



# Onset of Internally Heated Convection in a Porous Layer With Variable Gravity: A Brinkmann Model

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**Abstract.** The influence of heat source, and variable gravity field on the stability of convective phenomena in a porous layer is investigated numerically by considering Brinkmann's model. Three types of gravity variations, such as, linear, parabolic, and cubic functions are considered. For linear theory, the method of normal modes has been employed to solve governing dimensionless equations which led an eigenvalue problem. The onset of convection is delayed by increasing Darcy number and gravity variation parameter. An enhancement of internal heat source makes the system unstable.

**Keywords.** Linear stability, Porous layer, Variable gravity

**Mathematics Subject Classification (2020).** 76S05, 76E06

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## 1. Introduction

Convective instabilities in a porous layer is analyzed by Horton and Rogers [15], Lapwood [21]. Weakly nonlinear analysis in a rotating porous medium is studied by Bhadauria *et al.* [5] and showed that Nusselt number decreases as Taylor number increases. Homsy and Sherwood [14]

are observed the convective instabilities in a porous medium with through flow. Convective phenomena in a porous medium under different models are studied by many researchers like Babu *et al.* [4], Elder [11], Seema *et al.* [17], Kuznetsov *et al.* [18, 19], Nield [25], Nield and Bejan [26], Nield and Kuznetsov [27], Ravi *et al.* [29, 30], Suma *et al.* [35], and Yadav [36, 37].

Convective phenomena in a porous layer with variable gravity field has received considerable interest in recent decades due to its vast range of applications in geothermal processes. In addition, earlier, there were not many studies related to the convective flow phenomena in the porous medium with variable gravity field. Lack of attention to such studies could be mainly because of its complexity, as it has been difficult to accurately characterize this process. It is well understood that Earth's gravitational field differs with regard to height in the situation of convection. The gravity field on thermal convection was first examined by Pradhan and Samal [28]. Thermal instability in a reactive porous media with magnetic field and variable gravity is examined by Harfash and Alshara [13]. They studied the linear and non-linear theories for convection. The influence of linear and non-linear gravity field variations on onset of instability in a porous medium with heat source is studied by Rionero and Straughan [33].

Straughan [34] studied the non-linear theory of thermal convection with variable gravity field. Effect of variable gravity and heat source in a porous layer have been studied by Harfash [12]. Deepika and Narayana [10] examined the variable gravity field on Hadley-Prats flow in a porous medium with vertical throughflow using non-linear energy theory and deduce that when variable gravity effect is not present then horizontal throughflow and vertical throughflow have symmetric nature.

Mahabaleshwar *et al.* [22] deduced in their article that convective phenomena in a porous layer with the heat source and gravity variation, the size and shape of convection cell are not changing with the change in variable gravity and heat source, further Chand *et al.* [9] discussed the steady convection and came to know that decreasing gravity parameter has to stabilize effect. Magneto rotating convective phenomena in nanofluid layer is studied by Mahajan and Arora [23]. Mahajan and Sharma [24] extended the work done by Mahajan and Arora [23] by considering variable gravity. Kaloni and Qiao [16] used the non-linear theory to illustrate the variable gravity on convective phenomena in a porous medium. Many authors, such as, Alex *et al.* [1], Alex and Patil [2, 3], Chand [6, 7], Chand *et al.* [8], Yadav [38, 39], and Yadav [40] studied the effect variable gravity field.

The purpose of current analysis is to investigate the linear theory of magneto-instability in a porous layer with the impact of variable gravity and heat source. The organization of the present analysis is as follows: Section 2 describes the mathematical problem. The linear instability theory is focused in Section 3 and Section 4 provides an in-detail explanation of the employed numerical method. Section 5 deals with the discussion of results. Conclusions are written in last section.

## 2. Basic Equations

Let us consider a newtonian fluid saturated porous layer bounded by the planes  $z = 0$  and  $z = d$ . The inclination angle of the layer is  $\alpha$  with respect to the  $x$ -axis.  $z$ -axis is taken vertically upward. The vertical thermal difference along the walls is  $\Delta T$ . The governing equations are:

$$\nabla \cdot \mathbf{u} = 0, \quad (2.1)$$

$$\frac{\mu}{K} \mathbf{u} = -\nabla p + \tilde{\mu} \nabla^2 \mathbf{u} + \rho_0 \bar{g}(z) \beta (T - T_0) \hat{e}_z, \quad (2.2)$$

$$\sigma \frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \chi \nabla^2 T + q', \quad (2.3)$$

corresponding boundary conditions are

$$\mathbf{u} = 0, \quad T = T_0 + \Delta T, \quad \text{on } z = 0,$$

$$\mathbf{u} = 0, \quad T = T_0, \quad \text{on } z = 1. \quad (2.4)$$

where

$\mu$ : viscosity,

$K$ : permeability,

$\rho$ : density,

$p$ : pressure,

$\beta$ : thermal expansion coefficient,

$t$ : time,

$\tilde{\mu}$ : effective viscosity,

$\sigma$ : heat capacity ratio,

$\chi$ : thermal diffusivity,

$q'$ : internal heat source (where  $q' > 0$ ),

$\bar{g}(z) = g_0[1 + \delta G(z)]$ ,

$g_0$ : reference gravity,

$\delta$ : gravity variation parameter.

The following dimensionless quantities are introduced

$$\left. \begin{aligned} x &= x^* d, & y &= y^* d, & z &= z^* d, \\ u &= \frac{\chi}{d} u^*, & v &= \frac{\chi}{d} v^*, & w &= \frac{\chi}{d} w^*, \\ t &= \frac{\sigma d^2}{\chi} t^*, & T &= (T_0 + T \Delta T) T^*, \end{aligned} \right\} \quad (2.5)$$

with

$$Ra = \frac{g \rho_0 \beta \Delta T K d}{\xi \mu}, \quad Da = \frac{\tilde{\mu} \kappa}{\mu d^2}, \quad Q = \frac{q' d^2}{\chi \Delta T}. \quad (2.6)$$

where  $Ra$ ,  $Da$ , and  $Q$  are the Rayleigh number, Darcy number, and Internal heat source parameter. The governing equations in non-dimensional form is given by

$$\nabla \cdot \mathbf{u} = 0, \quad (2.7)$$

$$\mathbf{u} = -\nabla p + Da \nabla^2 \mathbf{u} + Ra T [1 + \delta G(z)], \quad (2.8)$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \nabla^2 T + Q, \quad (2.9)$$

$$\mathbf{u} = 0, \quad T = 1 \quad \text{on } z = 0,$$

$$\mathbf{u} = 0, \quad T = 0 \quad \text{on } z = 1. \quad (2.10)$$

## 2.1 Basic Flow

The basic flow of eqs. (2.7)-2.10 is written as,

$$u_b = 0, \quad (2.11)$$

$$T_b = \frac{Q}{2}(z - z^2) + 1 - z. \quad (2.12)$$

## 3. Linear Stability Analysis

The perturbation of basic state for the eqs. (2.11)-(2.12), as

$$\mathbf{u} = u_b + \mathbf{U},$$

$$\begin{aligned} T &= T_b + \Phi, \\ p &= P_b + P. \end{aligned} \quad (3.1)$$

By substituting eq. (3.1) into eqs. (2.7)-(2.10), we get

$$\nabla \cdot \mathbf{U} = 0, \quad (3.2)$$

$$\mathbf{U} = -\nabla P + Da \nabla^2 \mathbf{u} + [1 + \delta G(z)] Ra \Phi \hat{e}_z, \quad (3.3)$$

$$\frac{\partial \Phi}{\partial t} + \mathbf{U} \cdot \nabla T_b + u_b \cdot \nabla \Phi = \nabla^2 \Phi, \quad (3.4)$$

$$\mathbf{U} = 0, \Phi = 0 \text{ on } z = 0, 1. \quad (3.5)$$

Eliminating pressure by taking the third component of curl of curl eq. (3.3), one obtains

$$\nabla^2 w - Da \nabla^4 w - [1 + \delta G(z)] Ra \nabla_h^2 \Phi = 0, \quad (3.6)$$

$$\frac{\partial \Phi}{\partial t} + \mathbf{U} \cdot \nabla T_b + u_b \cdot \nabla \Phi = \nabla^2 \Phi, \quad (3.7)$$

$$z = 0, 1; w = 0, \Phi = 0. \quad (3.8)$$

We choose the normal modes in the form of

$$w = W(z) e^{i(mx+ly-\omega t)}, \quad \Phi = \Phi(z) e^{i(mx+ly-\omega t)}, \quad (3.9)$$

where  $m$  is the wave number along  $x$  direction,  $l$  is the wave number  $y$  direction, and  $q = \sqrt{m^2 + l^2}$  is the non-dimensional wave number and angular frequency is given by  $\omega$ . On using eq. (3.9), eqs. (3.6)-(3.8) become

$$((D^2 - q^2) - Da(D^2 - q^2)^2)W + Ra[1 + \delta G(z)]q^2 \Phi = 0, \quad (3.10)$$

$$(D^2 - q^2 - i\omega)\Phi - W \frac{dT_b}{dz} = 0, \quad (3.11)$$

$$z = 0, 1; W = \Phi = 0. \quad (3.12)$$

## 4. Solution Methodology

In the present study, `bvp4c` routine in MATLAB R2020b is employed for solving eqs. (3.10)-(3.12). Eqs. (3.10)-(3.12) are converted into a system of ordinary differential equations of first order.  $w'(0) = 1$  (normalization condition) is considered to obtain the non-zero solution. The eigenvalue  $Ra$  is determined using condition. In MATLAB R2020b, the `indexmin` command has been used for obtaining the critical  $Ra$  and the critical  $q$ . The absolute and relative tolerance are taken as  $10^{-9}$  and  $10^{-6}$  to gain higher-order accuracy.

**Table 1.** Comparison of  $Ra_c$  of the present theory with the results of Rionero and Straughan [33] for  $Q = 0$  and  $Ha = 0$

For Case A			For Case B		
$\delta$	Present $R$	Rionero and Straughan [33]	$\delta$	Present $R$	Rionero and Straughan [33]
0	39.478	39.478	0	39.478	39.478
1	77.079	77.020	0.2	41.832	41.832
1.5	132.020	132.020	0.4	44.455	44.455
1.8	189.908	189.908	0.6	47.389	47.389
1.9	212.280	212.280	0.8	50.682	50.682

**Table 2.** Evaluation of  $Ra_c$  for different values of  $Q$  and  $\delta$  and for the fixed value of  $Da = 0.2$

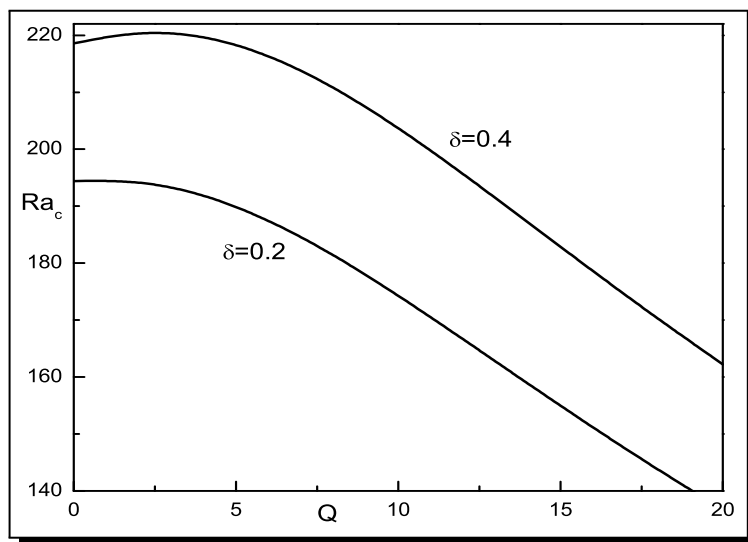
$\delta$	$Q$	Case A	Case B	Case C	$\delta$	Case A	Case B	Case C
0.2	0	194.372	185.418	181.248	0.4	218.596	197.186	187.997
0.2	2	194.520	185.508	181.096	0.4	220.787	198.860	189.001
0.2	4	192.182	183.254	178.683	0.4	220.082	197.988	187.622
0.2	6	187.684	178.966	174.324	0.4	216.670	194.757	184.091
0.2	8	181.550	173.135	168.508	0.4	211.042	189.609	178.849
0.2	10	174.362	166.309	161.764	0.4	203.840	183.103	172.430
0.2	12	166.644	158.980	154.565	0.4	195.699	175.792	165.341
0.2	14	158.795	151.526	147.268	0.4	187.146	168.133	157.990
0.2	16	151.093	144.207	140.122	0.4	178.563	160.455	150.675
0.2	18	143.707	137.186	133.276	0.4	170.203	152.9799	143.584
0.2	20	136.729	130.547	126.812	0.4	162.215	145.836	136.830

### 5. Discussion

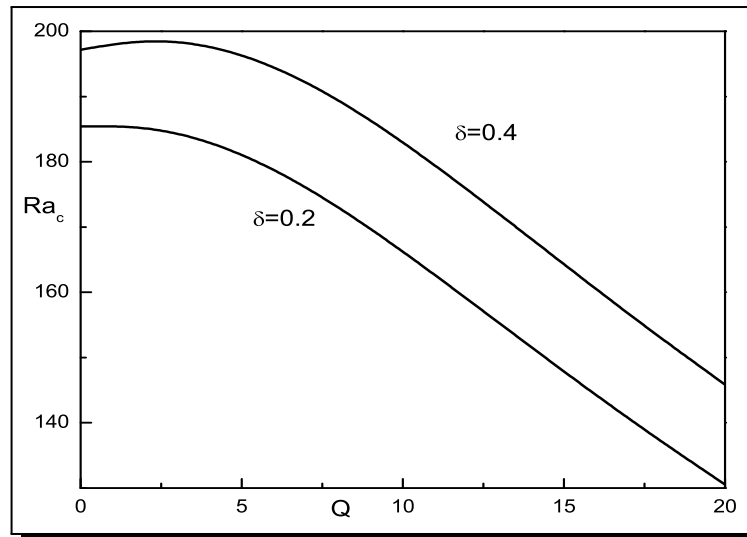
The influence of variable gravity field, internal heat source and magnetic field on Darcy-Bénard convection is studied. The eigenvalue problem for linear theory is examined using the `bvp4c` routine in MATLAB R2020b ([31, 32]). The effect of internal heat source parameter  $Q$ , Darcy number  $Da$ , and gravity variation parameter  $\delta$ , on critical Rayleigh number  $Ra_c$ , and critical wave number  $q_c$  are shown in Figures 1-6 and Tables 1-3. The following three cases of gravity field variance are considered:

- (A)  $G(z) = -z$ ,
- (B)  $G(z) = -z^2$ ,
- (C)  $G(z) = -z^3$ .

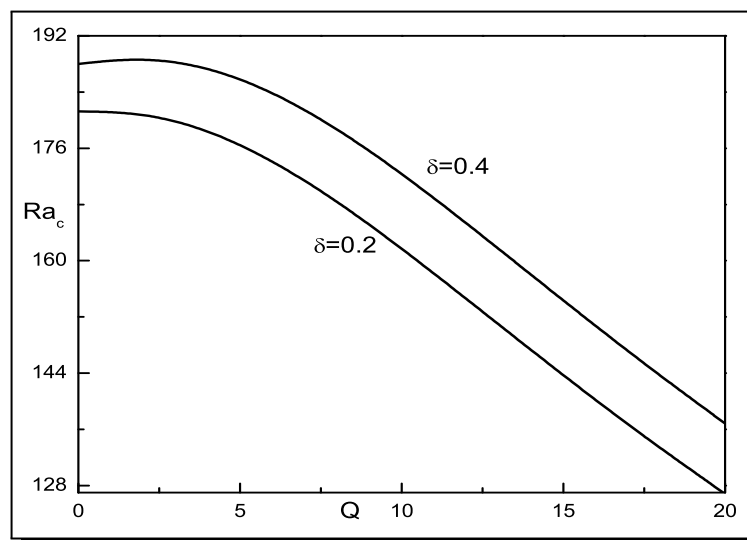
The comparison between present results (for  $Q = 0$  and  $Da = 0$ ) with the results of Rionero and Straughan [33] has been provided in Table 1. Table 1 clearly indicates a good agreement of present results with that of Rionero and Straughan [33].



**Figure 1.**  $Ra_c$  as a function of  $Q$  for  $\delta = 0.2, 0.4$ ,  $Da = 0.2$  and  $G(z) = -z$



**Figure 2.**  $Ra_c$  as a function of  $Q$  for  $\delta = 0.2, 0.4$ ,  $Da = 0.2$  and  $G(z) = -z^2$



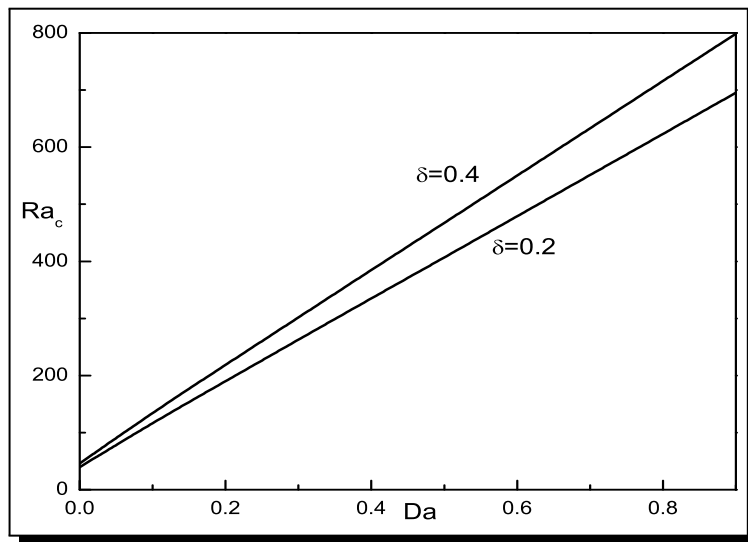
**Figure 3.**  $Ra_c$  as a function of  $Q$  for  $\delta = 0.2, 0.4$ ,  $Da = 0.2$  and  $G(z) = -z^3$

Table 2 shows the curve of critical  $Ra$  versus  $Q$  for different values of  $\delta$  for three cases. Visual representation of these values has given in Figures 1-3. From Table 2, it is observe that with an increase in  $Q$ , the  $Ra_c$  decreases, this is due to enhance the temperature in the global system. So  $Q$  has destabilizing effect, and it is also observed that with an increase in the  $\delta$ , the  $Ra_c$  increases. Hence  $\delta$  has a stabilizing effect. Finally, we also observe from Table 2 that the system become more stable in *Case A* ( $G(z) = -z$ ) and less stable in *Case C* ( $G(z) = -z^3$ ).

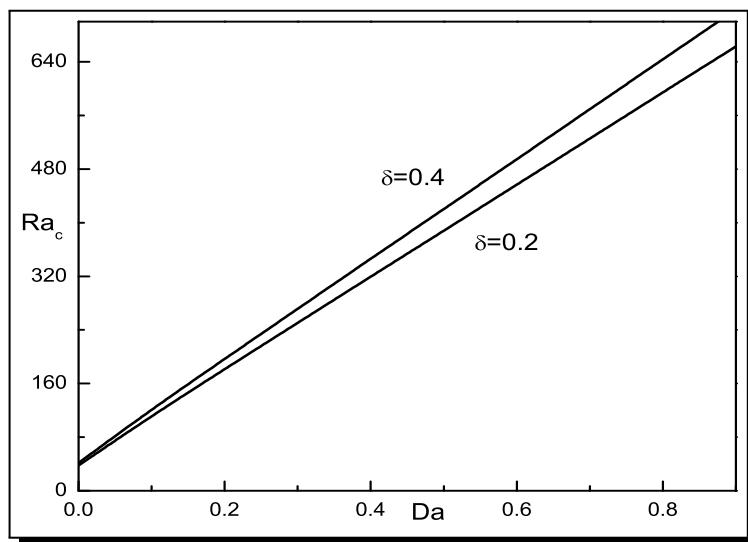
Table 3 shows the effect of  $Da$  on critical  $Ra$  for  $\delta = 0.2$  and  $0.4$  for three cases. Visual representation of this tabular values has given in Figures 4-6. From Table 3, we observe that with an increase in  $Da$  the  $Ra_c$  increases. So  $Da$  has stabilizing effect. And it is also found that with an increase in the value of  $\delta$ , the  $Ra_c$  increases. Therefore,  $\delta$ , has a stabilizing effect. Furthermore, one can also observe from this Table that the system become more stable in *Case A* ( $G(z) = -z$ ) and less stable in *Case C* ( $G(z) = -z^3$ ).

**Table 3.** Evaluation of  $Ra_c$  for different values of  $\delta$  and  $Da$  and for the fixed value of  $Q = 5$

$\delta$	$Da$	Case A	Case B	Case C	$\delta$	Case A	Case B	Case C
0.2	0	39.559	37.744	36.720	0.4	46.142	41.483	39.109
0.2	0.1	117.269	111.823	108.958	0.4	135.032	121.411	114.857
0.2	0.2	190.173	181.336	176.719	0.4	218.689	196.645	186.102
0.2	0.3	262.573	250.369	244.009	0.4	301.785	271.374	256.866
0.2	0.4	334.808	319.245	311.145	0.4	384.701	345.940	327.474
0.2	0.5	406.969	388.051	378.213	0.4	467.531	420.430	398.008
0.2	0.6	479.088	456.817	445.242	0.4	550.318	494.881	468.507
0.2	0.7	551.184	525.561	512.252	0.4	633.084	569.311	538.977
0.2	0.8	623.264	594.289	579.242	0.4	715.819	643.717	609.438
0.2	0.9	695.337	663.011	646.231	0.4	798.553	718.118	679.882



**Figure 4.**  $Ra_c$  as a function of  $Da$  for  $\delta = 0.2, 0.4$ ,  $Q = 5$  and  $G(z) = -z$



**Figure 5.**  $Ra_c$  as a function of  $Da$  for  $\delta = 0.2, 0.4$ ,  $Q = 5$  and  $G(z) = -z^2$

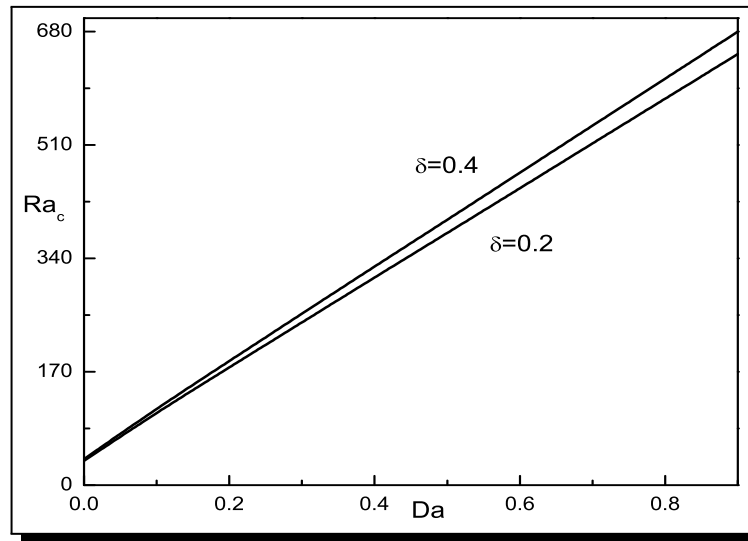


Figure 6.  $Ra_c$  as a function of  $Da$  for  $\delta = 0.2, 0.4$ ,  $Q = 5$  and  $G(z) = -z^3$

## 6. Conclusions

The onset of magnetoconvection in a porous layer with variable gravity have performed numerically using linear theory. We have analysed four cases of gravity field variation namely, (A)  $G(z) = -z$ , (B)  $G(z) = -z^2$ , and (C)  $G(z) = -z^3$ , and conclusions are listed below:

- Darcy number,  $Da$ , and variable gravity,  $\delta_1$  delay the onset of convection.
- Heat source parameter,  $Q$  destabilize the system.
- The system become more stable for linear variation, and less stable for cubic variation.

## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

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