**Communications in Mathematics and Applications**
Vol. 14, No. 5, pp. 1847–1856, 2023
ISSN 0975-8607 (online); 0976-5905 (print)
Published by RGN Publications
DOI: 10.26713/cma.v14i5.2072

http://www.rgnpublications.com

Research Article

# Weakly Nonlinear Convection of a Maxwell Fluid in a Porous Layer With Coriolis Effect

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**Received:** September 28, 2022 **Accepted:** March 23, 2023

**Abstract.** The linear and non-linear instability theories of a Maxwell fluid in a Darcy-Benard setup with coriolis effect is studied. For linear theory, the method of normal modes has been employed to solve governing dimensionless equations which led an eigenvalue problem and it is solved analytically. We obtained the expressions for steady and oscillatory thermal Rayleigh numbers. The effects of different physical parameters on steady and oscillatory convective phenomena are presented and described. In order to study the heat transport by convection the well-known equation, Landau-Ginzburg equation has been derived.

**Keywords.** Porous media, Rotation, Maxwell fluid, Non-inear stability analysis

**Mathematics Subject Classification (2020).** 35Q56, 76E30, 76S05



## 1. Introduction

Double-diffusive convection is of important study due its practical applications in many geological processes. Also, this study has wide range of geotechnical applications, in particular, liquid re-injection, the migration, underground disposal of nuclear wastes, and drying processes (Chandrasekhar [5], Beckermann and Viskanta [4], Coriell *et al*. [6], Prescott and Incropera

[14], Zhou and Zebib [20], and Babu *et al.* [2,3]). Similarly, Double-diffusive instability of a Newtonian fluid in a porous layer is well studied [13]. Similarly, convective phenomena in a porous layer saturated with non-Newtonian fluids is of interest due to its vast applications in science and engineering. Non-Newtonian fluids are frequently treated using the viscoelastic fluid model, power-law model, Maxwell fluid model etc. (see Dharmadhikari and Kale [7], Reddy and Ragoju [16], and Shenoy [17]).

Double-diffusive convection in a porous layer saturated by Oldroyd fluid was discussed by Malashetty and Swamy [11]. They obtained the analytical conditions for steady, overstable, and finite amplitude convective phenomena. Malashetty *et al.* [12] and Kumar and Bhadauria [10] extended the work done by Malashetty and Swamy [11] by considering thermal non-equilibrium effect. The double-diffusive convective motion of a Maxwell liquid was considered by Awad *et al.* [1]. They deduce that the minimal value of thermal Rayleigh number decreases with the Maxwell parameter. Wang and Tan [18] derived the condition for onset of convective instability of a non-Newtonian type liquid in a porous layer. They used modified Maxwell-Darcy model. Internally heated double-diffusive instability in a non-Newtonian type of coupled stress fluid in a porous layer was discussed by Gaikwad and Kouser [9].

Gaikwad and Dhanraj [8] discussed the effects of anisotropic and internal heating on the binary Maxwell liquid in a permeable layer. Reddy and Ragoju [15] studied the thermo solutal convection of a Maxwell fluid in a porous layer with the chemical reaction effect. They found that the Damkohler number has a contrast effect on steady and oscillatory convection. Yadav *et al.* [19] extended the work done by Reddy and Ragoju [15] by considering internal heat source.

In the present analysis, we consider linear and weakly non-linear instability theories of Maxwell fluid in a porous layer with coriolis effect. The organization of the present analysis is as follows: In Section 2, we describe the mathematical formulation. The linear stability analysis is described in Section 3. In Section 4, two dimensional amplitude equation is derived. The discussions of obtained results are presented in Section 5. In the last section, conclusions are written.

## 2. Mathematical Formulation

Let us assume a porous layer of maxwell fluid heated and salted from below, and zone placed between two infinitely, parallel, horizontal plates at $z = 0$ and $z = d$. The $z$-axis is oriented upward. It is rotating at a constant rate $\Omega$. The governing equations under the Oberbeck–Boussinesq approximation are

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\left(1+\lambda_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\phi}\frac{\partial \mathbf{u}}{\partial t}+\nabla P-\rho\mathbf{g}\right)+\frac{\mu}{\kappa}\mathbf{u}+\frac{2\rho_0\Omega}{\phi}\widehat{e_z}\times\mathbf{u}=0, \tag{2}$$

$$(\rho c)_m\frac{\partial\theta}{\partial t}+(\rho c)_f(\mathbf{u}\cdot\nabla)\theta=(\rho c)_m kT\nabla^2\theta, \tag{3}$$

$$\rho=\rho_0(1-\alpha(\theta-\theta_0)), \tag{4}$$

$$\mathbf{u} = \theta = 0 \text{ on } z = 0, 1. \tag{5}$$

The conductive state of the Maxwell fluid is as follows:

$$\mathbf{u}_b = 0, \quad \rho = \rho_b(z), \quad \theta_b = \theta_0 - \left(\frac{\Delta\theta}{d}\right). \tag{6}$$

We introduce the dimensionless parameters as follows

$$\begin{aligned}
&(x, y, z) = (x^* d, y^* d, z^* d), \\
&(u, v, w) = \left(\frac{\phi k}{d} u^*, \frac{\phi k}{d} v^*, \frac{\phi k}{d} w^*\right), \\
&t = \frac{d^2}{k} t^*, \quad p = \frac{\mu \phi k}{\kappa} p^*, \quad \theta = \Delta\theta \theta^*.
\end{aligned} \tag{7}$$

Then the governing equations for non-dimensional quantities can be written as

$$\nabla \cdot \mathbf{u} = 0, \tag{8}$$

$$\left(1 + \lambda \frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial \mathbf{u}}{\partial t} + \nabla P - R\widehat{e_z}\right) + \mathbf{u} + Ta\,\widehat{e_z} \times \mathbf{u} = 0, \tag{9}$$

$$\frac{\partial \theta}{\partial t} + \gamma(\mathbf{u} \cdot \nabla)\theta = w + \nabla^2 \theta, \tag{10}$$

$$\mathbf{u} = \theta = 0 \text{ on } z = 0, 1, \tag{11}$$

where

$$\begin{aligned}
&R = \frac{\rho_0 \delta\theta g \kappa d}{\mu \phi k_T}, \quad Ta = \left(\frac{2\rho_0 \Omega d^2}{\mu}\right)^2 \\
&Pr = \frac{\mu \phi}{\rho_0 k_T}, \quad \lambda = \frac{\lambda_1 k_T}{d^2}, \\
&\gamma = \frac{(\rho c)_f}{(\rho c)_m} \phi.
\end{aligned}$$

All quantities which are used in the above equations have been described in the nomenclature. We now eliminate the pressure by taking the third component of curl of (9) and curl of curl of (9), one obtains

$$\left(1 + \lambda \frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial}{\partial t}\right)\omega_z + \omega_z - Ta\frac{\partial w}{\partial z} = 0, \tag{12}$$

$$\left(1 + \lambda \frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial}{\partial t}\right)(-\nabla^2 w) - \nabla^2 w + R\theta\left(1 + \lambda \frac{\partial}{\partial t}\right)(\nabla_h)^2 - Ta\frac{\partial \omega_z}{\partial z} = 0, \tag{13}$$

where $\omega_z = (\nabla \times \mathbf{V}) \cdot \hat{e_z}$ and $\nabla_h^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

By removing $\omega_z$ and $\theta$ from eqs. (12), (13), (10) one obtains,

$$\mathcal{L} w = \mathcal{N}, \tag{14}$$

where

$$\begin{aligned}
\mathcal{L} = &\left(1 + \left(1 + \lambda \frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial}{\partial t}\right)\right)\left(\left(\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2 + \overset{2}{Ta}\left(\frac{\partial}{\partial t} - \nabla^2\right)\frac{\partial^2}{\partial z^2}\right. \\
&\left. + \left(1 + \lambda \frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t} - \nabla^2\right)\nabla^2 - R(\nabla_h)^2\right)\right),
\end{aligned} \tag{15}$$

$$\mathcal{N} = -R(\nabla_h)^2\left(1+\lambda\frac{\partial}{\partial t}\right)\left(1+\left(1+\lambda\frac{\partial}{\partial t}\right)\left(\frac{1}{Pr}\frac{\partial}{\partial t}\right)\right)\gamma(\mathbf{u}\cdot\nabla)\theta. \tag{16}$$

## 3. Linear Stability Analysis

Let us substitute $w = \sin\pi z e^{i(lx+my)+i\omega t}$ in $\mathcal{L}w = 0$, hence one obtains

$$R = \frac{(-i\delta^2+\omega)(\pi^2 Pr^2 Ta^2 + \delta^2(Pr+\omega(i-\lambda\omega))^2)}{Pr\, q^2(-i+\lambda\omega)(Pr+\omega(i-\lambda\omega))}. \tag{17}$$

### 3.1 Stationary Convection

First, we consider stationary instability, i.e., $\sigma = 0$ is real. The stationary Rayleigh number $R_s$ can be written as

$$R_s = \frac{\pi^2 Ta^2 \delta^2 + \delta^4}{q^2}. \tag{18}$$

### 3.2 Oscillatory Convection

Now consider the real and imaginary parts of $R$, which require the imaginary part of $R$ to vanish. Substituting $\omega^2$into the real part of $R$ yields the thermal Rayleigh number, $R_T^{oc}$, for oscillatory convection.

## 4. Weakly Non-linear Analysis

Let us introduce the following series expansion interms of $\epsilon$

$$\left.\begin{aligned} u &= \epsilon u_0 + \epsilon^2 u_1 + \epsilon^3 u_2 + \cdots, \\ v &= \epsilon v_0 + \epsilon^2 v_1 + \epsilon^3 v_2 + \cdots, \\ w &= \epsilon w_0 + \epsilon^2 w_1 + \epsilon^3 w_2 + \cdots, \\ \theta &= \epsilon\theta_0 + \epsilon^2\theta_1 + \epsilon^3\theta_2 + \cdots, \\ C &= \epsilon C_0 + \epsilon^2 C_1 + \epsilon^3 C_2 + \cdots, \end{aligned}\right\} \tag{19}$$

where

$$\epsilon^2 = \frac{R-R_{sc}}{R_{sc}} << 1.$$

The first approximations are

$$\left.\begin{aligned} u_0 &= \frac{i\pi}{l_{sc}}[Ae^{i(l_{sc}x+m_{sc}y)}\cos\pi z - c.c], \\ v_0 &= \frac{\pi}{il_{sc}}[Ae^{i(l_{sc}x+m_{sc}y)}\cos\pi z - c.c], \\ w_0 &= [Ae^{i(l_{sc}x+m_{sc}y)}\sin\pi z + c.c], \\ \theta_0 &= \frac{\gamma}{\delta_{sc}^2}[Ae^{i(l_{sc}x+m_{sc}y)}\sin\pi z + c.c, \end{aligned}\right\} \tag{20}$$

where the amplitude, $A = A(X,Y,Z,T)$, is the amplitude and the complex conjugate is denoted by $c.c$. The variables $X,Y,Z$ and $T$ can be scaled as

$$X = \epsilon x, \quad Y = \epsilon^{\frac{1}{2}} y, \quad Z = z, \; T = \epsilon^2 t,$$

Using the above scaling differential operators can be written as

$$\left.\begin{aligned}&\frac{\partial}{\partial x}\to\frac{\partial}{\partial x}+\epsilon\frac{\partial}{\partial X},\\&\frac{\partial}{\partial y}\to\frac{\partial}{\partial y}+\epsilon^{\frac{1}{2}}\frac{\partial}{\partial Y},\\&\frac{\partial}{\partial z}\to\frac{\partial}{\partial Z},\\&\frac{\partial}{\partial t}\to\epsilon^2\frac{\partial}{\partial T}.\end{aligned}\right\}\tag{21}$$

By using eq. (21), the operators $\mathcal{L}$ and $\mathcal{N}$ of eq. (14) can be written as

$$\left.\begin{aligned}&\mathcal{L}=\mathcal{L}_0+\epsilon\mathcal{L}_1+\epsilon^2\mathcal{L}_2\cdots,\\&\mathcal{N}=\mathcal{N}_0+\epsilon\mathcal{N}_1+\epsilon^2\mathcal{N}_2\cdots.\end{aligned}\right\}\tag{22}$$

On substituting eq. (22) into eq. (14), and comparing the coefficients of $\epsilon$, $\epsilon^2$ and $\epsilon^3$, one obtains

$$\mathcal{L}_0w_0=0,\tag{23}$$

$$\mathcal{L}_0w_1+\mathcal{L}_1w_0=\mathcal{N}_0,\tag{24}$$

$$\mathcal{L}_0w_2+\mathcal{L}_1w_1+\mathcal{L}_2w_0=\mathcal{N}_1.\tag{25}$$

where

$$\mathcal{L}_0=-\nabla^4-R\nabla_h^2-D^2\nabla^2\overset{2}{Ta},\tag{26}$$

$$\mathcal{L}_1=2\frac{\partial^2}{\partial x\partial X}(-2\nabla^2-R-D^2\overset{2}{Ta}),\tag{27}$$

$$\mathcal{L}_2=\frac{\partial^2}{\partial X^2}(-2\nabla^2-R-D^2\overset{2}{Ta})+\left(2\frac{\partial^2}{\partial x\partial X}\right)^2(\nabla^2-D^2\overset{2}{Ta})$$
$$+\frac{\partial}{\partial\tau}\left(\nabla^2+D^2\overset{2}{Ta}-R\lambda\nabla_h^2-\frac{1}{Pr}(2\nabla^4-R\nabla_h^2)\right).\tag{28}$$

Let us substitute the solution $w_0$ in to $\mathcal{L}_0w_0=0$. One obtains

$$R_{sc}=\frac{\pi^2Ta^2\delta^2+\delta^4}{q^2},\tag{29}$$

from the equation $\mathcal{L}_0w_1+\mathcal{L}_1w_0=\mathcal{N}_0$, $\mathcal{N}_1=0$ and $\mathcal{L}w_0=0$. The equation reduces to $w_1=0$, which implies $u_1=0$, and also,

$$\theta_1=\frac{\gamma}{2\pi\delta_{sc}^2\pi}|A|^2\sin 2\pi z.\tag{30}$$

On substituting the first order solutions into the eq. (25), we obtain Newell-Whitehead equation in the form of

$$\lambda_0\frac{\partial A}{\partial T}-\lambda_1\left(\frac{\partial}{\partial X}-\frac{i}{2q_{sc}}\frac{\partial^2}{\partial Y^2}\right)^2A-\lambda_2A+\lambda_3|A|^2A=0,\tag{31}$$

where

$$\lambda_0=R_{sc}\lambda q_{sc}^2-\delta_{sc}^2-\pi^2\overset{2}{Ta}-\frac{1}{Pr}(2\delta_{sc}^4+R_{sc}q_{sc}^2),$$

$$\lambda_1=2\delta_{sc}^2-R_{sc}+\pi^2\overset{2}{Ta},$$

$$\lambda_2 = -\delta_{sc}^4 + R_{sc} q_{sc}^2 - \pi^2 \overset{2}{Ta}\,\delta_{sc}^2,$$

$$\lambda_3 = \frac{\gamma}{2\pi\delta_{sc}^2} R_{sc} q_{sc}^2 \,. \tag{32}$$

Dropping $t$ and $y$-dependence terms in the eq. (31), one obtains

$$\frac{d^2A}{dX^2} + \frac{\lambda_2}{\lambda_1}\left(1 - \frac{\lambda_3}{\lambda_1}|A|^2\right)A = 0. \tag{33}$$

Therefore,

$$A(x) = A_0 \tanh\left(\frac{x}{\Lambda_0}\right), \tag{34}$$

where

$$A_0 = \left(\frac{\lambda_2}{\lambda_3}\right)^{\frac{1}{2}} \quad \text{and} \quad \Lambda_0 = \left(\frac{2\lambda_1}{\lambda_2}\right)^{\frac{1}{2}}.$$

### 4.1 Heat Transport by Convection

From equation (34), we obtain maximum of steady amplitude $A$ ($|A_{\max}|$) as

$$|A_{\max}| = \left(\frac{\epsilon^2\lambda_2}{\lambda_3}\right)^{\frac{1}{2}}. \tag{35}$$

We define the Nusselt number in terms of amplitude $A$ as

$$Nu = 1 + \frac{\epsilon^2}{\delta_{sc}^2}|A_{\max}|^2. \tag{36}$$

From eq. (36), we obtain convection for $R > R_{sc}$ and conduction for $R \le R_{sc}$. Eq. (31) is valid for $\lambda_3 > 0$ which is possible when $R > R_{sc}$, Thus we get

(i) convection for $Nu > 1$,

(ii) conduction for $Nu \le 1$ (see in Figure 1).

## 5. Results and Discussions

In the present analysis, the numerical results and conclusions are presented. The linear and nonlinear instability of Darcy-Benard setup saturated by a Maxwell fluid with rotation confined between two horizontal boundaries is studied. The linear instability threshold parameters consisting of the Rayleigh number $R$, Taylor number$Ta$, relaxation parameter $\lambda$ and Prandtl number $Pr$ are shown in Table 1-3.

**Table 1.** Critical Rayleigh number at the onset of stationary convection versus Taylor number

| $Ta$ | $R_c^{sc}$ | $Ta$ | $R_c^{sc}$ | $Ta$ | $R_c^{sc}$ |
|---|---|---|---|---|---|
| 400 | 1618520.3233 | 2400 | 58252032.2439 | 4400 | 195790561.1939 |
| 800 | 6472821.3451 | 2800 | 79287336.6715 | 4800 | 233006869.0274 |
| 1200 | 14563323.0480 | 3200 | 103558841.7804 | 5200 | 273459377.5421 |
| 1600 | 25890025.4321 | 3600 | 131066547.5704 | 5600 | 317148086.7380 |
| 2000 | 40452928.4974 | 4000 | 161810454.0415 | 6000 | 364072996.6150 |

**Table 2.** Critical Rayleigh number at the onset of oscillatory convection versus $Ta$ fixed at $\lambda = 0.8$

| | $Pe = 0.001$ | $Pe = 0.01$ | $Pe = 0.1$ |
|---|---|---|---|
| $Ta$ | $R_c^{oc}$ | $R_c^{oc}$ | $R_c^{oc}$ |
| 400 | 20213002.8335 | 1999590.0068 | 178835.4255 |
| 800 | 20212015.9025 | 1999403.4566 | 178917.9303 |
| 1200 | 20211054.0222 | 1999348.9338 | 179055.9854 |
| 1600 | 20210206.9685 | 1999337.3702 | 179248.6488 |
| 2000 | 20209476.0403 | 1999345.0151 | 179495.1632 |
| 2400 | 20208846.7657 | 1999363.9661 | 179794.6591 |
| 2800 | 20208303.9211 | 1999391.0889 | 180146.0610 |
| 3200 | 20207834.2074 | 1999424.9620 | 180548.1727 |
| 3600 | 20207426.4861 | 1999464.8709 | 180999.6829 |
| 4000 | 20207071.5092 | 1999510.4265 | 181499.0948 |

**Table 3.** Critical Rayleigh number at the onset of oscillatory convection versus $\lambda$ fixed at $Ta = 800$

| | $Pe = 0.001$ | $Pe = 0.01$ | $Pe = 0.1$ |
|---|---|---|---|
| $\lambda$ | $R_c^{oc}$ | $R_c^{oc}$ | $R_c^{oc}$ |
| 0.50 | 1190331.2286 | 116794.0248 | 13552.3204 |
| 0.55 | 2122849.7122 | 209444.1885 | 21095.4959 |
| 0.60 | 3588517.0008 | 354891.1087 | 33642.2843 |
| 0.65 | 5808631.4859 | 574942.4725 | 53115.7906 |
| 0.70 | 9066983.7370 | 897556.2319 | 81793.8314 |
| 0.75 | 13720716.0801 | 1357884.8303 | 122543.6830 |
| 0.80 | 20212015.9025 | 1999403.4566 | 178917.9303 |
| 0.85 | 29080674.0240 | 2875120.1685 | 255218.1061 |
| 0.90 | 40977506.7848 | 4048867.3903 | 356563.3972 |
| 0.95 | 56678623.8238 | 5596675.0544 | 488966.2117 |

The behavior of critical Rayleigh number at the onset of stationary convection $R_c^{sc}$ versus $Ta$ is displayed in Table 1. In this table, enhancing of $R_c^{sc}$ with the enhancement in the value of $Ta$ is observed. Hence $R_c^{sc}$ has a stabilizing effect on the system.

Table 2 shows that change of critical Rayleigh number at the onset of oscillatory convection $R_c^{oc}$ with $Ta$ for different values of $Pe$ and for fixed value of $\lambda = 0.8$. This table shows that, $Ta$ increases as $R_c^{oc}$ decreases at $Pe = 0.001$. Also, at $Pe = 0.01$, increasing of $Ta$ the value of $R_c^{oc}$ is monotonically decreasing as well as increasing at certain point, but it is different at $Pe = 0.1$, that is $Ta$ increases as $R_c^{oc}$ increases.

Table 3 depicts the variation of $R_c^{oc}$ versus $\lambda$ for different values of $Pe$ with other parameters held constant. In this table, the enhancement of $R_c^{oc}$ is observed with the increase in the value of $\lambda$ and $Pe$. As a result, $\lambda$ and $Pe$ have a stabilizing effect on the system.

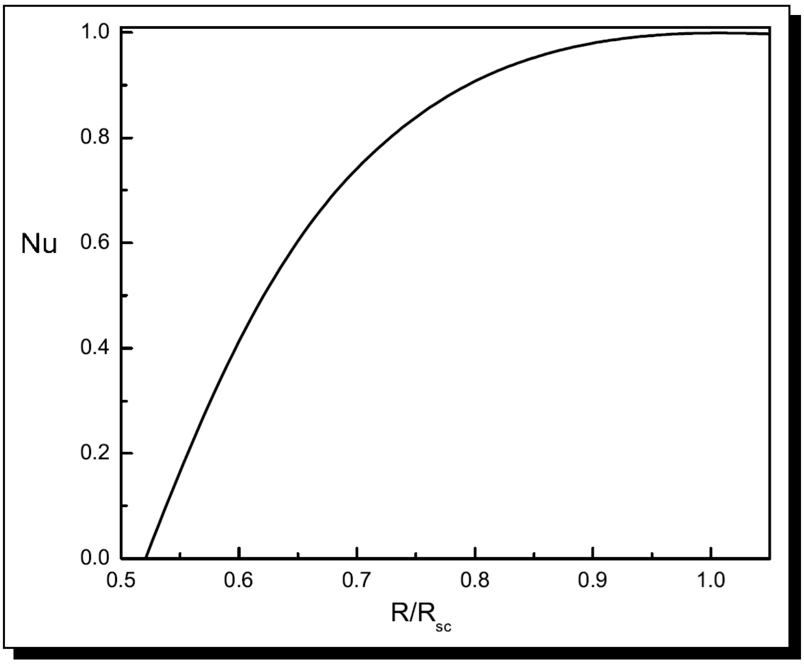


**Figure 1.** The figure is plotted for the fixed values of $Ta = 50$, $\lambda = 0.5$, $Pr = 5$

## 6. Conclusion

In this study, we have examined the linear and non-linear instability of Darcy-Benard setup saturated by a Maxwell fluid with rotation. The behaviour of various parameters like the Rayleigh number $R$, Taylor number $Ta$, relaxation parameter $\lambda$, wavenumber $q$, relaxation parameter $\gamma$ and Prandtl number $Pr$ have been analysed. The results can be summarized as follows:

- For oscillatory convection, the critical Rayleigh number increases with as increase in relaxation parameter by varing different Prandtl number, which is stabilizing factor to make the system more stable.
- For stationary convection, the critical thermal Rayleigh number increases with as increase in the Taylor number, which is stabilizing factor to make the system more stable.

## Nomenclature

| | |
|---|---|
| $\overline{V}$ | Fluid velocity |
| $u$, $v$, $w$ | velocity components |
| $\Omega$ | Angular Velocity |
| $\theta$ | Temperature |
| $t$ | Time |
| $P$ | Pressure |
| $g$ | acceleration due gravity |
| $k_T$ | Thermal diffusivity |
| $\kappa$ | Permeability |
| $d$ | Length |

*Dimensionless Parameters*

| | |
|---|---|
| $A$ | Complex Amplitude |
| $R$ | Rayleigh number |

| | | | |
|---|---|---|---|
| $Ta$ | Hartmann number | *Greek Symbols* | |
| $\lambda$ | relaxation parameter | $\alpha$ | Thermal expansion coefficient |
| $Pr$ | Prandtl number | $\epsilon$ | Porosity |
| $q$ | Wave number | $\rho$ | Fluid density |
| $Nu$ | Nusselt number | $\nu$ | Kinematic viscosity |

## Competing Interests

The authors declare that they have no competing interests.

## Authors' Contributions

All the authors contributed significantly in writing this article. The authors read and approved the final manuscript.

## References


[1] F. G. Awad, P. Sibanda and S. S. Motsa, On the linear stability analysis of a Maxwell fluid with double-diffusive convection, *Applied Mathematical Modelling* **34**(11) (2010), 3509 – 3517, DOI: 10.1016/j.apm.2010.02.038.

[2] A. B. Babu, G. S. K. Reddy and S. G. Tagare, Nonlinear magneto convection due to horizontal magnetic field and vertical axis of rotation due to thermal and compositional buoyancy, *Results in Physics* **12** (2019), 2078 – 2090, DOI: 10.1016/j.rinp.2019.02.022.

[3] A. B. Babu, G. S. K. Reddy and S. G. Tagare, Nonlinear magnetoconvection in a rotating fluid due to thermal and compositional buoyancy with anisotropic diffusivities, *Heat Transfer* **49**(1) (2020), 335 – 355, DOI: 10.1002/htj.21615.

[4] C. Beckermann and R. Viskanta, Double-diffusive convection during dendritic solidification of a binary mixture, *Physicochemical Hydrodynamics* **10**(2) (1988), 195 – 213.

[5] S. Chandrasekhar, *Hydrodynamic and Hydrodynamic Stability*, Clarendon Press, Oxford University, 652 pages (1961).

[6] S. R. Coriell, M. R. Cordes, W. J. Boettinger and R. F. Sekerka, Convective and interfacial instabilities during unidirectional solidification of a binary alloy, *Journal of Crystal Growth* **49**(1) (1980), 13 – 28, DOI: 10.1016/0022-0248(80)90056-1.

[7] R. V. Dharmadhikari and D. D. Kale, Flow of non-Newtonian fluids through porous media, *Chemical Engineering Science* **40**(3) (1985), 527 – 528, DOI: 10.1016/0009-2509(85)85113-7.

[8] S. Gaikwad and M. Dhanraj, Onset of double diffusive convection in a maxwell fluid saturated anisotropic porous layer with internal heat source, *Special Topics & Reviews in Porous Media: An International Journal* **4**(4) (2013), 359 – 374, DOI: 10.1615/SpecialTopicsRevPorousMedia.v4.i4.70.

[9] S. N. Gaikwad and S. Kouser, Double diffusive convection in a couple stress fluid saturated porous layer with internal heat source, *International Journal of Heat and Mass Transfer* **78** (2014), 1254 – 1264, DOI: 10.1016/j.ijheatmasstransfer.2014.07.021.

[10] A. Kumar and B. Bhadauria, Double diffusive convection in a porous layer saturated with viscoelastic fluid using a thermal non-equilibrium model, *Physics of Fluids* **23** (2011), 054101, DOI: 10.1063/1.3588836.

**[11]** M. S. Malashetty and M. Swamy, The onset of double diffusive convection in a viscoelastic fluid layer, *Journal of Non-Newtonian Fluid Mechanics* **165**(19-20) (2010), 1129 – 1138, DOI: 10.1016/j.jnnfm.2010.05.011.

**[12]** M. S. Malashetty, A. A. Hill and M. Swamy, Double diffusive convection in a viscoelastic fluid-saturated porous layer using a thermal non-equilibrium model, *Acta Mechanica* **223** (2012), 967 – 983, DOI: 10.1007/s00707-012-0616-1.

**[13]** D. A. Nield and A. Bejan, *Convection in Porous Media*, 4th edition, Springer, New York, xxvi + 778 pages (2013), DOI: 10.1007/978-1-4614-5541-7.

**[14]** P. Prescott and F. Incropera, Magnetically damped convection during solidification of a binary metal alloy, *ASME Journal of Heat and Mass Transfer* **115**(2) (1993), 302 – 310, DOI: 10.1115/1.2910680.

**[15]** G. S. K. Reddy and R. Ragoju, Thermal instability of a maxwell fluid saturated porous layer with chemical reaction, *Special Topics & Reviews in Porous Media: An International Journal* **13**(1) (2022), 33 – 47, DOI: 10.1615/SpecialTopicsRevPorousMedia.2021037410.

**[16]** G. S. K. Reddy and R. Ragoju, Thermal instability of a power-law fluid-saturated porous layer with an internal heat source and vertical throughflow, *Heat Transfer* **51**(2) (2022), 2181 – 2200, DOI: 10.1002/htj.22395.

**[17]** A. V. Shenoy, Non-Newtonian fluid heat transfer in porous media, *Advances in Heat Transfer* **24** (1994), 101 – 190, DOI: 10.1016/S0065-2717(08)70233-8.

**[18]** S. Wang and W. Tan, Stability analysis of soret-driven double-diffusive convection of Maxwell fluid in a porous medium, *International Journal of Heat and Fluid Flow* **32**(1) (2011), 88 – 94, DOI: 10.1016/j.ijheatfluidflow.2010.10.005.

**[19]** D. Yadav, M. Al-Siyabi, M.K. Awasthi, S. Al-Nadhairi, A. Al-Rahbi, M. Al-Subhi, R. Ragoju and K. Bhattacharyya, Chemical reaction and internal heating effects on the double diffusive convection in porous membrane enclosures soaked with maxwell fluid, *Membranes* **12**(3) (2022), 338, DOI: 10.3390/membranes12030338.

**[20]** H. Zhou and A. Zebib, Oscillatory double diffusive convection in crystal growth, *Journal of Crystal Growth* **135**(3-4) (1994), 587 – 593, DOI: 10.1016/0022-0248(94)90151-1.