq 2)$ consisting of $m$ copies $C_n$ with a common vertex $x$, then $S \subseteq V^*$ is a con-sfd-basis of $G^*$ if $S$ contains the common vertex and exactly one vertex from each copy of $C_n$ $(n \geq 3)$ of $D_n^{(m)}$.*

*Proof.* Consider $G^* = D_n^{(m)}$ be a Dutch windmill graph with $|D_n^{(m)}| = m(n-1)+1$ consisting of $m$ copies of $C_n$ $(n \geq 3)$ with the common vertex $x$. Let a set $S^*$ consist of $m$ vertices, exactly one vertex from each copy of $C_n$ in $D_n^{(m)}$. Then, we have three cases:

*Case* 1: Let $n$ be odd $(n \geq 3)$ and $S^*$ be a set of $m$ adjacent vertices of $x$, exactly one vertex from each copy of $C_n$. Then, all the vertices of $G$ of $D_n^{(m)}$ appear on *sfd*:u-v, which admits $x$ as central vertex with $D_{\square f}(u,v) = 2(n-1)$ and $D_{\square f}(x,u) = D_{\square f}(x,v) = n-1$. By *Case* 1 of Theorem 3.5, $|S^*| = m$ and so $S^*$ is a square free detour basis of $G$. Also, by Theorem 2.2, $S^*$ contains an element $x$. Moreover, since $G^*[S^* \cup x]$ is connected, $S$ is a *con-sfd*-basis of $G^*$ with $|S| = |S^*| + |x| = m+1$.

*Case* 2: Let $n$ be even $(n \geq 6)$ and $S^*$ consist of either $m$ adjacent vertices or $m$ antipodal vertices of $x$. We find that every vertex of $D_n^{(m)}$ appear on $u-v$ *sfd* of length $2(n-1)$ when $D_{\square f}(u,x) = D_{\square f}(v,x) = n-1$ and of length $\frac{3n-2}{2}$ when either $D_{\square f}(u,x)$ or $D_{\square f}(v,x)$ is $\frac{n}{2}$. Then by *Case* 2 of Theorem 3.5, $S^*$ is an *sfd*-basis of $D_{\square f}$, but $G^*[S^*]$ is not connected. Hence consider a set $S = S^* \cup x$. By Theorem 2.2, $S^*$ is a *con-sfd*-basis of $G$ with $|S^*| = m+1$.

*Case* 3: Let $n = 4$ and $S^*$ consist of m antipodal vertices of $x$ from each copy of $C_4^{(i)}$: $x_i, a_i, b_i, c_i, x_i (1 \leq i \leq m)$. We observe that all the vertices of $G^*$ appear on $b_i - b_{i+1} (1 \leq i \leq m-1)$ *sfd* of length 4, where $D_{\square f}(x,b_i) = D_{\square f}(x,b_{i+1}) = 2$. Thus $|S^*| = m$ and so $S^*$ is an *sfd*-basis of $G^*$. Then by *Case* 3 of Theorem 3.5, $S^*$ is an *sfd*-basis with $|S^*| = m$, yet not a *con-sfd*-basis. By including the common vertex $x$ to $S^*$, we obtain a *sfd*-basis $S$ of cardinality $m+1$. □

## 5. Vertex Square Free Detour Number of the Windmill Graphs

**Definition 5.1.** The set $S_u$ of vertices in a *con*-graph $G^*$ is said to be an *sfd*-set of $G^*$ if $I_{D_{\square f}}[S_u]$, where the interval $I_{D_{\square f}}[u,v]$ contains every vertex $y$ of $G^*$ appearing on any $sfd: u-v$ for some vertex $v$ in $S_u$. The minimum cardinality of its vertex *sfd*-sets is the vertex *sfd*-number $dn_{\square f_u}(G^*)$ of $G^*$. A vertex *sfd*-set $S_u$ is called vertex *sfd*-basis of $G^*$ if $|S_u| = dn_{\square f_u}(G^*)$.

**Example 5.2.** Consider the pictured graph $G_3$ in Figure 3, $S_{v_6} = \{v_1, v_2, v_4\}$ is a $v_6$-*sfd*-set. Hence $dn_{\square f_{v_6}}(G_3) = 3$. Also, the sets $S'_{v_6} = \{v_1, v_2, v_5\}$, $S''_{v_6} = \{v_1, v_3, v_4\}$ and $S'''_{v_6} = \{v_1, v_3, v_5\}$ are the $dn_{\square f_{v_6}}$-sets of $G_3$. Therefore, vertex *sfd*-set for any graph $G^*$ need not be unique.

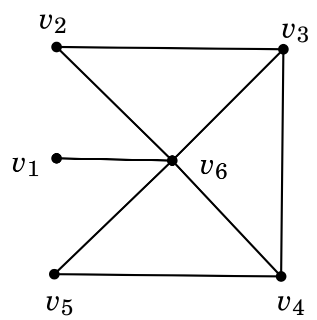


**Figure 3.** $G_3$

**Observation 5.3.** In any *con*-graph $G^*$ with order $n$,

(i) $2 \leq dn_u(G^*) \leq dn_{\square f_u}(G^*) \leq n$,

(ii) every vertex *sfd*-set is also a vertex detour set.

**Theorem 5.4.** *Let $G^* = (V^*, E^*)$ be a windmill $W_n^{(m)}$ $(m \geq 2, n \geq 3)$ consisting of $m$ copies of $K_n$ with common vertex $x$. Then $S_u$ is a $dn_{\square_{fu}}$-set of $G^*$ if $S_u$ consisting of*

(i) *$m$ vertices of $K_n^{(m)}$ of $W_n^{(m)}$, where $u$ is the common vertex of $W_n^{(m)}$,*

(ii) *$m-1$ vertices, taking exactly one vertex from $m-1$ copies of $K_n$, where $u$ is a non-common vertex of $G^*$.*

*Proof.* If $G^* = W_n^{(m)}$ is a Windmill with $m$ copies of $K_n$ and a common vertex $x$, then $|W_n^{(m)}| = m(n-1)+1$.

(i) Consider $u$ is the common vertex $x$ of $G$. Let $S_u = \{x_j \mid j = 1,2,3,\dots,m\}$ be a set of $m$ vertices, taking exactly one vertex from each copy of $K_n$. Thus, all the vertices of $W_n^{(m)}$ appear on $u = x - x_i$ *sfd* with $D_{\square_f}(x, x_i) = 2$. Then, by Theorem 2.3, $S_u$ is a $u$-*sfd*-set of $G^*$ with $|S_u| = m$. Thus $S_u$ is a $dn_{\square_{fu}}$-set of $G^*$.

(ii) Let $u = x_k$ $(1 \leq k \leq n-1)$ be a vertex on any $K_n^{(j)}$ $(j = 1,2,3,\dots,m)$ of $W_n^{(m)}$, say $K_n^{(1)}$. Suppose $S_u = \{x_l \mid 2 \leq l \leq m\}$ is a set with $m$ vertices taken from each copy of $K_n^{(l)}$ $(2 \leq l \leq m)$. Then, all the vertices of $W_n^{(m)}$ lie on $x_k - x_l$ $(1 \leq k \leq n-1; 2 \leq l \leq m)$ *sfd* of length 4. Thus $S_u$ is the $u$-*sfd*-set of $G^*$. Since $|S_u| = m-1$, $S_u$ is a $dn_{\square_{fu}}$-set of $W_n^{(m)}$. □

**Theorem 5.5.** *Let $G^* = (V^*, E^*)$ be a Dutch Windmill $D_4^{(m)}$ with $m$ copies of $C_4$ and common vertex $x$. Then $S_u \subseteq V^*(D_4^{(m)})$ is a $dn_{\square_{fu}}$-set of $D_4^{(m)}$ if $S_u$ consists of*

(i) *$m$ vertices of $G^*$, exactly one vertex of each copy of $C_4$, where $u$ is a common vertex $x$,*

(ii) *$m-1$ vertices of $G^*$, exactly one vertex from each copy of $C_4$ other than the copy in which $u$ lies with the antipodal vertex of $u$ in $C_4$ which holds $u$,*

(iii) *$m-1$ vertices of $G^*$, exactly one vertex from each copy of $C_4$ other than the copy in which $u$ lies.*

*Proof.* If $G^* = D_4^{(m)}$ is a Dutch windmill, then $|D_4^{(m)}| = 3m+1$ with $m$ copies of $C_4$ and a common vertex $x$.

(i) Let $u$ be the common vertex $x$ of $G^*$. Let $S_u = x_j \mid j = 1,2,3,\dots,m$ be a set of $m$ vertices, exactly one vertex antipodal to $u = x$ in each copy of $C_4$. Then, all the vertices of $D_4^{(m)}$ appear on $x - x_i$ $(1 \leq i \leq m)$ square free detour. Thus $S_u$ is a $u$-*sfd* set of $D_4^{(m)}$. Since $|S_u| = m$, $S_u$ is a $dn_{\square_{fu}}$-set of $D_4^{(m)}$.

(ii) Suppose $u$ is a vertex on any $D_4^{(i)}$ $(1 \leq i \leq m)$ of $G^*$. Assume that $u$ lies on the first copy $C_4^{(1)} : x, r_1, s_1, t_1, x$. When $u = r_1$ or $t_1$, let $S_u = \{s_j, x_1 \mid 2 \leq j \leq m, D_{\square_{fu}}(x, x_1) = 1\}$ be a set of $m$ vertices, exactly one vertex antipodal to $x$ in each copy of $C_4^{(j)}(2 \leq j \leq m)$ where $x_1$ is the antipodal vertex of $u$ in $C_4^{(1)}$. Then every vertex of $G$ except the antipodal vertex of $u$ in $C_4^{(1)}$ lies on $u - s_1$ square free detour. Obviously, that antipodal vertex in $C_4^{(1)}$ lies on $u - x_1$ square free detour with $D_{\square_{fu}}(x, x_1) = 1$. Thus $S_u$ is a $u$-*sfd* set of $G^*$. Since $|S_u| = (m-1)+1 = m$, $S_u$ is a $dn_{\square_{fu}}$-set of $D_4^{(m)}$.

(iii) Consider $u$ is a vertex on any $C_4^{(i)}$ $(1 \leq i \leq m)$ of $G^*$. Assume that $u$ lies on the first copy $C_4^{(1)} : x, r_1, s_1, t_1, x$. When $u = s_1$, suppose $S_u = \{s_j \mid 2 \leq j \leq m\}$ is a set with $m-1$ vertices, exactly one vertex antipodal to $x$ in each copy of $C_4^{(j)}$ $(2 \leq j \leq m)$. Then each vertex of $D_4^{(m)}$ appears on *sfd* : $u - s_j$ $(2 \leq j \leq m)$. Therefore, $S_u$ is a $u$-*sfd* set of $D_4^{(m)}$. Since $|S_u| = m-1$, $S_u$ is a $dn_{\square_{fu}}$-set of $D_4^{(m)}$. □

## 6. Conclusion

We defined the *sfd* number, the $csfd$ number and $u$-*sfd* number and investigated various square free parameters for windmill graph and Dutch windmill graph. We can extend the exhibited results for some other detour parameter that enables the applications in network topologies.

### Competing Interests

The authors declare that they have no competing interests.

### Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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